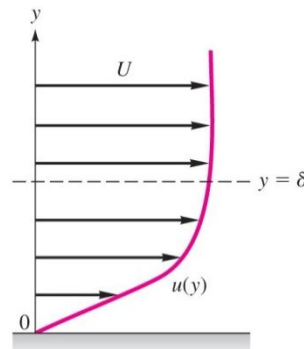

The exam is closed book and closed notes.

An approximation for the boundary-layer shape is the formula:

$$u(y) \approx U \sin\left(\frac{\pi y}{2\delta}\right), \quad 0 \leq y \leq \delta$$

Where U is the stream velocity far from the wall and δ is the boundary layer thickness, as in the Figure below. If the fluid is helium at 20°C and 1 atm ($\rho = 0.1664 \text{ kg/m}^3$; $\mu = 1.97\text{E-}5 \text{ kg/m-s}$), and if $U = 10.8 \text{ m/s}$ and $\delta = 3 \text{ cm}$, use the formula to (a) estimate the wall shear stress τ_w in Pa, and (b) find the position in the boundary layer where τ is one-half of τ_w .

Equation: $\tau = \mu \frac{du}{dy}$



Solution:

(a)

$$\tau_w = \mu \frac{\partial u}{\partial y} \Big|_{y=0} = \mu \left(U \frac{\pi}{2\delta} \cos \frac{\pi y}{2\delta} \right)_{y=0} = \frac{\pi \mu U}{2\delta} \quad +4$$

$$\text{Numerical values : } \tau_w = \frac{\pi (1.97 \text{ E} - 5 \text{ kg / m} - \text{s}) (10.8 \text{ m / s})}{2 (0.03 \text{ m})} = 0.011 \text{ Pa} \quad +3$$

(b)

The variation of shear stress across the boundary layer is a cosine wave, $\tau = \mu (du/dy)$:

$$\tau(y) = \frac{\pi \mu U}{2\delta} \cos\left(\frac{\pi y}{2\delta}\right) = \tau_w \cos\left(\frac{\pi y}{2\delta}\right) = \frac{\tau_w}{2} \text{ when } \frac{\pi y}{2\delta} = \frac{\pi}{3}, \text{ or : } y = \frac{2\delta}{3} = 0.02 \text{ m} \quad +3$$