
The exam is closed book and closed notes.

Suppose you buy a $1.22 \times 1.22 \ m$ sheet of plywood and put it on your roof rack, as in the figure. You drive home at 20 km/h. (a) If the board is perfectly aligned with the airflow, how thick is the boundary layer at the end? (Assume that the flow is laminar for $Re < 5E5$ and turbulent for larger Re.) (b) Estimate the drag force on both sides of the plate. (c) What is the air velocity at a point 1.45 mm normal to the end of the plate?

$\rho = 1.2 kg/m^3$
$\mu = 1.8 \times 10^{-5} kg/m \cdot s$
$1 km/h = 0.278 m/s$

1.22m
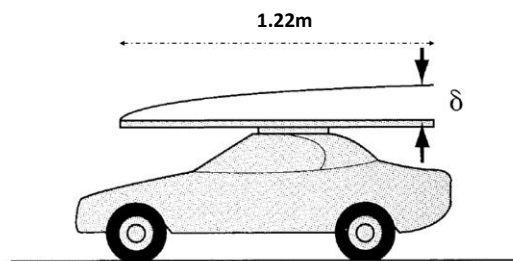
Equations:

- Laminar Boundary Layer: $c_f = \dfrac{2\tau_w}{\rho U^2} = \dfrac{0.664}{Re_x^{1/2}}$; $C_D = \dfrac{D}{\frac{1}{2}\rho A U^2} = \dfrac{1.328}{Re_L^{1/2}}$; $\dfrac{\delta}{x} \approx \dfrac{5.0}{Re_x^{1/2}}$; velocity profile

given in the Table below:

$y[U/(\nu x)]^{1/2}$	u/U	$y[U/(\nu x)]^{1/2}$	u/U
0.0	0.0	2.8	0.81152
0.2	0.06641	3.0	0.84605
0.4	0.13277	3.2	0.87609
0.6	0.19894	3.4	0.90177
0.8	0.26471	3.6	0.92333
1.0	0.32979	3.8	0.94112
1.2	0.39378	4.0	0.95552
1.4	0.45627	4.2	0.96696
1.6	0.51676	4.4	0.97587
1.8	0.57477	4.6	0.98269
2.0	0.62977	4.8	0.98779
2.2	0.68132	5.0	0.99155
2.4	0.72899	∞	1.00000
2.6	0.77246		

Solution:

(a) Calculate Reynolds number

$$20 \ km/h = 5.56 m/s$$

$$Re = \frac{\rho U L}{\mu} = \frac{1.2(5.56)(1.22)}{1.8 \times 10^{-5}} = 4.52 \times 10^5 \qquad \text{(+3)}$$

$$\rightarrow \text{Laminar Flow}$$

$$\therefore \frac{\delta}{x} = \frac{\delta}{1.22} = \frac{5}{\sqrt[2]{4.52 \times 10^5}} = 0.0074m \qquad \text{(+3)}$$

$$\delta = 0.009 \text{m} = 9 \text{mm}$$

(b) Drag force – laminar condition

$$F_{laminar} = C_D \frac{\rho}{2} U^2 A = \frac{1.328}{\sqrt{Re}} \frac{\rho}{2} U^2 A$$

$$= \frac{1.328}{\sqrt{4.52 \times 10^5}} \frac{1.2}{2}(5.56)^2(1.22 \times 1.22 \times 2) = 0.109N \qquad \text{(+2.5)}$$

(c) At $y = 1.45 mm$,

$$\eta = y\sqrt{\frac{U}{\nu x}} = (0.00145m)\sqrt{\frac{5.56}{(1.5E-5)(1.22)}} = 0.8 \qquad \text{(+1.5)}$$

From the table: at $\eta = 0.8$, read $u/U \sim 0.265$, hence $u = 0.265 \times 5.56 \sim 1.47 m/s$