

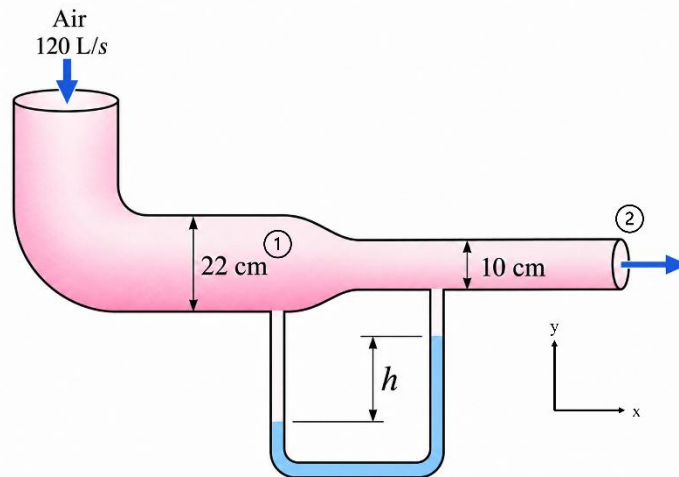
1. In the figure below, air enters vertically at section 1, turns 90°, and exits horizontally to the atmosphere at section 2. The flow rate is 120 L/s, and a water manometer measures the pressure difference between the two horizontal sections.

$\rho_{water} = 1000 \text{ kg/m}^3$, $\rho_{air} = 1.20 \text{ kg/m}^3$, and $g = 9.81 \text{ m/s}^2$. The loss coefficient is $K_c = 0.20$, and the energy correction factor is $\alpha = 1$. Assume uniform velocity at the inlet and outlet. Neglect bend losses and gravitational effects on the airflow. The outlet pressure is atmospheric. (a) the outlet velocity, V_2 . (b) the pressure difference, $p_1 - p_2$. (c) the difference in the water levels, h . Assume negligible pressure loss between the downstream manometer tap and section 2 ($p_{tap} = p_2$). (d) the horizontal and vertical components of the force exerted by the air on the pipe.

Continuity equation: $-\frac{d}{dt} \int_{cv} \rho \, dV = \int_{cs} \rho (\mathbf{V} \cdot \mathbf{n}) \, dA$

Momentum equation: $\Sigma \mathbf{F} = \frac{d}{dt} \int_{cv} \rho \mathbf{V} \, dV + \int_{cs} \rho \mathbf{V} (\mathbf{V} \cdot \mathbf{n}) \, dA$

$$\left(\frac{p}{\rho g} + \frac{\alpha V^2}{2g} + z \right)_1 = \left(\frac{p}{\rho g} + \frac{\alpha V^2}{2g} + z \right)_2 + h_{loss}, \quad h_{loss} = K_c \frac{V_2^2}{2g}$$



Solution:

a) Outlet velocity

From $Q = AV$,

$$V_1 = \frac{0.120}{\pi(0.22)^2/4} = 3.16 \text{ m/s}, \quad \boxed{V_2 = \frac{0.120}{\pi(0.10)^2/4} = 15.28 \text{ m/s}}. \quad +3$$

b) Pressure difference

With $h_L = K_c V_2^2 / (2g)$ and $\alpha = 1$, the energy equation gives

$$\frac{p_1}{\rho_{\text{air}} g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho_{\text{air}} g} + \frac{V_2^2}{2g} + K_c \frac{V_2^2}{2g}.$$

Thus, for $K_c = 0.20$,

$$p_1 - p_2 = \frac{\rho_{\text{air}}}{2} [(1 + K_c)V_2^2 - V_1^2].$$

$$p_1 - p_2 = \frac{1.20}{2} [1.20(15.28)^2 - (3.16)^2] = \boxed{162 \text{ Pa}}. \quad +1$$

Since $p_2 = 0$ gauge, $p_1 = 162 \text{ Pa}$ gauge.

c) Manometer reading

Considering air hydrostatic effects, the water manometer gives

$$p_1 - p_2 = (\rho_{\text{water}} - \rho_{\text{air}})gh. \quad +2$$

Therefore,

$$\boxed{h = \frac{162.1}{(1000 - 1.20)(9.81)} = 0.0165 \text{ m} = 16.5 \text{ mm}}. \quad +1$$

d) Force exerted by the air on the pipe

For the entire pipe, from vertical inlet 0 to outlet 2, bend losses and air weight are neglected, so $V_0 = V_1$ and $p_0 = p_1 = 162 \text{ Pa}$ gauge. Also, $p_2 = 0$ gauge.

$$\dot{m} = \rho_{\text{air}} Q = (1.20)(0.120) = 0.144 \text{ kg/s}, \quad A_0 = \frac{\pi(0.22)^2}{4} = 0.0380 \text{ m}^2.$$

Let (R_x, R_y) be the force of the pipe on the air. Applying momentum equation in the directions shown,

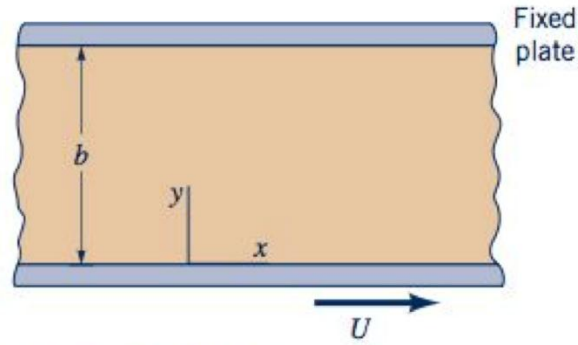
$$R_x = \dot{m} V_2 = (0.144)(15.28) = 2.20 \text{ N}, \quad +2$$

$$R_y - p_0 A_0 = \dot{m} V_1, \quad R_y = (0.144)(3.16) + (162.1)(0.0380) = 6.62 \text{ N}. \quad +1$$

The force of the air on the pipe (left and down) is $(-R_x, -R_y)$:

$$\boxed{F_x = -2.20 \text{ N}, \quad F_y = -6.62 \text{ N}}.$$

2. The viscous, incompressible flow between the parallel plates shown in Figure is caused by both the motion of the bottom plate and a constant pressure gradient $\frac{\partial p}{\partial x}$. Assuming steady, 2D, and parallel flow and using differential analysis: (a) Show that the flow is fully developed using continuity equation; (b) Find the velocity profile $u(y)$ using Navier-Stokes equations with appropriate boundary conditions; (c) Find wall shear stress at bottom wall; **Explicitly state all assumptions.**



$$\rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \rho \left(u \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right)$$

$$\rho g_y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) = \rho \left(v \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right)$$

$$\rho g_z - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) = \rho \left(w \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right)$$

Solution 2:

KNOWN: *Flow condition, Boundary condition*

FIND: **+1**

- Show that the flow is fully developed
- velocity profile $u(y)$
- Find wall shear stress at bottom wall
- Find the flow rate

ASSUMPTIONS:

- steady: $\frac{\partial}{\partial t} = 0$
- 2D flow: $w = 0$; $\frac{\partial}{\partial z} = 0$
- Parallel flow: $v = w = 0$
- $\frac{\partial p}{\partial x} = \text{constant}$

ANALYSIS:

(a) Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u}{\partial x} + 0 + 0 = 0 \rightarrow \frac{\partial u}{\partial x} = 0 \rightarrow \text{fully developed } \mathbf{+2}$$

(b) x -momentum equation

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{\partial p}{\partial x} \quad \mathbf{+2}$$

Integrate twice

$$u(y) = \frac{1}{2\mu} \frac{\partial p}{\partial x} y^2 + c_1 y + c_2 \quad \mathbf{+2}$$

Apply boundary conditions

$$y = 0 \rightarrow u = U \rightarrow c_2 = U$$

$$y = b \rightarrow u = 0 \rightarrow 0 = \frac{1}{2\mu} \left(\frac{\partial p}{\partial x} \right) b^2 + c_1 b + U \rightarrow c_1 = -\frac{1}{2\mu} \left(\frac{\partial p}{\partial x} \right) b - \frac{U}{b} \quad \mathbf{+2}$$

Therefore,

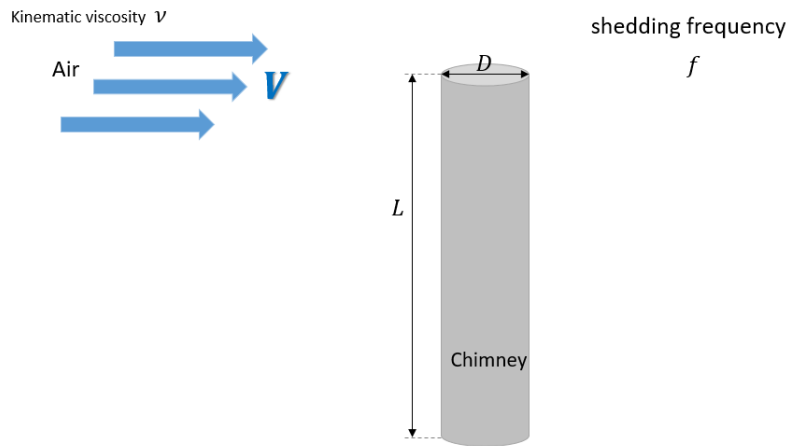
$$u(y) = \frac{1}{2\mu} \left(\frac{\partial p}{\partial x} \right) (y^2 - by) + U \left(1 - \frac{y}{b} \right)$$

(c) Shear stress at bottom wall

$$\tau = \mu \frac{du}{dy} = \frac{1}{2} \left(\frac{\partial p}{\partial x} \right) (2y - b) - \frac{U}{b} \mu$$

$$\tau_{wall} = \tau(0) = -\frac{b}{2} \left(\frac{\partial p}{\partial x} \right) - \frac{U}{b} \mu \quad \textcolor{red}{+1}$$

3. As wind blows over a chimney, vortices are shedding in the wake as shown in the Figure below. The dimensional shedding frequency f depends on chimney diameter D , chimney length L , wind velocity V , and air kinematic viscosity ν . (a) Find dimensionless f which depends on dimensionless groups.



Quantity	Symbol	Dimensions	
		$MLT\Theta$	$FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ, τ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω, Ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Thermal expansion coefficient	β	Θ^{-1}	Θ^{-1}

Solution 3:

ASSUMPTIONS: the problem is only a function of the above dimensional variables

ANALYSIS:

(a)

$$f = fcn(D, L, V, \nu) ; \quad n = 5$$

$$f = \{T^{-1}\} \quad D = \{L\} \quad L = \{L\} \quad V = \{LT^{-1}\} \quad \nu = \{L^2T^{-1}\} ; \quad j = 2$$

$$\therefore k = n - j = 3 \quad \textcolor{red}{+2.5}$$

repeating variables = D, V

$$\pi_1 = fD^{a_1}V^{b_1} = \{(T^{-1})(L)^{a_1}(LT^{-1})^{b_1}\} = \{L^0T^0\}$$

$$a_1 = 1; \quad b_1 = -1$$

$$\pi_1 = \frac{fD}{V} \quad \textcolor{red}{+2.5}$$

$$\pi_2 = LD^{a_2}V^{b_2} = \{(L)(L)^{a_2}(LT^{-1})^{b_2}\} = \{L^0T^0\}$$

$$a_2 = -1; \quad b_2 = 0$$

$$\pi_2 = \frac{L}{D} \quad \textcolor{red}{+2.5}$$

$$\pi_3 = \nu D^{a_3}V^{b_3} = \{(L^2T^{-1})(L)^{a_3}(LT^{-1})^{b_3}\} = \{L^0T^0\}$$

$$a_3 = -1; \quad b_3 = -1$$

$$\pi_3 = \frac{\nu}{VD} \quad \textcolor{red}{+2.5}$$

$$\frac{fD}{V} = fcn\left(\frac{L}{D}, \frac{\nu}{VD}\right)$$