P7.7 (Momentum integral estimates)

Air at *20*°C and 1 atm enters a 40-cm-square duct as in Fig. P7.7. Using the "displacement thickness" concept of Fig. 7.4, estimate (a) the mean velocity and (b) the mean pressure in the core of the flow at the position $x = 3$ m. (c) What is the average gradient, in Pa/m, in this section?

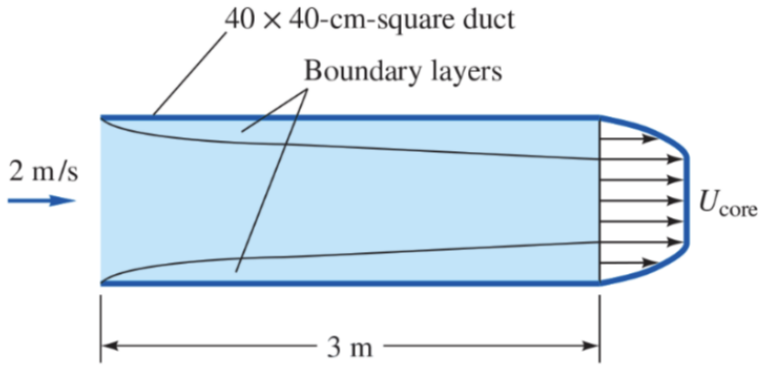


Fig. P7.7

P7.20 (Laminar flate-plate flow)

Air at *20*°C and 1 atm flows at 10 m/s past the flat plate in Fig. P7.20. A pitot stagnation tube, placed 2 mm from the wall, develops a manometer head $h = 5$ mm of Meriam red oil, SG = *0.827*. Use this information to estimate the downstream position x of the pitot tube. Assume laminar flow.

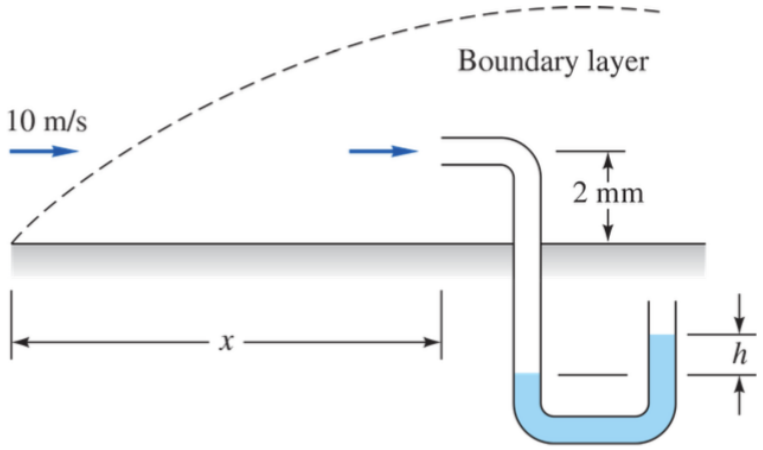


Fig. P7.20

P7.23 (Laminar flate-plate flow)

Suppose you buy a 4- by 8-ft sheet of plywood and put it on your roof rack. (See Fig. P7.23.) You drive home at 35 mi/h. (a) Assuming the board is perfectly aligned with the airflow, how thick is the boundary layer at the end of the board? (b) Estimate the drag on the sheet of plywood if the boundary layer remains laminar. ~~(c) Estimate the drag on the sheet of plywood if the boundary layer is turbulent (assume the wood is smooth), and compare the result to that of the laminar boundary layer case.~~

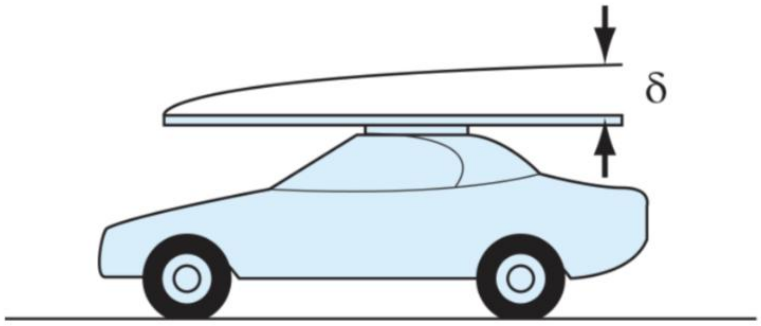


Fig. P7.23

P7.7

Solution: For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E}{-5}$ kg/m·s. Using laminar boundary-layer theory, compute the displacement thickness at x = 3 m:

$$\text{Re}_x = \frac{\rho U x}{\mu} = \frac{1.2(2)(3)}{1.8\text{E}{-5}} = 4\text{E}5 \text{ (laminar)}, \quad \delta^* = \frac{1.721x}{\text{Re}_x^{1/2}} = \frac{1.721(3)}{(4\text{E}5)^{1/2}} \approx 0.0082 \text{ m}$$

$$\text{Then, by continuity,} \quad V_{exit} = V\left(\frac{L_o}{L_o - 2\delta^*}\right)^2 = (2.0)\left(\frac{0.4}{0.4 - 0.0164}\right)^2$$

$$\approx \mathbf{2.175} \ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.} \text{ (a)}$$

The pressure change in the (frictionless) core flow is estimated from Bernoulli's equation:

$$p_{exit} + \frac{\rho}{2}V_{exit}^2 = p_o + \frac{\rho}{2}V_o^2, \quad \text{or:} \quad p_{exit} + \frac{1.2}{2}(2.175)^2 = 1 \text{ atm} + \frac{1.2}{2}(2.0)^2$$

$$\text{Solve for} \quad p|_{x=3m} = 1 \text{ atm} - 0.44 \text{ Pa} = \mathbf{-0.44\,Pa} \ (\textit{gage}) \quad \textit{Ans.} \text{ (b)}$$

The average pressure gradient is $\Delta p/x = (-0.44 \text{ Pa}/3.0 \text{ m}) \approx \mathbf{-0.15\ Pa/m}$ *Ans.* (c)

P7.20

Solution: For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E}{-5}$ kg/m·s. Assume constant stream pressure, then the manometer can be used to estimate the local velocity u at the position of the pitot inlet:

$$\Delta p_{\text{mano}} = p_0 - p_\infty = (\rho_{\text{oil}} - \rho_{\text{air}})gh_{\text{mano}} = [0.827(998) - 1.2](9.81)(0.005) \approx 40 \text{ Pa}$$

$$\text{Then} \quad u_{\text{pitot inlet}} \approx [2\Delta p/\rho]^{\frac{1}{2}} = [2(40)/1.2]^{\frac{1}{2}} \approx 8.2 \text{ m/s}$$

Now, with u known, assuming the B.L is laminar, and the Blasius solution uses u/U to determine the position η:

$$\frac{u}{U} = \frac{8.2}{10} = 0.821, \ \text{Table 7.1 read} \ \eta \approx 2.854 = y\left(\frac{U}{\nu x}\right)^{\frac{1}{2}}$$

$$\text{or:} \ x = \left(\frac{U}{\nu}\right)\left(\frac{y}{\eta}\right)^2 = \left(\frac{10}{1.5 \times 10^{-5}}\right)\left(\frac{0.002}{2.854}\right)^2 \approx \mathbf{0.327\ m} \quad \textit{Ans.}$$

Check $Re_x = (10)(0.327)/(1.5E{-}5) \approx 2.18E5 < 5E5$, OK, B.L is indeed laminar.

Table.7.1 The Blasius Velocity Profile [1 to 3]

$y[U/(vx)]^{1/2}$	u/U	$y[U/(vx)]^{1/2}$	u/U
0.0	0.0	2.8	0.81152
0.2	0.06641	3.0	0.84605
0.4	0.13277	3.2	0.87609
0.6	0.19894	3.4	0.90177
0.8	0.26471	3.6	0.92333
1.0	0.32979	3.8	0.94112
1.2	0.39378	4.0	0.95552
1.4	0.45627	4.2	0.96696
1.6	0.51676	4.4	0.97587
1.8	0.57477	4.6	0.98269
2.0	0.62977	4.8	0.98779
2.2	0.68132	5.0	0.99155
2.4	0.72899	∞	1.00000
2.6	0.77246		

P7.23

Solution: For air take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. Convert L = 8 ft = 2.44 m and U = 35 mi/h = 15.6 m/s. Evaluate the Reynolds number, is it laminar or turbulent?

$$\mathrm{Re}_L = \frac{\rho UL}{\mu} = \frac{1.2(15.6)(2.44)}{1.8E{-}5} = 2.55E6 \quad probably\ laminar + turbulent$$

(a) Evaluate the range of boundary-layer thickness between laminar and turbulent:

$$Laminar: \quad \frac{\delta}{L} = \frac{\delta}{2.44\ m} \approx \frac{5.0}{\sqrt{2.55E6}} = 0.00313, \quad or: \quad \delta \approx 0.00765\ m = \mathbf{0.30\ in}$$

$$Turbulent: \quad \frac{\delta}{2.44} \approx \frac{0.16}{(2.55E6)^{1/7}} = 0.0195, \quad or: \quad \delta \approx 0.047\ m = \mathbf{1.9\ in} \quad Ans.\ (a)$$

(b, c) Evaluate the range of boundary-layer drag for both laminar and turbulent flow. Note that, for flow over both sides, the appropriate area A = 2bL:

$$F_{lam} = C_D \frac{\rho}{2} U^2 A \approx \left(\frac{1.328}{\sqrt{2.55E6}} \right) \frac{1.2}{2} (15.6)^2 (2.44 \times 1.22 \times 2 \ sides) = \mathbf{0.73\ N} \quad \textit{Ans.}\ \text{(b)}$$

$$F_{turb} \approx \left(\frac{0.031}{(2.55E6)^{1/7}} \right) \frac{1.2}{2} (15.6)^2 (2.44 \times 1.22 \times 2 \ sides) = \mathbf{3.3\ N} \quad \textit{Ans.}\ \text{(c)}$$

We see that the turbulent drag is about 4 times larger than laminar drag.