Appendix A

SPHERICAL COORDINATES

Coordinates $\mathbf{r} = (r, \theta, \omega)$

Velocity $\mathbf{u} = (u_r, u_\theta, u_\omega)$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(u_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial u_\omega}{\partial\omega} = 0$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial\theta} + \frac{u_\omega}{r\sin\theta}\frac{\partial u_r}{\partial\omega} - \frac{u_\theta^2 + u_\omega^2}{r}\right)$$

$$= -\frac{\partial p}{\partial r} + \mu\left(\nabla^2 u_r - \frac{2}{r^2}u_r - \frac{2}{r^2}\frac{\partial u_\theta}{\partial\theta} - \frac{2}{r^2}u_\theta\cot\theta - \frac{2}{r^2\sin\theta}\frac{\partial u_\omega}{\partial\omega}\right) + \rho f_r$$

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_\omega}{r\sin\theta}\frac{\partial u_\theta}{\partial\omega} + \frac{u_r u_\theta}{r} - \frac{u_\omega^2}{r}\cot\theta\right)$$

$$= -\frac{1}{r}\frac{\partial p}{\partial\theta} + \mu\left(\nabla^2 u_\theta + \frac{2}{r^2}\frac{\partial u_r}{\partial\omega} - \frac{u_\theta}{r^2\sin^2\theta} - \frac{2}{r^2}\frac{\cot\theta}{\sin\theta}\frac{\partial u_\omega}{\partial\omega}\right) + \rho f_\theta$$

$$\rho\left(\frac{\partial u_\omega}{\partial t} + u_r\frac{\partial u_\omega}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\omega}{\partial\theta} + \frac{u_\omega}{r\sin\theta}\frac{\partial u_\omega}{\partial\omega} + \frac{u_r u_\omega}{r} + \frac{u_\theta u_\omega}{r}\cot\theta\right)$$

$$= -\frac{1}{r\sin\theta}\frac{\partial p}{\partial\omega} + \mu\left(\nabla^2 u_\omega + \frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial\omega} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u_\theta}{\partial\omega} - \frac{u_\omega}{r^2\sin^2\theta}\right) + \rho f_\omega$$

$$\rho C_v\left(\frac{\partial T}{\partial t} + u_r\frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial\theta} + \frac{u_\omega}{r\sin\theta}\frac{\partial T}{\partial\omega}\right)$$

$$= k\nabla^2 T + 2\mu\left[\left(\frac{\partial u_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_r}{r}\right)^2 + \left(\frac{1}{r\sin\theta}\frac{\partial u_\omega}{\partial\omega} + \frac{u_r}{r} + \frac{u_\theta}{r}\cot\theta\right)^2\right]$$

$$+ \mu\left\{\left[\frac{1}{r}\frac{\partial u_r}{\partial\theta} + r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right]^2 + \left[\frac{1}{r\sin\theta}\frac{\partial u_r}{\partial\omega} + r\frac{\partial}{\partial r}\left(\frac{u_\omega}{r}\right)\right]^2 + \left[\frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial\omega} + \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{u_\omega}{\sin\theta}\right)\right]^2\right\}$$

where

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\omega^2}$$

$$\tau_{rr} = 2\mu\frac{\partial u_r}{\partial r}$$

$$\tau_{\theta\theta} = 2\mu\left(\frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_r}{r}\right)$$

$$\tau_{\omega\omega} = 2\mu\left(\frac{1}{r\sin\theta}\frac{\partial u_\omega}{\partial\omega} + \frac{u_r}{r} + \frac{u_\theta\cot\theta}{r}\right)$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial\theta}\right]$$

$$\tau_{\theta\omega} = \tau_{\omega\theta} = \mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{u_\omega}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial\omega}\right]$$

$$\tau_{\omega r} = \tau_{r\omega} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{u_\omega}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial u_r}{\partial\omega}\right]$$