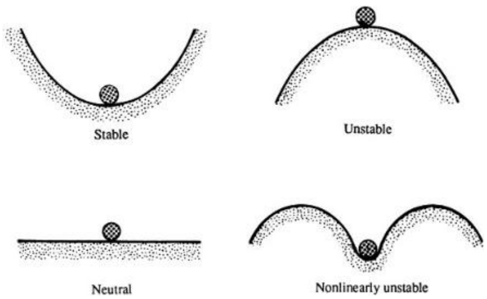
Chapter 5 Introduction to The Stability of Laminar Flows

1. Introduction
   a. Basic Concepts

For any given laminar flow, if Re is greater than $Re_c \approx 1000$ (when referred to a transverse thickness) the flow will become turbulent, ie, disorderly, randomly unsteady, apparently impossible to analyze exactly, but fortunately amendable to study of its average values. Almost all practical engineering flows are, in fact, turbulent. The two key concepts in this chapter are stability and transition. Stability: can a given physical state withstand a disturbance and still return to its original state. Transition: the change over space and time and a certain Re range of a laminar flow into a turbulent flow. Note that stability theory predicts the smallest Re at which disturbances can be amplified which should not be confused with the point of transition to turbulence ($X_{trans} \approx$ 10-20 times the distance x_{crit}).



Stability vs. Instability

Top left: stable

Top right: unstable

Bottom right: neutrally stable

Bottom right: stable small but unstable large disturbances

Broadly speaking, there are two types of stability and transition for laminar flows:

(1) Transition from laminar-to-laminar flow
(2) Transition from laminar to turbulent flow

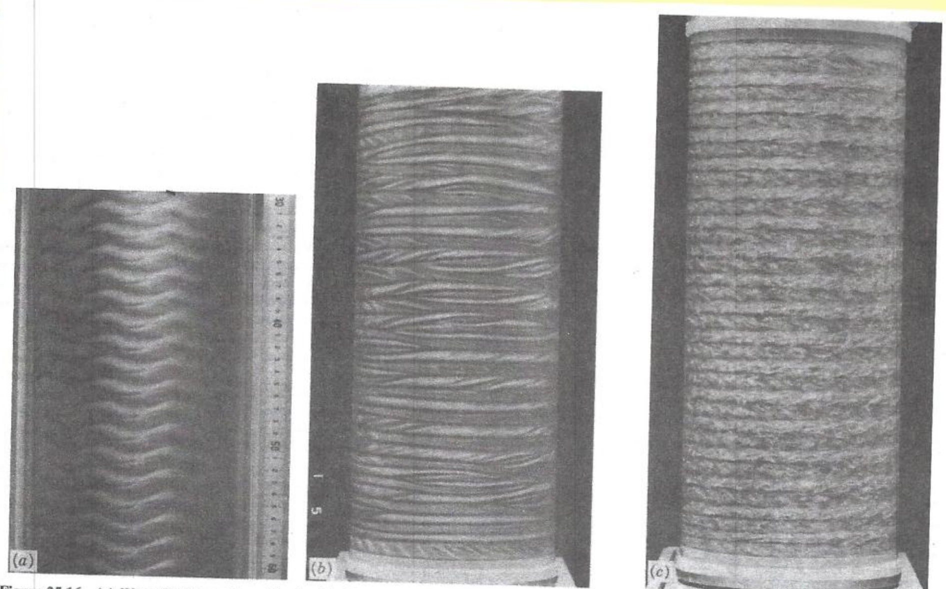


Figure 25.16 (*a*) Wavy Taylor vortices. Reprinted with permission from Koschmieder (1979). (*b*) Braided Taylor vortices. From Andereck et al. (1983). (*c*) Turbulent Taylor vortices. Courtesy of Zhang and Swinney (1985), University of Texas. Reprinted with permission.

b. Historical Background

Historical Perspective:

Researcher	Year	Result
Reynolds	1883	Classical pipe flow exp. showed that transition was the result of instability of the laminar flow & Re was the criterion for the onset of instability
Rayleigh	1879	showed theoretically that a necessary condition for inviscid (or dynamic) instability is an inflection point in the velocity profile
Orr Sommerfeld	1907 1908	independently derived the complete stability equation which governs the behavior of small disturbances.
Tollmien	1929	First to solve Orr-Sommerfeld eq. and showed that the Blasius profile is unstable for $Re > Re_c$ inspite of the absence of an inflection point. The destabilizing effect of viscosity at $Re > Re_c$ evidently due to the transfer of energy from the mean flow to a disturbance
Schlichting	~1940	Re_c established for a wide variety of velocity profiles: $Re_c \approx 100$ separation profiles; ~600 Blasius; and ~70,000 asymptotic suction
Schubauer & Skramstad (WBS)	1943	experimental verification of Tollmien-Schlichting Stability theory
Lin, Lee, etc	1950-1960	critically examined & improved theory

Aero,

Gortler (1938): dynamic instability on concave surfaces

RAE (1952): Cross flow instability

Present Research: non linear theory, 3D flow, high speed flow, hypersonic flow

c. Small Disturbance Stability Analysis

All small-disturbance stability analyses follow the same general line of attack, which may be listed in seven steps.

1. We seek to examine the stability of a basic solution to the physical problem, Q_0, which may be a scalar or vector function.
2. Add a disturbance variable Q' and substitute $(Q_0 + Q')$ into the basic equations which govern the problem.
3. From the equation(s) resulting from step 2, subtract the basic terms that Q_0 satisfies identically. What remains is the *disturbance equation.*
4. Linearize the disturbance equation by assuming *small* disturbances, that is, $Q' \ll Q_0$, and neglect terms such as Q'^2 and Q'^3, etc.
5. If the linearized disturbance equation is complicated and multidimensional, it can be simplified by assuming a form for the disturbances, such as a traveling wave or a perturbation in only one direction.
6. The linearized disturbance equation should be homogeneous and have homogeneous boundary conditions. It can thus be solved only for certain specific values of the equation's parameters. In other words, it is an *eigenvalue* problem.
7. The eigenvalues found in step 6 are examined to determine when they grow (are unstable), decay (are stable), or remain constant (neutrally stable). Typically the analysis ends with a chart showing regions of stability separated from unstable regions by *neutral curves.*