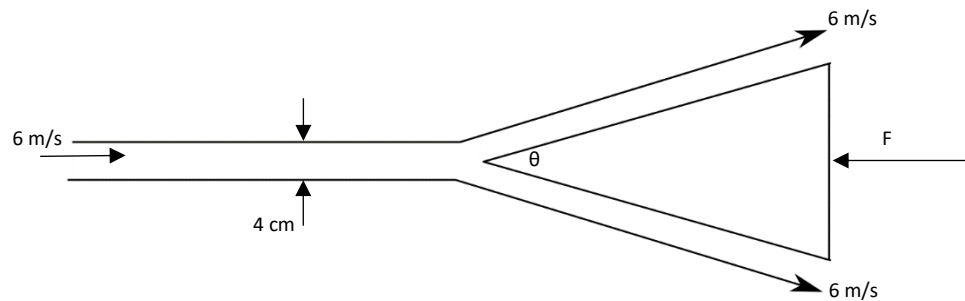


- 1) A wedge splits a sheet of water, as shown in the figure. Both wedge and sheet are very long into the paper. The angle of the wedge θ is 48° . (a) What is the thickness of the water sheets at each side of the wedge? (b) What is the mass flow rate per unit depth? (c) What is the force F_x (per meter of depth into the paper) required to hold the wedge stationary? (d) Explain why F_y is equal to zero. (Use water density $\rho = 998 \text{ kg/m}^3$)



Continuity equation: $-\frac{d}{dt} \int_{CV} \rho dV = \int_{CS} \rho \underline{V}_R \cdot \underline{n} dA$

Momentum equation: $\sum F = \frac{d}{dt} \int_{CV} \rho \underline{V} dV + \int_{CS} \rho \underline{V} \underline{V}_R \cdot \underline{n} dA$

Hint: 1) the force required to hold the wedge should be in the x-direction.

2) $\cos(24 \text{ deg}) = 0.9135$

Assumption: non-deforming CV, steady uniform flow with one inlet and two outlets.

Solution:

(a) Apply continuity to find thickness of exit sheets t_{exit} (D is the depth into the paper)

If the control volume has only a number of one-dimensional inlets and outlets, we can rewrite the continuity equation algebraically.

$$\int_{CV} \frac{\partial \rho}{\partial t} dV + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0$$

From steady flow assumption, $\int_{CV} \frac{\partial \rho}{\partial t} dV = 0$,

$$\sum_i (\rho_i A_i V_i)_{in} = \sum_i (\rho_i A_i V_i)_{out}$$

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 + \rho_3 A_3 V_3$, where 1 is inlet, 2,3 are outlets. Since $\rho_1 = \rho_2 = \rho_3 = \rho_{\text{water}}$,

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

$$\therefore V_1 = V_2 = V_3 = V, \quad A_2 = A_3 = A \quad +3$$

$$A_1 V = 2AV \quad (V = 6 \text{ m/s})$$

$$A_1 = 2A, \quad Dt_1 = 2Dt$$

$$t = \frac{t_1}{2} = \frac{4 \text{ cm}}{2 \text{ cm}} = 0.02 \text{ m} \quad +2$$

(b) The mass flow per unit depth is

$$\frac{\dot{m}}{D} = \rho V t_{in} = \left(998 \frac{\text{kg}}{\text{m}^3}\right) \left(6 \frac{\text{m}}{\text{s}}\right) (0.04 \text{ m}) = 239.5 \frac{\text{kg}}{\text{m s}} \quad +2$$

(c) Apply x-momentum integral relation over a control volume surrounding the wedge:

if the control volume has only one-dimensional inlets and outlets,

$$\sum F = \frac{d}{dt} \left(\int_{\text{CV}} V \rho d\mathcal{V} \right) + \sum (\dot{m}_i V_i)_{\text{out}} - \sum (\dot{m}_i V_i)_{\text{in}}$$

For the x-direction, assuming steady flow,

$$\sum F_x = + \sum (\dot{m}_i V_i)_{\text{out}} - \sum (\dot{m}_i V_i)_{\text{in}}$$

$$\sum F_x = -F = \dot{m}_2 V_2 + \dot{m}_3 V_3 - \dot{m}_1 V_1 \text{ (per unit depth)}$$

$$\therefore \dot{m}_2 = \dot{m}_3 = \frac{\dot{m}}{2},$$

$$-F = \frac{\dot{m}}{2} V + \frac{\dot{m}}{2} V - \dot{m} V = \dot{m} V \cos \frac{\theta}{2} - \dot{m} V = \dot{m} V \left(\cos \frac{\theta}{2} - 1 \right) \quad +2$$

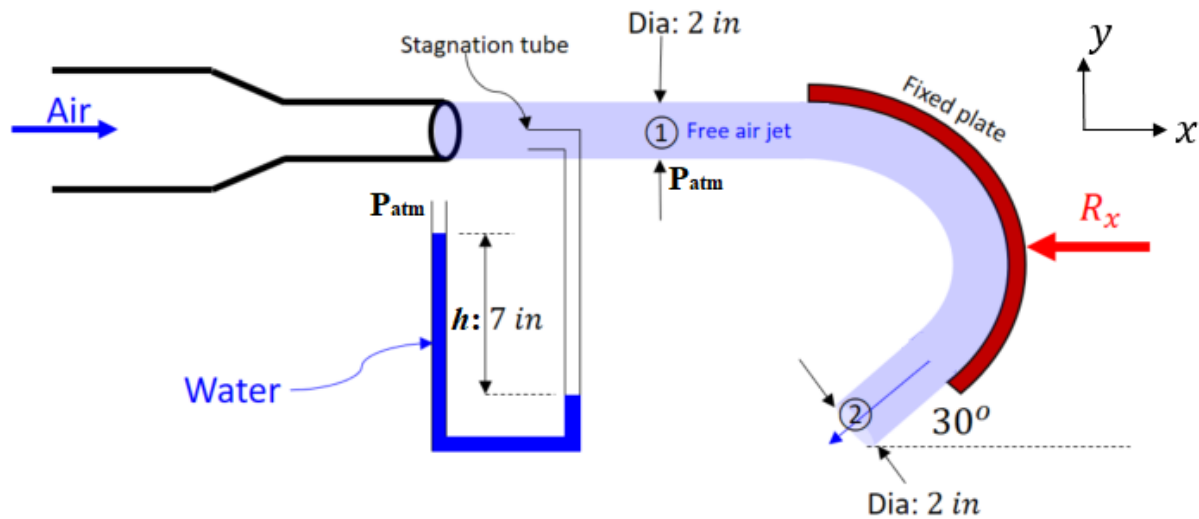
Substitute the values:

$$F = - \left(239.5 \frac{\text{kg}}{\text{m s}} \right) \left(6 \frac{\text{m}}{\text{s}} \right) (0.9135 - 1) = 124 \text{ N (per unit depth)} \quad +0.5$$

(d) The flow is symmetric with respect to the x-axis since the wedge forms an isosceles triangle and the sheets have the same thickness t_{exit} . **+0.5**

2) Air discharge from a 2 in diameter nozzle and strikes a curved vane, which is in a vertical plane as shown in below figure. A stagnation tube connected to a water U-tube manometer is located in the free air jet. Determine the horizontal component of the force that the air jet exerts on the vane. Neglect the weight of the air and all friction.

$\gamma_{\text{water}}: 62.4 \text{ lb/ft}^3$



Solution:

KNOWN: Inlet and outlet diameter, Outlet flow angle, manometer height.

FIND:

- Force R_x that requires to hold the plate stationary

ASSUMPTIONS:

Neglect weight of air

Neglect friction loss

The momentum equation in the x-direction. $\sum F_x = + \sum (\dot{m}_i V_i)_{\text{out}} - \sum (\dot{m}_i V_i)_{\text{in}}$

$$\begin{aligned} -R_x &= \dot{m}V_2 - \dot{m}V_1 \\ R_x &= \dot{m}V_1 - \dot{m}V_2 \end{aligned} \quad (1)$$

Diameter of section ① and ② is same

$$\therefore |V_1| = |V_2| \quad (1)$$

V_2 Direction is negative x and has $\cos(30^\circ)$

$$\therefore R_x = \dot{m}V_1 - \dot{m}(-V_1 \cos 30^\circ)$$

$$R_x = \dot{m}V_1(1 + \cos 30^\circ) \quad (2)$$

Calculate V_1 using Bernoulli equation (Apply stagnation point and section ①)

$$\frac{P_{stag}}{\rho_{air}} + \frac{V_{stag}^2}{2} + gz_{stag} = \frac{P_1}{\rho_{air}} + \frac{V_1^2}{2} + gz_1$$

$$\because z_{stag} = z_1,$$

$$\frac{P_{stag}}{\rho_{air}} + \frac{V_{stag}^2}{2} = \frac{P_1}{\rho_{air}} + \frac{V_1^2}{2}$$

$$\frac{P_{stag}}{\rho_{air}} + \frac{\cancel{V_{stag}^2}}{2} = \frac{P_1}{\rho_{air}} + \frac{V_1^2}{2}$$

$$\frac{P_{stag}}{\rho_{air}} = \frac{P_1}{\rho_{air}} + \frac{V_1^2}{2} \quad (1) \quad (2)$$

If H is the vertical distance between the stagnation point S and the air–water interface,

From the manometer,

$$P_{stag} + \gamma_{air}H - \gamma_{water}h = P_{atm}$$

Neglect weight of air

$$P_{stag} + \cancel{\gamma_{air}H} - \gamma_{water}h = P_{atm}$$

$$P_{stag} = P_{atm} + h\gamma_{water} \quad (2)$$

Combine Eq. (1) and Eq. (2) and $P_1 = P_{atm}$,

$$\frac{P_{atm} + h\gamma_{water}}{\rho_{air}} = \frac{P_{atm}}{\rho_{air}} + \frac{V_1^2}{2}$$

$$\therefore \frac{V_1^2}{2} = \frac{h\gamma_{water}}{\rho_{air}}, \quad V_1 = \sqrt{2h \frac{\gamma_{water}}{\rho_{air}}} \quad (2)$$

Plug in V_1 to the R_x

$$\therefore \dot{m} = \rho_{air} \frac{\pi D^2}{4} V_1,$$

$$R_x = \dot{m} V_1 (1 + \cos 30^\circ) = \rho_{air} \frac{\pi D^2}{4} V_1^2 (1 + \cos 30^\circ)$$

$$R_x = \rho_{air} \frac{\pi D^2}{4} 2h \frac{\gamma_{water}}{\rho_{air}} (1 + \cos 30^\circ)$$

$$R_x = \frac{\pi h D^2}{2} \gamma_{water} (1 + \cos 30^\circ)$$

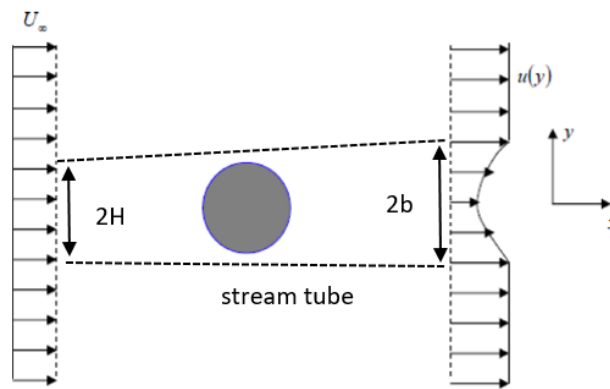
$$R_x = \frac{\pi(7/12)}{2} \left(\frac{2}{12}\right)^2 62.4 (1 + \cos 30^\circ)$$

$$R_x = 2.963 \text{ lb} \quad (2)$$

3) Consider an experiment in which the drag on a two-dimensional body immersed in a steady incompressible flow can be determined from measurement of velocity distribution far upstream and downstream of the body as shown in Figure below. Velocity far upstream is the uniform flow U_∞ , and that in the wake of the body is measured to be $u(y) = \frac{U_\infty}{2} \left(\frac{y^2}{b^2} + 1 \right)$, which is less than U_∞ due to the drag of the body. Assume that there is a stream tube with inlet height of $2H$ and outlet height of $2b$ as shown in Figure below. (a) Determine the relationship between H and b using the continuity equation. (b) Find the drag per unit length of the body as a function of U_∞ , b and ρ .

(Hint : Momentum Equation $\sum F_x = \int u \rho (\underline{V} \cdot \underline{n}) dA$)

Continuity Equation $-\frac{d}{dt} \int_{CV} \rho dV = \int_{CS} \rho \underline{V} \cdot \underline{n} dA$



Assumption: steady incompressible flow with one inlet and one outlet.

Solution:

a) Continuity:

For incompressible flow,

$$\begin{aligned}
 0 &= \rho \int_{CS} \underline{V} \cdot \underline{n} dA \\
 \rho \left(\int_{-H}^H -U_\infty dy + \int_{-b}^b u(y) dy \right) &= 0 \\
 \int_{-H}^H -U_\infty dy + \int_{-b}^b \frac{U_\infty}{2} \left(\frac{y^2}{b^2} + 1 \right) dy &= 0 \\
 [-U_\infty y]_{-H}^H + \int_{-b}^b \frac{U_\infty}{2} \left(\frac{y^2}{b^2} + 1 \right) dy &= 0 \\
 -2HU_\infty + \frac{U_\infty}{2} \int_{-b}^b \left(\frac{y^2}{b^2} + 1 \right) dy &= 0 \\
 \frac{U_\infty}{2} \left[\left(\frac{y^3}{3b^2} + y \right) \right]_{-b}^b &= 2HU_\infty
 \end{aligned} \tag{2}$$

$$\frac{1}{2} \left(\frac{b^3}{3b^2} + b + \frac{b^3}{3b^2} + b \right) = 2H$$

$$\left(\frac{b}{3} + b + \frac{b}{3} + b \right) = \left(\frac{8}{3} b \right) = 4H \quad (1) \quad \therefore H = \frac{2b}{3} \quad (1)$$

b) x-momentum:

$$\Sigma F_x = \int u \rho (\underline{V} \cdot \underline{n}) dA$$

Drag per unit length:

$$-F_D = -\rho U_\infty^2 (2H) + \rho \int_{-b}^b u^2(y) dy \quad (1)$$

$$F_D = \rho U_\infty^2 (2H) - \rho \int_{-b}^b \left[\frac{U_\infty}{2} \left(\frac{y^2}{b^2} + 1 \right) \right]^2 dy = \rho U_\infty^2 (2H) - \rho \frac{U_\infty^2}{4} \int_{-b}^b \left(\frac{y^2}{b^2} + 1 \right)^2 dy \quad (2)$$

Calculating integral:

$$\int_{-b}^b \left(\frac{y^2}{b^2} + 1 \right)^2 dy = \int_{-b}^b \left(\frac{y^4}{b^4} + \frac{2y^2}{b^2} + 1 \right) dy = \frac{y^5}{5b^4} + \frac{2y^3}{3b^2} + y \Big|_{-b}^b = 2 \left(\frac{b}{5} + \frac{2b}{3} + b \right) = \frac{56}{15} b \quad (1)$$

Entering into the momentum equation:

$$F_D = 2\rho H U_\infty^2 - \frac{1}{4} \rho U_\infty^2 \left(\frac{56}{15} b \right) = \rho U_\infty^2 \left(\frac{4}{3} b - \frac{14}{15} b \right) = \rho U_\infty^2 \frac{2b}{5} \quad (1)$$