

## Görtler Instabilities

In fluid dynamics, Görtler vortices are secondary flows that appear in a boundary layer flow along a concave wall. If the boundary layer is thin compared to the radius of curvature of the wall, the pressure remains constant across the boundary layer. On the other hand, if the boundary layer thickness is comparable to the radius of curvature, the centrifugal action creates a pressure variation across the boundary layer. This leads to the centrifugal instability (Görtler instability) of the boundary layer and consequent formation of Görtler vortices.

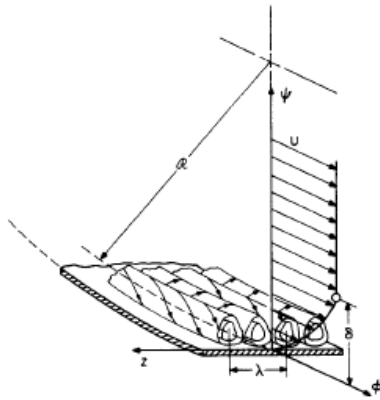


FIG. 1. Vortexlike disturbances in a boundary-layer flow over a curved wall with the axis of the vortices parallel to the principal flow direction.

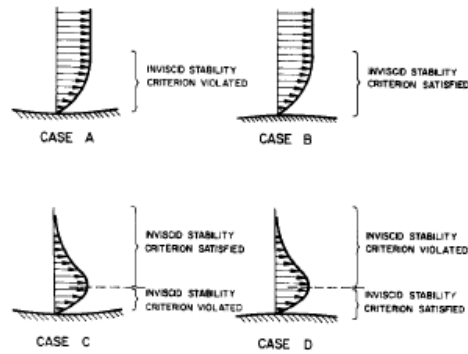


FIG. 3. Shear layers and their stability according to the inviscid stability criterion. Flows in cases A and B correspond to the Blasius boundary layer while flows in cases C and D correspond to a wall jet. Cases A, C, and D are potentially unstable, case B is stable.

An inviscid stability criterion was developed by Floryan (1986) to identify boundary layer and wall jet flows which are potentially unstable with respect to the Görtler type disturbances as follows: a flow is stable if velocity magnitude decrease everywhere with distance away from the wall in the case of a concave wall, and increase in the case of convex wall. The inviscid stability criterion can be represented as

$$dV_{\theta}^2/dr < 0 \quad \text{stable for concave wall}$$

$$dV_{\theta}^2/dr > 0 \quad \text{stable for convex wall}$$

wherein  $r$  is the radius in the polar coordinate system with origin at the center of curvature of the flow boundary, and the  $V_{\theta}$  is tangential velocity component.

Floryan and Saric (1982) provide a viscous instability model for Görtler vortices in which the Görtler number for wall jet flows is defined as

$$G = \frac{U_s \delta}{\nu} \left( \frac{\delta}{R} \right)^{\frac{1}{2}} \quad (27)$$

Where  $U_s$  is the maximal velocity in the velocity profile of the wall jet;  $\delta$  is the boundary layer thickness;  $\nu$  is the kinematic viscosity and  $R$  is the radius of curvature of the wall. The wave number of the Görtler type disturbances with wave length  $\lambda_V$  is defined as

$$\alpha = 2\pi\delta/\lambda_V \quad (28)$$

The critical Görtler number that is used as stability criterion is determined by the wave number of the Görtler type disturbances in different cases of flow (e.g. wall jet and boundary layer) and curved surface (e.g. convex and concave).

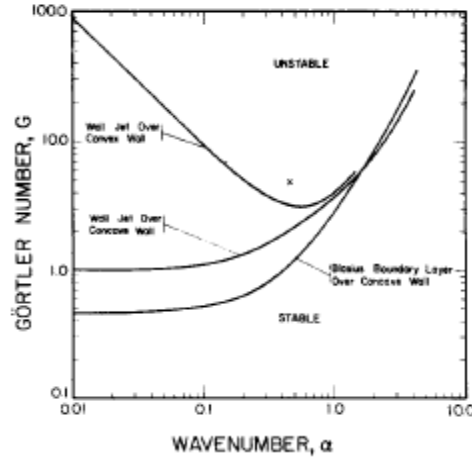


FIG. 4. Neutral stability curves for the Blasius boundary layer over a concave wall and wall jet over concave and convex walls. The cross denotes conditions under which the disturbance velocity fields to be displayed in Fig. 6 were determined.

L., Gui, H., Yoon and F., Stern, “Techniques for measuring bulge-scar pattern of free surface deformation and related velocity distribution in shallow water flow over a bump,” *Experiments in Fluids*, 55 (4), 1-15, 2014.

Wang, Z., Yang, J., and Stern, F., “High-fidelity simulations of bubble, droplet, and spray formation in breaking waves, *JFM*, 2016, vol. 792, pp. 307-327.

Timur Kent Dogan, Zhaoyuan Wang and Frederick Stern, “DNS of Air-Water Interface Instabilities for Flow over a Bump in a Shallow Water Flume,” *Symposium on Naval Hydrodynamics Osaka, Japan*, 31 May – 5 June 2020