Appendix

$$u_x + v_y + w_z = 0$$

$$u_t + u u_x + v u_y + w u_z = -p_x/\rho + \nu(u_{xx} + u_{yy} + u_{zz})$$

$$v_t + u v_x + v v_y + w v_z = -p_y/\rho + \nu(v_{xx} + v_{yy} + v_{zz})$$

$$w_t + u w_x + v w_y + w w_z = -p_z/\rho + \nu(w_{xx} + w_{yy} + w_{zz})$$

$$(x^*, z^*) = (x, z)/L \qquad y^* = y/h_0 \qquad t^* = U t/L$$

$$(u^*, w^*) = (u, w)/U \qquad v^* = vL/Uh_0 \qquad p^* = p/p_0$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t^*}\frac{\partial t^*}{\partial t} = \frac{U}{L}\frac{\partial}{\partial t^*} \qquad \frac{\partial}{\partial x} = \frac{\partial}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{1}{L}\frac{\partial}{\partial x^*} \qquad \frac{\partial}{\partial z} = \frac{1}{L}\frac{\partial}{\partial z^*} \qquad \frac{\partial}{\partial y} = \frac{1}{h_0}\frac{\partial}{\partial y^*}$$

$$(u, w) = U(u^*, w^*) \qquad v = \frac{U h_0}{L} v^* \qquad p = p_0 p^* \qquad t = \frac{L}{U} t^*$$

$$= U \varepsilon v^* \qquad \varepsilon = h_0/L << 1$$

$$\frac{1}{L}\frac{\partial}{\partial x^*} U u^* + \frac{1}{\cancel{h_0}}\frac{\partial}{\partial y^*}\frac{U \cancel{h_0}}{L} v^* + \frac{1}{L}\frac{\partial}{\partial z^*} U w = 0$$

$$\frac{U}{L}\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*}\right) = 0 \quad \text{ie no change continuity}$$

$Re_L = \frac{\rho U L}{\mu}$

$$\frac{U}{L}\frac{\partial}{\partial t^*} U u^* + \frac{U^2}{L}\left(u^* \frac{\partial u^*}{\partial x^*}\right) + \frac{U h_0}{L} v^* \frac{1}{h_0}\frac{\partial}{\partial y^*} U u^* + U w^* \frac{1}{L}\frac{\partial}{\partial z^*} U u^*$$

$\varepsilon = h_0/L$

$$= -\frac{p_0}{\rho L}\frac{\partial p^*}{\partial x^*} + \nu\left(\frac{1}{L^2}\frac{\partial^2}{\partial x^{*2}} U u^* + \frac{1}{h_0^2}\frac{\partial^2}{\partial y^{*2}} U u^* + \frac{1}{L^2}\frac{\partial^2}{\partial z^{*2}} U u^*\right)$$

$\varepsilon^2 Re_L = \frac{h_0^2}{L^2}\frac{\rho U L}{\mu}$

$$\frac{U^2}{L}\left(\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} + w^*\frac{\partial u^*}{\partial z^*}\right) = -\frac{p_0}{\rho L}\frac{\partial p^*}{\partial x^*} + \nu\left(\frac{U}{L^2}\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{U}{h_0^2}\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{U}{L^2}\frac{\partial^2 u^*}{\partial z^{*2}}\right)$$

$\frac{h_0^2 \rho U}{L \mu}$

$$\frac{\rho U^2}{L}\frac{Du^*}{Dt} = -\frac{p_0}{L}\frac{\partial p^*}{\partial x^*} + \mu\frac{U}{L^2}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{L^2}{h_0^2}\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right)$$

$\Lambda = \frac{\mu U L}{p_0 h_0^2}$

$$\frac{\rho U L}{\mu}\frac{Du^*}{Dt} = \frac{-p_0 L}{\mu U}\frac{\partial p^*}{\partial x^*} + \frac{\partial^2 u^*}{\partial x^{*2}} + \varepsilon^{-2}\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}$$

$$\varepsilon^2 Re_L \frac{Du^*}{Dt} = \underbrace{\frac{-p_0 L h_0^2}{\mu U L^2}}\frac{\partial p^*}{\partial x^*} + \varepsilon^2\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) + \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$\frac{p_0 h_0^2}{\mu U L} = \Lambda^{-1} \qquad \Lambda = \frac{\mu U L}{p_0 h_0^2} = \frac{\text{viscous}}{\text{pressure}} \text{ forces}$$

bearing #

$\varepsilon \ll 1$ Re_L moderate $\Lambda \sim$ order unity

$\therefore$ $$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$$ drop *

Similarly for z $$0 = -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial z^2}$$

$$\frac{U}{L}\frac{\partial}{\partial t^*}\frac{U h_0}{L} v^* + \frac{U u^*}{L}\frac{\partial}{\partial x}\frac{U h_0}{L} v^* + \frac{U h_0}{L} v^* \frac{1}{h_0}\frac{\partial}{\partial y^*}\frac{U h_0}{L} v^* + \frac{U w^*}{L}\frac{\partial}{\partial z^*}\frac{U h_0}{L} v^*$$
$$= -\frac{p_0}{h_0}\frac{\partial p^*}{\partial y^*} + \nu\left(\frac{1}{L^2}\frac{\partial^2}{\partial x^{*2}}\frac{U h_0}{L} v^* + \frac{1}{h_0^2}\frac{\partial^2}{\partial y^{*2}}\frac{U h_0}{L} v^* + \frac{1}{L^2}\frac{\partial^2}{\partial z^{*2}}\frac{U h_0}{L} v^*\right)$$

$$\frac{U^2 h_0}{L^2}\left[\frac{\partial v^*}{\partial t} + u^*\frac{\partial v^*}{\partial x} + v^*\frac{\partial v^*}{\partial y^*} + w^*\frac{\partial v^*}{\partial z^*}\right] = -\frac{p_0}{\rho h_0}\frac{\partial p^*}{\partial y^*} + \nu\frac{U}{h_0 L}\frac{\partial^2 v^*}{\partial y^{*2}}$$
$$+ \nu\frac{U h_0}{L^3}\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right)$$

$$\frac{\rho U L}{\mu}\frac{D v^*}{Dt} = \frac{-p_0 L^3}{\mu U h_0^2} + \frac{L^2}{h_0^2}\frac{\partial^2 v^*}{\partial y^{*2}} + \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right)$$

$$\varepsilon^4 Re_L \frac{D v^*}{Dt} = \underbrace{\frac{-p_0 L^3}{\mu U h_0^2}\frac{h_0^4}{L^4}}\frac{\partial p^*}{\partial y^*} + \varepsilon^2\frac{\partial^2 v^*}{\partial y^{*2}} + \varepsilon^4\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right)$$

$$\frac{-p_0 h_0^2}{\mu U L} = \Lambda^{-1}$$

$\therefore$ $$0 = -\frac{\partial p^*}{\partial y^*}$$

note $\varepsilon^2 Re_L = .001$

room temperature 30-weight oil

with $\nu = 4 \times 10^{-4}$ m²/s

at $h_0 = .1$ mm $L = 25$ cm

$\Delta U = 10$ m/s

If external time

Scale τ = period

mechanical oscillation,

$\rho h^2/\mu\tau \ll 1$