Sinusoidal traveling gravity waves

$\eta(x,t) = a\cos\left[\frac{2\pi}{\lambda}(x - ct)\right]$ x = horizontal z = vertical = $\eta(x,t)$ = wave form

a = amplitude (+ crest − trough)

λ = wavelength

k = wave number (rad/m)

c = phase speed

$\frac{2\pi}{\lambda}(x - ct)$ = phase

$\lambda = \frac{2\pi}{\mathrm{k}}$ = distance between crests, $T = \frac{2\pi}{\mathrm{kc}} = \frac{\lambda}{c}$ = period, i.e., Δt between passage crests

f = cyclic frequency = T^{-1} Hz

$\omega = 2\pi f$ = radian frequency rad/s

$$\eta(x,t) = a\cos(kx - \omega t), \quad \omega = 2\pi \times \frac{kc}{2\pi} = kc$$

Speed where crest $\eta = a \Rightarrow$ phase $= 2\pi n$ n = integer

$$\frac{2\pi}{\lambda}(x_{\mathrm{crest}} - ct) = 2n\pi = kx_{\mathrm{crest}} - \omega t$$

$$x_{\mathrm{crest}} = \frac{\omega}{k}t + \frac{2n\pi}{k}$$

$$\Delta x_{\text{crest}} = \left(\frac{\omega}{k}\right)\Delta t$$

$$\lambda = \left(\frac{\omega}{k}\right)T = \left(\frac{\omega}{k}\right)\frac{\lambda}{c}$$

$$c = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f$$

Phase speed specifies the travel speed of constant-phase wave features such as troughs or crests.

3D generalization

$$\eta = a\cos(kx + ly + mz - ct) = a\cos(\underline{K}\cdot\underline{x} - \omega t)$$

$$\underline{K} = (k, l, m)\ K^2 = k^2 + l^2 + m^2 \quad \lambda = 2\pi/K$$

$$c = \frac{\omega}{K} = \text{phase velocity} \quad \underline{c} = \left(\frac{\omega}{K}\right)\widehat{e_k} \qquad \widehat{e_K} = \underline{K}/\left|\underline{K}\right| = \underline{K}/K$$

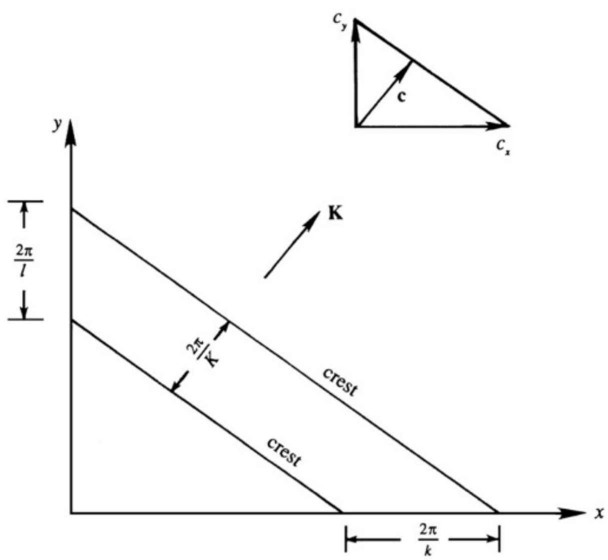

FIGURE 7.1 Wave crests propagating in the *x*-*y* plane. The crest spacing along the coordinate axes is larger than the wavelength $\lambda = 2\pi/K$. The inset shows how the trace velocities c_x and c_y are combined to give the phase velocity vector **c**.

trace velocity

$$c_x = \frac{\omega}{k} \qquad c_y = \frac{\omega}{l} \qquad c_z = \frac{\omega}{m}$$

all $> c = \omega/K$

Since k,l,m individually < K when all three $\neq 0$.

Thus c_x, c_y, c_z not vector components but reflect that constant phase surfaces appear to travel faster along directions not coinciding $\underline{K}$

If waves exist in fluid moving $\underline{U} \Rightarrow \underline{c_0} = \underline{c} + \underline{U}$ = observed phase speed

$$\underline{c_0} \cdot \underline{K} \Rightarrow \omega_0 = \omega + \underbrace{\underline{U} \cdot \underline{K}}_{Doppler\ shifted} \qquad \omega_0 = \text{observed frequency}$$

ω = intrinsic (measured moving with flow)

Say $\omega = 0$ but flow has periodicity in x with $\lambda = \frac{2\pi}{k}$ and moves at speed U, then frequency at point is $\omega_0 = Uk$.

https://en.wikipedia.org/wiki/Doppler_effect

The **Doppler effect** (also **Doppler shift**) is the change in the frequency or, equivalently, the period of a wave in relation to an observer who is moving relative to the source of the wave.[1][2][3] It is named after the physicist Christian Doppler, who described the phenomenon in 1842. A common example of Doppler shift is the change of pitch heard when a vehicle approaches and recedes from an observer. Compared to the emitted sound, the received sound has a higher pitch during the approach, identical at the instant of passing by, and lower pitch during the recession.[4]

Preliminary discussion: propagating vs standing gravity waves

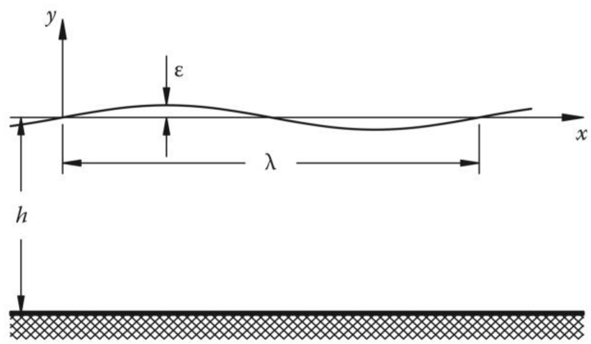

FIGURE 6.2
Parameters for a pure sinusoidal wave.

$$\eta(x,t) = \varepsilon \sin\left(\frac{2\pi}{\lambda}(x - ct)\right)$$

$k = \frac{2\pi}{\lambda} =$ wave number

$c \; =$ wave speed

Wave speed depends on water depth

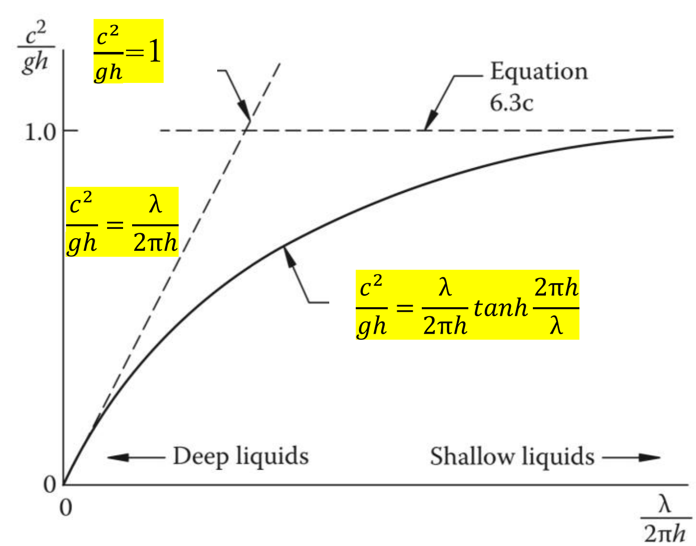

FIGURE 6.3
Propagation speed c for small-amplitude surface waves of sinusoidal form.

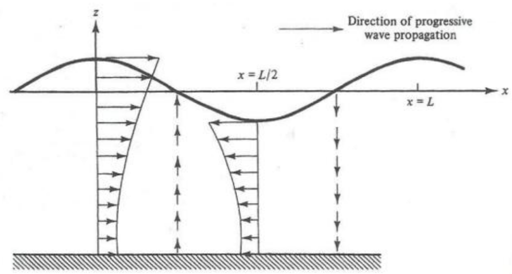

Figure 4.1 Water particle velocities in a progressive wave.

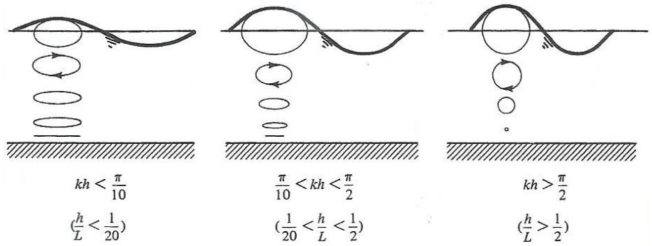

Figure 4.3 Water particle trajectories in progressive water waves of different relative depths.

Standing wave: superpose two traveling waves moving in opposite directions

$$\eta(x,t) = \frac{1}{2}\varepsilon\left[\sin\left(\frac{2\pi}{\lambda}(x-ct)\right) + \sin\left(\frac{2\pi}{\lambda}(x+ct)\right)\right]$$

$$= \varepsilon\sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi ct}{\lambda}\right)$$

$$\omega = \frac{2\pi c}{\lambda} \qquad f = \frac{\omega}{2\pi}$$

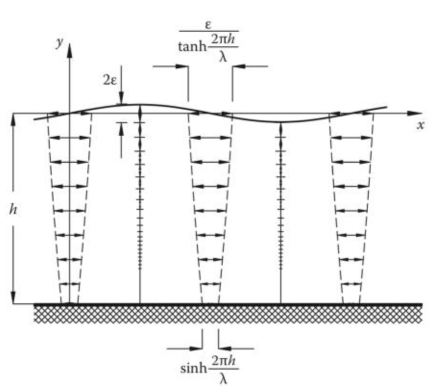

FIGURE 6.8
Particle trajectories induced by a sinusoidal standing wave of amplitude ε and wavelength λ.