

Name : _____

Quiz: No. 5

Time: 15 minutes

Student ID# : _____

Course: ME 5160, Fall 2026

The exam is closed book and closed notes.

Consider steady, two-dimensional, incompressible viscous flow of a Newtonian fluid between parallel plates a distance h apart, as shown in the Figure below. The upper plate moves at constant velocity U while the lower plate is at rest. The pressure varies linearly in the x direction: $p = p(x) = Cx$. If the plates are very wide and very long and the flow is fully developed, neglect gravity and find the velocity distribution between the plates.

Incompressible Continuity Equation:

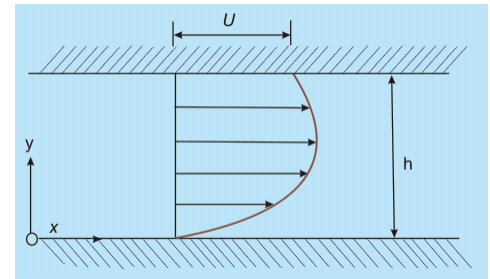
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Incompressible Navier-Stokes Equations in Cartesian Coordinates:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right)$$

$$\rho g_y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) = \rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right)$$

$$\rho g_z - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) = \rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right)$$



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Solution:KNOWN: $p(x)$ FIND: $u(y)$

ASSUMPTIONS: incompressible, steady, Newtonian flow, no gravity effects

ANALYSIS:

The plate is very wide and very long and the flow is fully developed, therefore the flow is essentially axial: $v = w = 0$.

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\frac{\partial u}{\partial x} + \underbrace{0}_{v=0} + \underbrace{0}_{w=0} = 0 \quad (1)$$

$$\Rightarrow u = u(y) \text{ only} \quad (2)$$

 x -momentum:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) \quad (1)$$

$$\underbrace{0}_{\text{no gravity}} - \frac{\partial p}{\partial x} + \mu \left(\underbrace{0}_{u=u(y)} + \frac{\partial^2 u}{\partial y^2} + \underbrace{0}_{u=u(y)} \right) = \rho \left(\underbrace{0}_{\text{steady}} + \underbrace{0}_{u=u(y)} + \underbrace{0}_{v=0} + \underbrace{0}_{w=0} \right) \quad (1)$$

$$\frac{\partial p}{\partial x} = C \quad (1)$$

$$\Rightarrow \mu \frac{d^2 u}{dy^2} = C$$

Integrate twice:

$$u = \frac{1}{\mu} C \frac{y^2}{2} + C_1 y + C_2 \quad (1)$$

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Boundary conditions:

$$\text{at } y = 0: u(y) = 0 \rightarrow C_2 = 0 \quad (0.5)$$

$$\text{at } y = h: u(y) = U \rightarrow C_1 = \frac{U}{h} - \frac{Ch}{2\mu} \quad (0.5)$$

Replace and find:

$$u = \frac{1}{\mu} C \frac{y^2}{2} + \left(\frac{U}{h} - \frac{Ch}{2\mu} \right) y \quad (1)$$