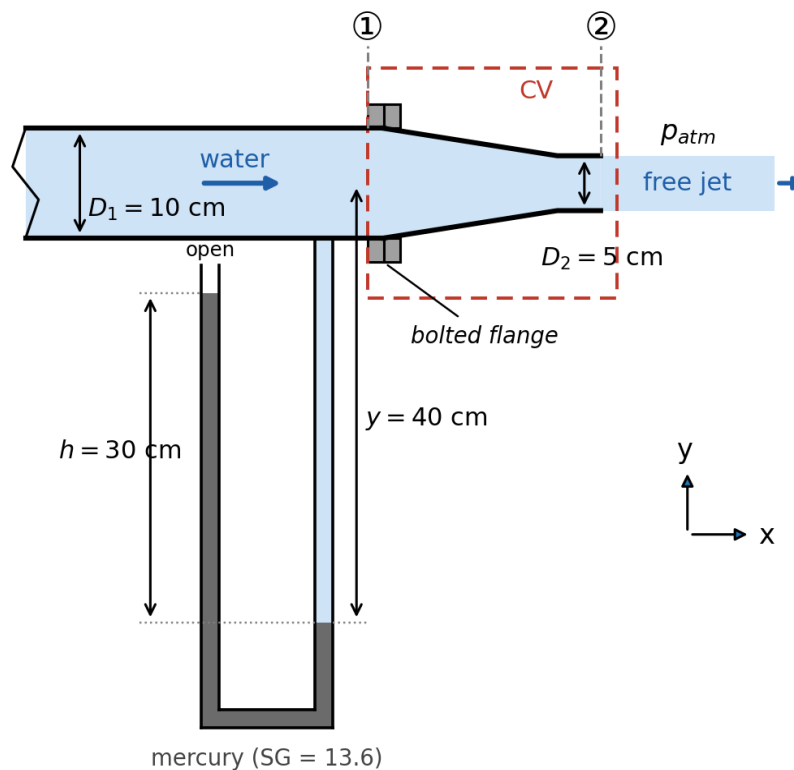


The exam is closed book and closed notes.

Water ($\rho = 998 \text{ kg/m}^3$) flows steadily through a horizontal nozzle bolted to the end of a pipe and discharges as a free jet into the atmosphere, as shown in the figure. The inlet of the nozzle (section ①) has diameter $D_1 = 10 \text{ cm}$ and the exit (section ②) has diameter $D_2 = 5 \text{ cm}$. A mercury U-tube manometer (SG = 13.6) is connected to a pressure tap just upstream of the flange; its other leg is open to the atmosphere. On the pipe side, water fills the manometer tube from the pipe centerline down a distance $y = 40 \text{ cm}$ to the mercury interface, and the mercury in the open leg stands $h = 30 \text{ cm}$ above that interface. (a) Determine the gauge pressure at section ①. (b) Using continuity and the Bernoulli equation, find the velocities V_1 and V_2 . (c) What is the mass flow rate? (d) What horizontal force F_x must the flange bolts exert to hold the nozzle in place? Give its direction. (e) Explain why the pressure at the exit is taken as atmospheric, and why no vertical force is required to hold the nozzle (other than supporting its weight). Neglect friction and the weight of the water in the nozzle. Use $g = 9.81 \text{ m/s}^2$.



Continuity equation: $-\frac{d}{dt} \int_{cv} \rho dV = \int_{cs} \rho (\mathbf{V} \cdot \mathbf{n}) dA$

Momentum equation: $\Sigma \mathbf{F} = \frac{d}{dt} \int_{cv} \rho \mathbf{V} dV + \int_{cs} \rho \mathbf{V} (\mathbf{V} \cdot \mathbf{n}) dA$

Hint: Choose a control volume that surrounds the nozzle and cuts through the flange at ① and through the jet at ②. Work in gauge pressure.

Assumption:

Steady, incompressible, frictionless flow; uniform velocity at ① and ②; non-deforming control volume with one inlet and one outlet; $z_1 = z_2$; the tap pressure equals p_1 (same diameter, no losses between them).

Solution:

(a) Manometer. Start at the pipe centerline and move through the manometer to the open end:

$$p_1 + \gamma_w y - \gamma_{Hg} h = p_{atm}$$

$$p_{1,gage} = \gamma_{Hg} h - \gamma_w y = (13.6)(9790)(0.30) - (9790)(0.40)$$

$$p_{1,gage} = 39,945 - 3,916 = 36,030 \text{ Pa} \approx 36.0 \text{ kPa} \quad +2$$

(b) Continuity. For steady incompressible flow with one inlet and one outlet,

$$\rho A_1 V_1 = \rho A_2 V_2$$

$$A_1 = \pi(0.10)^2/4 = 7.854 \times 10^{-3} \text{ m}^2, \quad A_2 = \pi(0.05)^2/4 = 1.963 \times 10^{-3} \text{ m}^2$$

$$V_1 = V_2(D_2/D_1)^2 = V_2/4 \quad +2$$

Bernoulli from ① to ② along the centerline streamline, with $p_2 = p_{atm}$ (0 gage) and $z_1 = z_2$:

$$p_1/\rho + V_1^2/2 = p_2/\rho + V_2^2/2$$

$$V_2 = \sqrt{\{ 2p_1 / (\rho(1 - (D_2/D_1)^4)) \}} = \sqrt{\{ 2(36,030) / (998(1 - 1/16)) \}} = 8.78 \text{ m/s} \quad +2$$

$$V_1 = 8.78/4 = 2.19 \text{ m/s} \quad +1$$

(c) Mass flow rate:

$$\dot{m} = \rho A_2 V_2 = (998 \text{ kg/m}^3)(1.963 \times 10^{-3} \text{ m}^2)(8.78 \text{ m/s}) = 17.2 \text{ kg/s} \quad +1$$

(d) Apply the x-momentum integral relation over the control volume. With only one-dimensional inlets and outlets and steady flow,

$$\Sigma F_x = \Sigma(\dot{m}_i V_i)_{out} - \Sigma(\dot{m}_i V_i)_{in}$$

The x-forces on the CV are the gage pressure force at ①, $p_1 A_1$ (acting in +x), and the bolt force R_x . At ② the jet is at atmospheric pressure, so its gage pressure force is zero.

$$p_1 A_1 + R_x = \dot{m}(V_2 - V_1) \quad +2$$

$$R_x = \dot{m}(V_2 - V_1) - p_1 A_1 = (17.2)(8.78 - 2.19) - (36,030)(7.854 \times 10^{-3})$$

$$R_x = 113.2 - 283.0 \approx -170 \text{ N} \quad +2$$

The negative sign means the bolts must pull the nozzle upstream (−x) with about 170 N.

Equivalently, the water pushes the nozzle downstream with 170 N, which is why a nozzle tends to blow off the end of a hose.

(e) A subsonic free jet cannot sustain a pressure difference across its boundary, so the jet leaves at the surrounding pressure: $p_2 = p_{atm} = 0$ gage. $+1$

Both the inflow and outflow momentum fluxes are purely horizontal, and the nozzle is axisymmetric about the x-axis, so the wall pressure forces cancel in the y-direction and there is no

Name:.....

Quiz: No. 3

Time: 20 minutes

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net change in y-momentum. With the water weight neglected, $F_y = 0$. **+1**