

Name : _____

Quiz: No. 6

Time: 15 minutes

Student ID# : _____

Course: ME 5160, Fall 2026

The exam is closed book and closed notes.

A large tank of liquid under pressure is drained through a smoothly contoured nozzle of area A . The mass flow rate $\dot{m}$ is thought to depend on the nozzle area A , the liquid density ρ , the difference in height between the liquid surface and the nozzle h , the change in pressure Δp , and the gravitational acceleration g . Taking ρ , A , and g as repeating variables, find an expression for the mass flow rate $\dot{m}$ as a function of the other parameters in the problem in terms of dimensionless Pi groups.

Quantity	Symbol	Dimensions	
		$MLT\Theta$	$FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ, τ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω, Ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Thermal expansion coefficient	β	Θ^{-1}	Θ^{-1}

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Solution:

Assumptions: the problem is only a function of the given dimensional variables.

$$\dot{m} = f(A, \rho, h, \Delta p, g)$$

$$n = 6$$

$$\dot{m} = \{MT^{-1}\}; \quad A = \{L^2\}; \quad \rho = \{ML^{-3}\}; \quad h = \{L\};$$

$$\Delta p = \{ML^{-1}T^{-2}\}; \quad g = \{LT^{-2}\} \quad (3)$$

$$j = 3 \rightarrow k = n - j = 3 \quad (1)$$

The repeating variables are ρ , A , and g ; adding each remaining variable in turn, we find the Pi groups:

$$\Pi_1 = \rho^a A^b g^c \dot{m} = \{(ML^{-3})^a (L^2)^b (LT^{-2})^c (MT^{-1})\} = \{M^0 L^0 T^0 \theta^0\}$$

$$a = -1; \quad b = -5/4; \quad c = -1/2$$

$$\Pi_1 = \frac{\dot{m}}{\rho A^{5/4} g^{1/2}} \quad (2)$$

$$\Pi_2 = \rho^a A^b g^c h = \{(ML^{-3})^a (L^2)^b (LT^{-2})^c (L)\} = \{M^0 L^0 T^0 \theta^0\}$$

$$a = 0; \quad b = -1/2; \quad c = 0$$

$$\Pi_2 = \frac{h}{A^{1/2}} \quad (2)$$

$$-3a + 2b + c - 1$$

$$\Pi_3 = \rho^a A^b g^c \Delta p = \{(ML^{-3})^a (L^2)^b (LT^{-2})^c (ML^{-1}T^{-2})\} = \{M^0 L^0 T^0 \theta^0\}$$

$$a = -1; \quad b = -1/2; \quad c = -1$$

$$\Pi_3 = \frac{\Delta p}{\rho A^{1/2} g} \quad (2)$$

Thus the arrangement of the dimensionless variables is:

$$\frac{\dot{m}}{\rho A^{5/4} g^{1/2}} = f\left(\frac{h}{A^{1/2}}, \frac{\Delta p}{\rho A^{1/2} g}\right)$$