Chapter 3 (6): Suction and Injection Flows

Flow over a porous wall

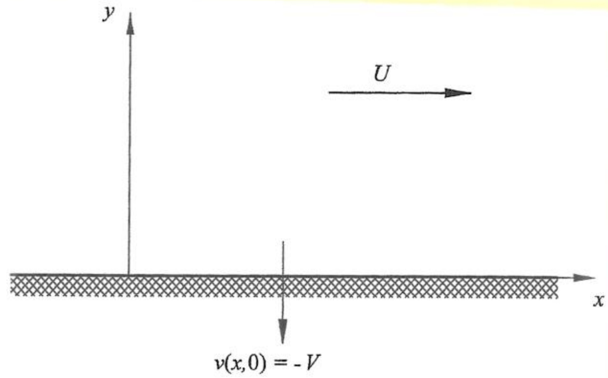

FIGURE 7.8 Uniform flow over a plane boundary with suction.

Suction can be used to prevent BL separation, e.g., on wings and other lifting surfaces.

Assume $p =$ constant $\quad u = u(y)$

$$u_x + v_y = 0 \Rightarrow v_y = 0$$

$$v u_y = \nu u_{yy}$$

$$u v_x + v v_y = \nu (v_{xx} + v_{yy})$$

$$u(0) = 0 \quad v(x,0) = -V \quad u(\infty) = U$$

and $v_y = 0 \Rightarrow v = -V$

$$\underbrace{-V u_y}_{\text{linearized convection}} = \nu u_{yy}$$

$$\frac{d}{dy}(u_y) = -\frac{V}{\nu}\frac{du}{dy}$$

$$u_y = -\frac{V}{\nu} u + A = -au + b$$

constant / equilibrium solution: $u_y = 0 = -au + b$

$u = b/a$

+ transient $\quad u(y) = b/a + v(y)$

$$\frac{du}{dy} = v' = -a\left(\frac{b}{a} + v\right) + b = -av$$

$$v = c\,e^{-ay}$$

$$u = b/a + c\,e^{-ay} \qquad a = V/\nu$$

$$= A + B e^{-Vy/\nu} \qquad b = A$$

$$b/a = A \quad c = B$$

$u(0) = 0 \Rightarrow B = -A$

$u(\infty) = V \Rightarrow A = V \qquad u(y) = V(1 - e^{-Vy/\nu})$

Thickness viscous layer: $\; .99 = 1 - e^{-Vy/\nu}$

$y = \delta = 4.6\nu/V$

Blowing solution diverges (i.e. $v = V$)

Consider $\underline{\omega} = (0, 0, \omega_z)$ of vorticity transport equation $-V\frac{d\omega_z}{dy} = \nu\frac{\partial^2 \omega_z}{\partial y^2}$

or $\quad -V\omega_z = \nu\frac{d\omega_z}{dy}$

convection toward wall $\qquad$ diffusion away from wall

this balance make suction solution possible

3.62 Flow between plates with bottom injection and top suction

v_w = constant

$w = 0$

$u = u(y)$

$$\rho v_w u_y = -\hat{p}_x + \mu u_{yy}$$

$$u(\pm h) = 0$$

$$u/u_{max} = \frac{2}{Re}\left(y/h - 1 + \frac{e^{Re} - e^{Re\,y/h}}{\sinh Re} \right) \qquad Re = v_w h/\nu$$

$\underbrace{h^2(-\hat{p}_x)/2\mu}_{u_{\mathbb{C}} \text{ for parabolic flow}}$

$u/u_{max}\Big|_{\text{Small } Re}$ = parabolic flow

$u/u_{max}\Big|_{\text{large } Re} = 2(1+y/h)/Re$ Straight-line variation which suddenly drops to zero at the upper wall

$\ast$ u_{max} ↓ as Re ↑

ie C_f ↑ as v_w ↑

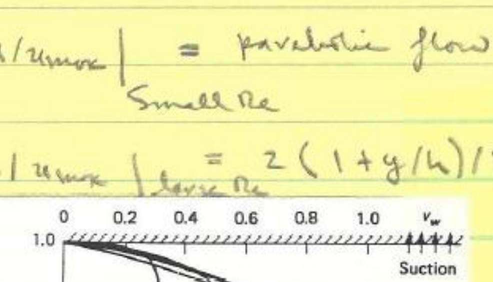

FIGURE 3-18
Velocity profiles for flow between parallel plates with equal and opposite porous walls, Eq. (3-127).

Solutions also possible for axial flow in circular annulus or flow between rotating cylinders