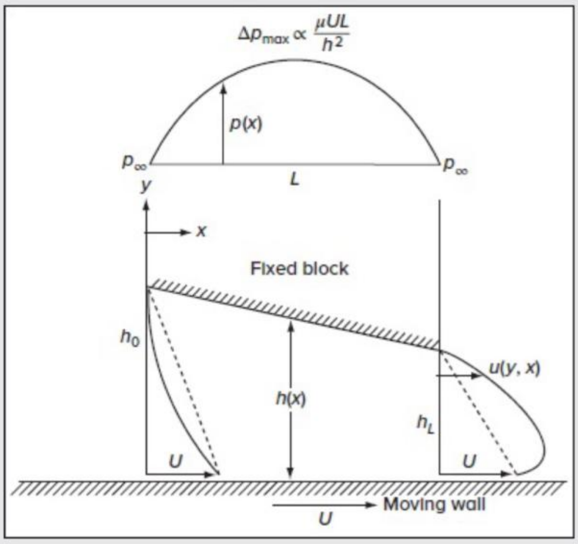
Lubrication Theory (also applicable porous media, filtration, adhesion, biological, and non-Newtonian flows)

Lubrication, i.e., friction reduction of two bodies in near contact is generally accomplished by a viscous fluid moving through a narrow but variable gap between the two bodies with one or both moving (Reynolds, 1886).



Assume 2D $\frac{\partial}{\partial z} = 0$ and Stokes flow such that inertia negligible, i.e.,

$\rho\, u\, u_x \ll \mu\, u_{yy}$

$\rho U \frac{U}{L} \ll \mu \left(\frac{U}{h^2}\right)$

$\frac{\rho U L}{\mu} \left(\frac{h}{L}\right)^2 \ll 1$ Re can be large if h/L small

$U = 10$ m/s, $L = 4$ cm, $h = 0.1$ mm

SAE-50 lubricating oil $\nu = 7 \times 10^{-4}$ m^2/s

$Re_L = 570$ but $Re_L \left(\frac{h}{L}\right)^2 = 0.004$ $\therefore$ OK

FIGURE 3-48

Low Reynolds number Couette flow in a varying gap: To maintain continuity, the gap pressure rises to a maximum and superimposes Poiseuille flow toward both ends of the gap.

$$\varepsilon = \frac{h}{L} \ll 1 \text{ and } Re \text{ moderate}$$

Like Stokes flow since Re fairly small and inertia negligible flow is usually quasi-steady.

Like BL flow in that $\frac{\partial p}{\partial y} = 0$ and $\frac{\partial p}{\partial x}$ important

However, the pressure and viscous stress scale differently than Stokes and BL flow.

Reynolds Equation for Bearing Theory

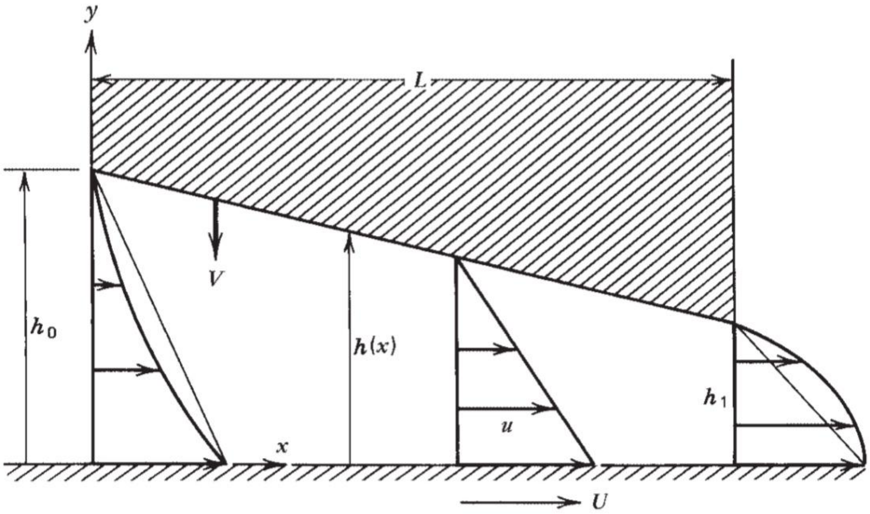


Figure 22.3 Flow in a slipper pad bearing is locally the sum of a Couette flow and a Poiseuille flow.

Events:

1. Moving wall sweeps fluid into narrowing passage due to viscous shear forces, which induces Couette flow $u = \frac{Uy}{h}$ and $q_c = \frac{1}{2}Uh$ (per unit span).

2. Continuity requires $Q =$ constant; therefore, p_x required that induces Poiseuille u component that redistributes fluid and maintains Q $=$ constant.

In general, $h(x,t)$ and for simplicity upper wall vertical motion only V which may be $V(t)$, lower wall $U =$ constant, and allow for w due p_z

$$x, z \sim L, \qquad y \sim h_0, \qquad t \sim \frac{L}{U}, \qquad u, w \sim U, \qquad v \sim \frac{Uh_0}{L}, \qquad p \sim \frac{\mu UL}{h_0^2}$$

y-momentum $\lim_{h_0/L \to 0} \Rightarrow \frac{\partial p}{\partial y} = 0$, i.e., $p = p(x,z)$

x- and z-momentum equations yield quasi-steady equations (Appendix A)

$$0 = -p_x + \mu u_{yy}$$

$$0 = -p_z + \mu w_{yy}$$

$$u(y=0) = U \qquad v(y=0) = 0 \qquad w(y=0) = 0$$

$$u(y=h) = 0 \qquad v(y=h) = V \qquad w(y=h) = 0$$

Partial integration over y of equations and application BC:

$$u = \frac{1}{2\mu} p_x \left(y^2 - yh\right) + \left(1 - \frac{y}{h}\right) U$$

$$w = \frac{1}{2\mu} p_z \left(y^2 - yh\right)$$

Combination Poiseuille & Couette flow function pressure gradients, h and U.

Need to determine pressure distribution that will support the load with the bearing.

The Reynolds equation for pressure is derived by integrating the continuity equation over the y-direction.

$$\frac{d}{dt}\int_{a(t)}^{b(t)} f(x,t)\ dx = \int_a^b \frac{\partial f}{\partial t}\ dx + \frac{db}{dt} f(b,t) - \frac{da}{dt} f(a,t)$$

$$\int_0^h u_x\ dy + \int_0^h w_z\ dy = -\int_0^h v_y\ dy = V = \frac{\partial h}{\partial t}$$

$$\bar{u} = \frac{q_x}{h} \qquad \frac{\partial}{\partial x}\underbrace{\int_0^h u\ dy}_{q_x} = \int_0^h \frac{\partial u}{\partial x}\ dy + \frac{dh}{dx} u(y=h) - \frac{d0}{dx} u(y=0) = \int_0^h \frac{\partial u}{\partial x}\ dy$$

$$\bar{w} = \frac{q_z}{h} \qquad \frac{\partial}{\partial z}\underbrace{\int_0^h w\ dy}_{q_z} = \int_0^h \frac{\partial w}{\partial z}\ dy + \frac{dh}{dz} w(y=h) - \frac{d0}{dz} w(y=0) = \int_0^h \frac{\partial w}{\partial z}\ dy$$

$$u(y) = \frac{1}{2\mu} p_x\,(y^2 - yh) + \left(1 - \frac{y}{h}\right) U = a\,(y^2 - yh) + U - \frac{Uy}{h}$$

$$q_x = \int_0^h \left[a(y^2 - yh) + U - \frac{Uy}{h}\right] dy$$

$$= \left[\frac{ay^3}{3} - \frac{ahy^2}{2} + Uy - \frac{Uy^2}{2h}\right]_0^h = \frac{ah^3}{3} - \frac{ah^3}{2} + Uh - \frac{Uh}{2} = -\frac{ah^3}{6} + \frac{Uh}{2}$$

$$w(y) = \frac{1}{2\mu} p_z\,(y^2 - yh) = b\,(y^2 - yh)$$

$$q_z = b\int_0^h (y^2 - yh)\ dy = b\left[\frac{y^3}{3} - \frac{y^2 h}{2}\right]_0^h = -\frac{bh^3}{6}$$

$$\frac{\partial}{\partial x}\left[-\frac{ah^3}{6} + \frac{Uh}{2}\right] + \frac{\partial}{\partial z}\left[-\frac{bh^3}{6}\right] = \frac{\partial h}{\partial t}$$

$$\frac{\partial}{\partial x}[-ah^3 + 3Uh] + \frac{\partial}{\partial z}[-bh^3] = 6\frac{\partial h}{\partial t}$$

$$\frac{\partial}{\partial x}\left[-\frac{1}{2\mu} p_x h^3 + 3Uh\right] + \frac{\partial}{\partial z}\left[-\frac{1}{2\mu} p_z h^3\right] = 6\frac{\partial h}{\partial t}$$

$$\frac{1}{\mu}\left[\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)\right] = 6U\frac{\partial h}{\partial x} + 12\frac{\partial h}{\partial t}$$

Reynolds equation lubrication in channel $h(x,t)$ with upper wall moving V(t) and lower wall moving constant speed U. $p(x,z,t)$ found as function (geometry by motion walls). Once pressure known proportion Poiseuille vs. Couette in the velocity profile (u,w)

Slider Bearing

Non-dimensional variables:

$$p^* = \frac{p - p_0}{\mu UL/h_0^2} \qquad x^* = \frac{x}{L} \qquad h^* = \frac{h}{h_0} = 1 - Ax^*$$

$$p = \frac{\mu UL}{h_0^2} p^* + p_0 \qquad A = \frac{\alpha L}{h_0} = \frac{h_0 - h_1}{h_0}, \ \alpha = \frac{h_0 - h_1}{h_0}$$

Assume 1D flow, i.e., $\frac{\partial}{\partial z} = 0 \quad w = 0 \quad \frac{\partial}{\partial t} = 0$, i.e., $V = 0$

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) = 6U\mu \frac{\partial h}{\partial x}$$

$$h^3 \frac{\partial p}{\partial x} = 6U\mu h + c$$

$c = -6U\mu h_m$ where $h_m = h\left(\frac{\partial p}{\partial x} = 0\right)$, i.e., where Poiseuille flow = 0

$$h^3 \frac{\partial p}{\partial x} = 6U\mu(h - h_m)$$

$$h^3 = h_0^3 h^{*3} \qquad \frac{\partial p}{\partial x} = \frac{\partial x^*}{\partial x}\frac{\partial p}{\partial p^*}\frac{\partial p^*}{\partial x^*} = \frac{1}{\cancel{L}} \times \frac{\mu U \cancel{L}}{h_0^2} p^*_{x^*}$$

$$p_x^* \cancel{h_0^3} h^{*3} \frac{\cancel{\mu U}}{\cancel{h_0^2}} = 6\cancel{\mu U}(\cancel{h_0} h^* - \cancel{h_0} h_m^*)$$

$$h^{*3} p_x^* = 6(h^* - h_m^*)$$

Note at h_m^* $u = \left(1 - \frac{y}{h}\right) U = \frac{U}{h}(h - y)$

$$h_m^* = \frac{2q}{Uh_0} = q^* = \frac{h_m}{h_0} \qquad q = \int_0^{h_m} u \, dy = \frac{U}{h}\left[hy - \frac{y^2}{2}\right]_0^{h_m} = \frac{Uh_m}{2}$$

$$dp^* = \frac{6}{h^{*3}}(h^* - h_m^*)dx^* = 6(h^{*-2} - h_m^* h^{*-3})dx^*$$

$$dp^* = 6[(1 - Ax^*)^{-2} - h_m^*(1 - Ax^*)^{-3}]dx^*$$

Let $x = 1 - Ax^* = h^*$ $\quad dx = -A\,dx^*$ $\quad \int x^n dx = \frac{x^{n+1}}{n+1}$ $\quad n \neq -1$

$$dp^* = -\frac{6}{A}[x^{-2} - h_m^* x^{-3}]dx$$

$$p^* = -\frac{6}{A}\left[\frac{x^{-1}}{-1} - h_m^* \frac{x^{-2}}{-2}\right] + C = 6A^{-1}\left(h^{*-1} - \frac{h_m^*}{2}h^{*-2}\right) + C$$

$$h_0 h^* = h = h_0(1 - Ax^*) = h_0\left[1 - \frac{h_0 - h_1}{h_0}\frac{x}{L}\right]$$

$$h(0) = h_0 \qquad h = h_0 - (h_0 - h_1)\frac{x}{L}$$

$$h(L) = h_1$$

$$h^* = 1 \Rightarrow h = h_0, \quad p = p_0 \quad \therefore p^* = 0$$

$$0 = 6A^{-1}\left(1 - \frac{h_m^*}{2}\right) + C$$

$$C = 6A^{-1}\left(-1 + \frac{h_m^*}{2}\right)$$

$$p^* = 6A^{-1}\left[1 - h^{*-1} + \frac{h_m^*}{2}(-h^{*-2} + 1)\right]$$

$$p^* = 6A^{-1}(h^{*-1} - 1) - 3A^{-1}h_m^*(h^{*-2} - 1)$$

$p^* = 0$ at $h^* = 1 - A = a$, i.e., $h = h_0 - (h_0 - h_1)\frac{x}{L}$ such that

$h(L) = h_1, \quad x^* = 1 \quad h^* = 1 - A, \quad p = p_0, \quad p^* = 0$

$$0 = 6A^{-1}\left(\frac{1}{a} - 1\right) - 3A^{-1}h_m^*\left(\frac{1}{a^2} - 1\right)$$

$$\frac{3h_m^*}{\cancel{A}}\left(\frac{1 - a^2}{a^2}\right) = \frac{6}{\cancel{A}}\left(\frac{1 - a}{a}\right)$$

$$h_m^*\cancel{(1 - a)}(1 + a) = 2\cancel{(1 - a)}a$$

$$h_m^* = \frac{2a}{1 + a} = \frac{2(1 - A)}{1 + 1 - A}$$

$$(1 - a)(1 + a) = 1 - a^2$$

$$(1 - A)(1 - A) = 1 - 2A + A^2$$

$$h_m^* = \frac{2(1 - A)}{2 - A}$$

$$p^* = 6A^{-1}\left(\frac{1 - h^*}{h^*}\right) - 3A^{-1}h_m^*\left(\frac{1 - h^{*2}}{h^{*2}}\right)$$

$$= 6A^{-1}h^*(1 - h^*) - 3A^{-1}h_m^*h^{*2}(1 - h^{*2})$$

$$= 6A^{-1}h^*\left[(1 - h^*) - \frac{h_m^*h^*}{2}(1 - h^{*2})\right]$$

$$= 6A^{-1}h^*(1 - h^*)\left[1 - \frac{h_m^*h^*}{2}(1 + h^*)\right]$$

$$p^*(x^*) = \frac{6x^*}{1 - Ax^*}\left[1 - \frac{1 - A}{2 - A} \times \frac{2 - Ax^*}{1 - Ax^*}\right]$$

$$h_m^* = 1 - Ax_m^* = \frac{2(1 - A)}{2 - A}$$

$$h^* = 1 - Ax^*$$

$$\frac{h^* - h^{*2}}{A}$$

$$\frac{1 - h^*}{h^*A} = \frac{\cancel{A}x^*}{\cancel{A}h^*} = \frac{x^*}{1 - Ax^*}$$

$$h^*(1 + h^*) = h^* + h^{*2}$$

$$= \frac{1 + h^*}{h^*} = \frac{2 - Ax^*}{1 - Ax^*}$$

$h^* = 1 - Ax^*$ and $h^{*3} p^*_{x^*} = 6(h^* - h^*_m)$

and $h^*_m = \frac{2(1-A)}{2-A}$ from 1st integration equation

$p^*_{x^*} = 0$ when $h^* = h^*_m$

$$\therefore 2\frac{1-A}{2-A} = 1 - Ax^*_m$$

$Ax^*_m = 1 - 2\frac{1-A}{2-A} = \frac{2-A-2(1-A)}{2-A} = \frac{\cancel{2}-A-\cancel{2}+2A}{2-A}$

$= \frac{A}{2-A}$ i.e. $x^*_m = \frac{1}{2-A}$

$1 - Ax^*_m = 1 - \frac{A}{2-A} = \frac{2-A-A}{2-A} = \frac{2(1-A)}{2-A}$

$2 - Ax^*_m = 2 - \frac{A}{2-A} = \frac{4-2A-A}{2-A} = \frac{4-3A}{2-A}$

$$p^*_m = \frac{6\cancel{(2-A)}}{\cancel{(2-A)}\,2(1-A)}\left[1 - \frac{\cancel{1-A}}{\cancel{2-A}}\frac{4-3A}{2-A}\frac{\cancel{2-A}}{2(\cancel{1-A})}\right]$$

$$\cancel{4} - 2A - \cancel{4} + 3A = A$$

$p^*_m = \frac{3}{1-A}\left[1 - \frac{4-3A}{2(2-A)}\right] = \frac{3}{1-A}\left[\frac{2(2-A)-4+3A}{2(2-A)}\right]$

$$p^*_m = \frac{3A}{2(1-A)(2-A)}$$

$A = 0$ walls $\parallel$ $A = \frac{h_0 - h_1}{h_0}$

Couette flow $p =$ constant $= p_0$

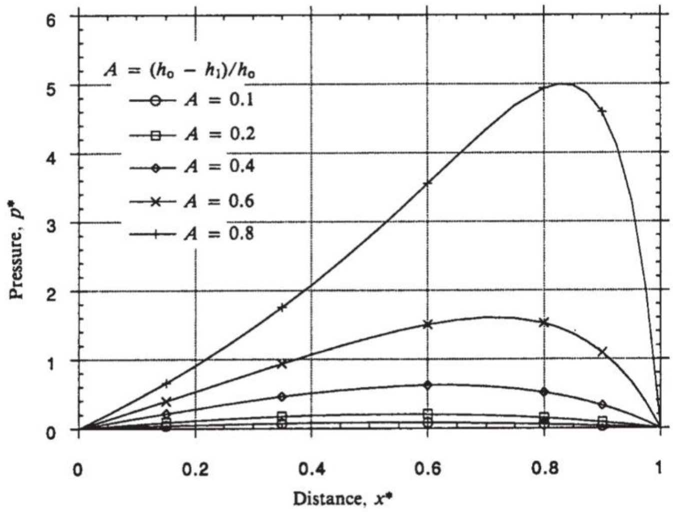


Figure 22.4 Pressure in a slipper pad bearing.

$A > 0$ but very small

$$p_m^* = \frac{3A}{4\left(1 - \frac{A}{2}\right)(1 - A)}$$

$$= \frac{3A}{4}\left(1 - \frac{A}{2}\right)^{-1} (1 - A)^{-1}$$

$$= \frac{3A}{4}\left(1 + \frac{A}{2}\right)(1 + A) \approx \frac{3A}{4}$$

$$x_m^* = \frac{1}{2}\left(1 - \frac{A}{2}\right)^{-1} = \frac{1}{2}\left(1 + \frac{A}{2}\right) \approx \frac{1}{2}$$

$$\underbrace{\frac{dp^*}{dx^*} > 0 \quad x^* < x_m^*}_{\text{Poiseuille flow opposes Couette flow and vice versa}} \qquad \text{and} \quad \frac{dp^*}{dx^*} < 0 \quad x^* > x_m^*$$

Process: fluid dragged into converging channel via viscous shear forces piles up to create high pressure $x^* = \frac{1}{2}$ after which $\frac{dp^*}{dx^*}$ changes sign and pushes flow towards exit. $\frac{dp^*}{dx^*}$ between center and either end induces Poiseuille flow towards both ends of the bearing, which subtracts Couette flow first half and adds second half.

$$\frac{p_m - p_0}{\mu UL/h_0^2} = \frac{3A}{4} + O[A^2]$$

$$p_m - p_0 = \frac{\mu UL}{h_0^2} \times \frac{3A}{4}$$

$$p_m - p_0 = \frac{3\mu UL}{4}\left(\frac{h_0 - h_1}{h_0^3}\right) = \frac{3\mu UL}{4}\frac{(1 - h_1/h_0)}{h_0^2}$$

Shows importance h_0.

SAE oil with $U = 10$ m/s, $L = 4$ cm, and $h_0 = 0.1$ mm.

$p_m - p_0$ of order $\frac{\mu UL}{h_0^2} \approx 2.5 \times 10^7$ Pa ≈ 250 atm

i.e., very high force to slipper block, which enables it to support large load without block touching wall.

Stokes flow is linear and ∴ reversible. If reverse wall motion $U < 0$ then $\Delta p < 0$, i.e., will cavitate and form vapor void in gap (G. I. Taylor, film low-Re hydrodynamics) ∴ flow in expanding narrow gap may not support large loads and provide good lubrication. Issue for rotating journal bearing where gap contracts/expands as rotor rotates and often leads to partial cavitation.

Squeeze film lubrication: viscous adhesion.

Wringing together smooth surfaces relationship crankshaft bearing, i.e., power stroke piston causes $V(t)$, which dominates over hydrodynamic journal-bearing effects, whereas separating smooth surfaces by pulling normal to increase gap height is difficult, although sliding is easy.

Assume $V = \frac{\partial h}{\partial t}$ and $U = 0$ Also w and $\frac{\partial}{\partial z} = 0$

Take $x = 0$ such that bearing pad ends at $\pm\frac{L}{2}$

Reynolds pressure equation: $\frac{1}{\mu}\frac{d}{dx}\left(h^3\frac{dp}{dx}\right) = 12\frac{\partial h}{\partial t} = 12V(t)$

$\therefore \frac{\partial p}{\partial x} = \frac{12\mu}{h^3}\frac{\partial h}{\partial t}x$ $\quad u = \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - yh)$ at $x = 0 = 0$ and maximum at x = $\pm L/2$. Thus, all flow into or out gap must cross the ends

$$p - p_0 = \frac{12\mu}{h^3}\frac{\partial h}{\partial t}\left[\frac{x^2}{2}\right]_{-L/2}^{x}$$

$$= \frac{12\mu}{h^3}\frac{\partial h}{\partial t}\left[\frac{x^2}{2} - \frac{L^2/4}{2}\right] = \frac{12\mu}{h^3}\frac{\partial h}{\partial t}\left[\frac{x^2}{2} - \frac{L^2}{8}\right]$$

$$= \frac{12\mu}{h^3}\frac{\partial h}{\partial t}\frac{1}{2}\frac{L^2}{4}\left(\frac{x^2}{L^2/4} - 1\right)$$

$$p - p_0 = -\frac{3\mu L^2}{2h^3}\frac{\partial h}{\partial t}\left[1 - \left(\frac{x}{L/2}\right)^2\right] \qquad p \propto h^{-3} \text{ Very large pressure}$$

$$p - p_0 = \frac{3\mu L^2}{2h^3}V\left[1 - \left(\frac{x}{L/2}\right)^2\right]$$

$W = \int_{-L/2}^{L/2} (p - p_0)\ dx$ load capacity per unit span

$$= \frac{3\mu L^2}{2h^3} V \int_{-L/2}^{L/2} \left[1 - \left(\frac{x}{L/2}\right)^2\right] dx$$

$$x' = \frac{x}{L/2} \qquad dx' = \frac{dx}{L/2} \qquad \frac{L}{2} dx' = dx$$

$$\int_{-1}^{1} (1 - x'^2)\ dx' = \left[x' - \frac{x'^3}{3}\right]_{-1}^{1} = \frac{2}{3} - \underbrace{\left(-1 + \frac{1}{3}\right)}_{-2/3} = \frac{4}{3} \times \frac{L}{2} = \frac{2L}{3}$$

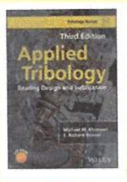
$$W = \frac{\cancel{3}\mu L^2}{\cancel{2}h^3} V \times \frac{\cancel{2}L}{\cancel{3}} = \mu \left(\frac{L}{h}\right)^3 V = -\mu \left(\frac{L}{h}\right)^3 \frac{dh}{dt} \qquad \mu = \frac{Ns}{m^2} \quad \left(\times \frac{m}{s} = \frac{W}{m}\right)$$

$$\int_{t_1}^{t_2} dt = -\frac{\mu L^3}{W} \int_{h_1}^{h_2} \frac{dh}{h^3} \qquad \int h^{-3} dh = -\frac{1}{2} h^{-2}$$

$$\Delta t = -\frac{\mu L^3}{W} \left[-\frac{1}{2} h^{-2}\right]_{h_1}^{h_2}$$

$$\Delta t = \frac{\mu L^3}{2W} \left(\frac{1}{h_2^2} - \frac{1}{h_1^2}\right)$$

Δt = time of approach for film gap to reduce from h_1 to h_2. $\Delta t \to \infty$ and $h_2 \to 0$, i.e., takes infinite time squeeze out all the fluid!



Applied Tribology: Bearing Design and Lubrication: Bearing Design and Lubrication

Author(s):
Michael M. Khonsari, E. Richard Booser

First published:8 August 2017
Print ISBN:9781118637241 |Online ISBN:9781118700280 |DOI:10.1002/9781118700280

9.4 Generalization for planar squeeze film

For planar squeeze-film problems, the time of approach has the following form (Moore, 1993):

$$\Delta t = K\frac{\mu A^2}{W}\left(\frac{1}{h_2^2}-\frac{1}{h_1^2}\right) \tag{9.13}$$

where K is the shape function. Taking the time derivative of Equation (9.13), the surface approach velocity can be obtained:

$$V_s = \frac{1}{2K}\frac{h^3 W}{\mu A^2} \tag{9.14}$$

where A is the plate area and h is the squeeze-film thickness.

Forms of the function K for a series of planar squeeze-film geometries are shown in Table 9.1 and plotted in Figure 9.5 for convenience (Khonsari and Jang, 1997). Using either the table or the figure, one can readily evaluate K. Then, for a given load, W, Equation (9.13) gives the time of approach for the film thickness to drop from an initial h_1 to a final h_2.

Example 9.1 Estimate the time of approach in a wet clutch system modeled as two rigid concentric annuli with $R_i = 0.047\,\text{m}$ and $R_o = 0.06\,\text{m}$ submerged in a lubricant with viscosity of $\mu = 0.006\,\text{Pa s}$. Hydraulic pressure is $P = 1.25\,\text{MPa}$, and the initial separation gap is $h_1 = 25 \times 10^{-6}\,\text{m}$. Estimate the initial squeeze velocity and the length of time necessary for the film thickness to drop to $h_2 = 5 \times 10^{-6}\,\text{m}$.

Table 9.1 Types of planar squeeze

Type	Ratio, r	Configuration	Constant, K
Circular section	—		$\frac{3}{4\pi}$
Elliptical section	$\Re = \frac{b}{a}$		$\frac{3\Re}{2\pi(1+\Re^2)}$
Rectangular section	$\Re = \frac{B}{L}$		$\frac{1}{2\Re}\left[1-\frac{192}{\pi^5\Re}\sum_{n=1,3,5,\ldots}^{\infty}\frac{\tanh(n\pi\Re/2)}{n^5}\right]$
Triangular section	—		$\frac{\sqrt{3}}{10}$
Circular sector	$\Re = \frac{\alpha}{2\pi}$		$\sum_{n=1,3,5,\ldots}^{\infty}\frac{24}{n^2\pi^3\Re\left[2+\left(\frac{n}{2\Re}\right)\right]^2}$
Concentric annulus	$\Re = \frac{D_i}{D_o}$		$\frac{3}{4\pi}\left[\frac{\ln\Re-\Re^4\ln\Re+(1-\Re^2)^2}{(1-\Re^2)^2\ln\Re}\right]$

The shape factor K for the concentric annulus can be evaluated from Table 9.1:

$$A = \pi\left(R_o^2 - R_i^2\right) = 0.0044\,\text{m}^2$$

$$\Re = \frac{R_i}{R_o} = \frac{0.047}{0.06} = 0.783$$

$$K = \frac{3}{4\pi}\frac{\ln(\Re)-\Re^4\ln\Re+(1-\Re^2)^2}{(1-\Re^2)^2\ln(\Re)}$$

$$= \frac{3}{4\pi}\frac{\ln(0.783)-(0.783)^4\ln(0.783)+(1-0.783^2)^2}{(1-0.783^2)^2\ln(0.783)} = 0.019$$

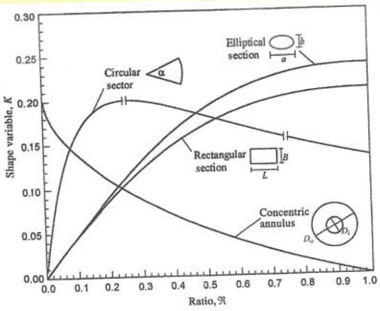


Figure 9.5 Variation of constant K with shape ratio $\Re$

Using Equation (9.13), the time of approach is

$$\Delta t = K\frac{\mu A^2}{W}\left(\frac{1}{h_2^2}-\frac{1}{h_1^2}\right) = K\frac{\mu A}{P}\left(\frac{1}{h_2^2}-\frac{1}{h_1^2}\right)$$

$$= \frac{0.019(0.006)(0.0044)}{1.25\times 10^6}\left[\frac{1}{(5\times 10^{-6})^2}-\frac{1}{(25\times 10^{-6})^2}\right] = 0.02\,\text{s}$$

The squeeze velocity can be estimated using Equation (9.14):

$$V_s = \frac{1}{2K}\frac{h_1^3 W}{\mu A^2} = \frac{1}{2K}\frac{h_1^3 P}{\mu A}$$

$$= \frac{1}{2(0.019)}\frac{(25\times 10^{-6})^3 1.25\times 10^6}{(0.006)(0.0044)} = 0.02\,\text{m/s}$$

The time of approach predicted above is a representation of the first stage of the engagement duration while the clutch operates in the hydrodynamic regime. This engagement begins when pressure is applied hydraulically by means of a piston and hydrodynamic pressure is developed in the ATF as a result of squeeze action which supports most of the applied load. Since the surfaces are separated by a relatively thick film of fluid, behavior of the clutch is governed by the theory of hydrodynamic lubrication. This period lasts only 0.02 s.

During engagement, the fluid film thickness drops to the extent that surface asperities come into contact. As a result, contact pressure at the asperity level begins to support a major portion of the imposed load, significantly influencing the behavior of the wet clutch. The film thickness is further reduced as the friction-lining material is compressed and deforms elastically. The surfaces are subsequently pressed together and 'locked' when their relative speed drops to zero. The timescale of the engagement process is typically of the order of 1 s, during which the squeeze action is of paramount importance. It follows, therefore, that in a typical engagement cycle, the lubrication regime undergoes a transition from hydrodynamic to mixed or boundary lubrication. This shift in the lubrication regime has important implications on the signature of the total torque, i.e. the combination of the viscous torque and contact torque. The total torque reaches a peak value when the film thickness drops to a minimum. After this peak value, the torque initially remains relatively flat for a short period of time and then begins to rise gradually as the relative speed between the clutch disks decreases. This increase in the torque is a direct consequence of the change in the coefficient of friction as a function of speed, in accordance with the Stribeck friction curve. The most interesting torque signature is a highly undesirable sudden spike or 'rooster's tail' toward the end of the engagement. Its occurrence can be predicted analytically (Jang and Khonsari, 1999) and can be treated to minimize its effect by altering the friction behavior.

Note that the friction material is rough, porous, and deformable. Also, the large disk diameters used in wet clutch systems may require consideration of the centrifugal forces. These elements can affect the engagement time and torque-transfer characteristics, as well as the temperature field in the ATF and on the surface of the separator. The interested reader may refer to Jang and Khonsari (1999) for detailed analysis of automotive wet clutch systems. More recent studies of wet clutches have included parametric analysis of variance and experimental results (Mansouri *et al.*, 2001, 2002; Marklund *et al.*, 2007).

Journal Bearing

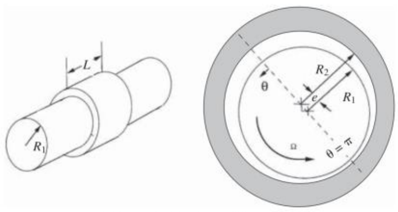


Figure 22.5 Journal bearing.

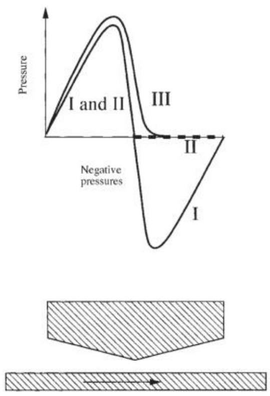


Figure 22.6 Similarity of journal bearing and double slider block. Pressure distribution I, full Sommerfeld; pressure distribution II, half Sommerfeld; pressure distribution III, Swift–Steiber.

Rotating shaft, journal, radius R_1, in bearing housing radius R_2.

$\frac{L}{R_1} < 0.5$ = short $\frac{L}{R_1} > 2$ = long

aspect ratio

journal offset bearing distance

e= eccentricity

$c = R_2 - R_1 =$ clearance

$\varepsilon = e/c$ = eccentricity ratio = measure clearance film height

with min $= c - e$, max $= c + e$

Rotates CCW such that drags fluid wide to narrow passage, which creates high pressure support journal. Amount of load determines e and c. Physics similar double slider. Converging/diverging passages similar double slider with reversed motion expanding half of since lubrication equations reversible solution is velocity profile reversed and $p = -p$. However large $-p$ implies cavitation. Therefore, p assumed constant or other approximations.