

$p \sin((b)x+c) - q \cos((b)x+d)$ combine as single wave

To combine

$$p \sin(bx + c) - q \cos(bx + d)$$

into a **single sinusoidal wave**, follow these steps.

Step 1: Expand using angle identities

$$\sin(bx + c) = \sin bx \cos c + \cos bx \sin c$$

$$\cos(bx + d) = \cos bx \cos d - \sin bx \sin d$$

Substitute:

$$p(\sin bx \cos c + \cos bx \sin c) - q(\cos bx \cos d - \sin bx \sin d)$$

Step 2: Collect like terms

$$= (p \cos c + q \sin d) \sin bx + (p \sin c - q \cos d) \cos bx$$

Let

$$A = p \cos c + q \sin d$$

$$B = p \sin c - q \cos d$$

So the expression becomes:

$$A \sin bx + B \cos bx$$

Step 3: Convert to single sine form

Any expression of the form:

$$A \sin bx + B \cos bx$$

can be written as:

$$R \sin(bx + \phi)$$

where

$$R = \sqrt{A^2 + B^2}$$

$$\tan \phi = \frac{B}{A}$$

Final Answer

$$p \sin(bx + c) - q \cos(bx + d) = R \sin(bx + \phi)$$

where

$$R = \sqrt{(p \cos c + q \sin d)^2 + (p \sin c - q \cos d)^2}$$

$$\phi = \tan^{-1} \left(\frac{p \sin c - q \cos d}{p \cos c + q \sin d} \right)$$

If you'd like, I can also express it in **cosine form** instead.

