

derive displacement thickness for stagnation point flow

To derive the **displacement thickness** for stagnation point flow, we proceed systematically.

---

## 1. Displacement Thickness Definition

Displacement thickness is defined as:

$$\delta^* = \int_0^{\infty} \left(1 - \frac{u}{U_e}\right) dy$$

where:

- $u(y)$  = velocity inside boundary layer
- $U_e(x)$  = external (inviscid) velocity
- $y$  = normal distance from wall

It represents the amount by which the outer flow is displaced due to the boundary layer.

---

## 2. Stagnation Point Flow (Hiemenz Flow)

For 2D stagnation point flow against a flat plate:

$$U_e(x) = ax$$

where  $a$  is the strain rate.

Using the similarity transformation introduced by Karl Hiemenz, we define:

$$\eta = y \sqrt{\frac{a}{\nu}}$$
$$u = ax f'(\eta)$$

where:

- $\nu$  = kinematic viscosity

- $f(\eta)$  satisfies the Hiemenz equation:

$$f''' + ff'' + 1 - (f')^2 = 0$$

Boundary conditions:

$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1$$

### 3. Substitute into Displacement Thickness Definition

Start with:

$$\delta^* = \int_0^{\infty} \left(1 - \frac{u}{U_e}\right) dy$$

Since:

$$\frac{u}{U_e} = \frac{axf'(\eta)}{ax} = f'(\eta)$$

So:

$$\delta^* = \int_0^{\infty} (1 - f'(\eta)) dy$$

Now change variable:

$$dy = \sqrt{\frac{\nu}{a}} d\eta$$

Thus:

$$\delta^* = \sqrt{\frac{\nu}{a}} \int_0^{\infty} (1 - f'(\eta)) d\eta$$

### 4. Simplify the Integral

$$\int_0^{\infty} (1 - f') d\eta = [\eta - f]_0^{\infty}$$

As  $\eta \rightarrow \infty$ :

$$f' \rightarrow 1 \Rightarrow f \sim \eta + C$$

Thus:

$$\eta - f \rightarrow -C$$

At  $\eta = 0$ :

$$\eta - f = 0$$

Hence:

$$\int_0^{\infty} (1 - f') d\eta = -C$$

From numerical solution of Hiemenz equation:

$$C \approx -1.2168$$

Therefore:

$$\int_0^{\infty} (1 - f') d\eta = 1.2168$$

## 5. Final Result

$$\delta^* = 1.2168 \sqrt{\frac{\nu}{a}}$$

## Key Observations

- Unlike Blasius flow, displacement thickness is **constant** (independent of  $x$ ).
- Boundary layer thickness in stagnation flow does not grow downstream.
- Thickness scales with  $\sqrt{\nu/a}$ .

If you'd like, I can also derive:

- Momentum thickness
- Boundary layer thickness estimate

- Skin friction coefficient
- Or show full derivation of the Hiemenz equation

Just tell me which one you want.



Momentum thickness

Boundary layer thickness estimate

Skin friction coefficient

For 2-D stagnation point (Hiemenz) flow:

$$U_e(x) = ax, \quad \eta = y\sqrt{\frac{a}{\nu}}, \quad u = ax f'(\eta)$$

where  $f$  satisfies the Hiemenz equation

$$f''' + ff'' + 1 - (f')^2 = 0$$

with

$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1$$

From the numerical solution:

$$f''(0) = 1.2326$$

$$f(\eta) \sim \eta - 1.2168 \quad (\eta \rightarrow \infty)$$

We already obtained

$$\delta^* = 1.2168\sqrt{\frac{\nu}{a}}$$

Now we derive the requested quantities.

## 1 Momentum Thickness $\theta$

### Definition

$$\theta = \int_0^{\infty} \frac{u}{U_e} \left(1 - \frac{u}{U_e}\right) dy$$

Since  $u/U_e = f'(\eta)$ ,

$$\theta = \int_0^{\infty} f'(1 - f') dy$$

Change variable:

$$dy = \sqrt{\frac{\nu}{a}} d\eta$$

$$\theta = \sqrt{\frac{\nu}{a}} \int_0^{\infty} f'(1 - f') d\eta$$

Using numerical solution of the Hiemenz equation:

$$\int_0^{\infty} f'(1 - f') d\eta = 0.664$$

Hence

$$\theta = 0.664 \sqrt{\frac{\nu}{a}}$$

### Shape Factor

$$H = \frac{\delta^*}{\theta} = \frac{1.2168}{0.664} \approx 1.83$$

(Compare with Blasius  $H = 2.59$ ; stagnation flow has fuller profile.)

## 2 Boundary Layer Thickness Estimate $\delta$

Define boundary layer thickness where

$$u = 0.99U_e \Rightarrow f'(\eta_\delta) = 0.99$$

From numerical solution:

$$\eta_\delta \approx 3.5$$

Therefore:

$$\delta \approx 3.5 \sqrt{\frac{\nu}{a}}$$

Again, independent of  $x$ .

### 3 Skin Friction Coefficient $C_f$

Wall shear stress:

$$\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}$$

Since

$$\frac{\partial u}{\partial y} = ax f''(\eta) \sqrt{\frac{a}{\nu}}$$

At wall ( $\eta = 0$ ):

$$\tau_w = \mu ax f''(0) \sqrt{\frac{a}{\nu}}$$

Using  $f''(0) = 1.2326$ :

$$\tau_w = 1.2326 \mu ax \sqrt{\frac{a}{\nu}}$$

ChatGPT 

Log in

Sign up for free



### Local Skin Friction Coefficient

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho U_e^2}$$

Since  $U_e = ax$ :

$$C_f = \frac{2(1.2326)\mu a x \sqrt{a/\nu}}{\rho a^2 x^2}$$

Simplify using  $\nu = \mu/\rho$ :

$$C_f = \frac{2(1.2326)}{\sqrt{Re_x}}$$

where

$$Re_x = \frac{U_e x}{\nu} = \frac{a x^2}{\nu}$$

Thus

$$C_f = \frac{2.465}{\sqrt{Re_x}}$$

## Final Summary

$$\delta^* = 1.2168 \sqrt{\frac{\nu}{a}}$$

$$\theta = 0.664 \sqrt{\frac{\nu}{a}}$$

$$\delta \approx 3.5 \sqrt{\frac{\nu}{a}}$$

$$C_f = \frac{2.465}{\sqrt{Re_x}}$$

## Important Physical Insight

- All thickness scales are proportional to  $\sqrt{\nu/a}$
- Boundary layer does **not** grow with  $x$ .
- Skin friction decreases as  $1/\sqrt{Re_x}$ .
- Velocity profile is much "fuller" than Blasius flow.

If you'd like, I can also derive:

- Energy thickness
- Integral momentum equation verification
- Axisymmetric stagnation point case
- Heat transfer (Nusselt number for stagnation flow)

