The identity

$$\nabla_j A_i\, x_j = (x_j \nabla_j) A_i$$

Step 1: What does ∇_j mean?

$$\nabla_j \equiv \frac{\partial}{\partial x_j}$$

So the left-hand side is:

$$\nabla_j A_i\, x_j = \frac{\partial A_i}{\partial x_j}\, x_j$$

Step 2: Scalars commute with multiplication

Both

- x_j (a coordinate component),
- $\frac{\partial A_i}{\partial x_j}$ (a scalar field),

are **scalars**.

So:

$$\frac{\partial A_i}{\partial x_j}\, x_j = x_j \frac{\partial A_i}{\partial x_j}$$

No tricks. Just commutativity of scalar multiplication.

Step 3: Recognize the directional derivative

Now rewrite:

$$x_j \frac{\partial A_i}{\partial x_j} = (x_j \nabla_j) A_i$$

That’s exactly the definition of the directional derivative

$$(\mathbf{x} \cdot \nabla) A_i$$

Important subtlety (why this *doesn’t* always work)

This equality holds **because the derivative acts only on** A_i.

If the derivative acted on the product, you’d get:

$$\nabla_j(A_i x_j) = x_j \nabla_j A_i + A_i \nabla_j x_j$$

⚠️ That is **not** what you wrote.

Bottom line (clean takeaway)

$$\boxed{\nabla_j A_i \, x_j = x_j \nabla_j A_i = (x_j \nabla_j) A_i}$$

Because:

- $\nabla_j A_i$ is a scalar,
- scalars commute with x_j,
- and the derivative acts only on A_i.

This is exactly the step used when rewriting

$$(\nabla \mathbf{A})\,\mathbf{x} \;\leftrightarrow\; (\mathbf{x} \cdot \nabla)\mathbf{A}$$

in continuum mechanics and CFD.

If you want, next we can:

- contrast this with $\nabla \cdot (\mathbf{x}A)$,
- or show why $(\mathbf{u} \cdot \nabla)\mathbf{u}$ is *not* a divergence,
- or do a **2D index-by-index expansion**.