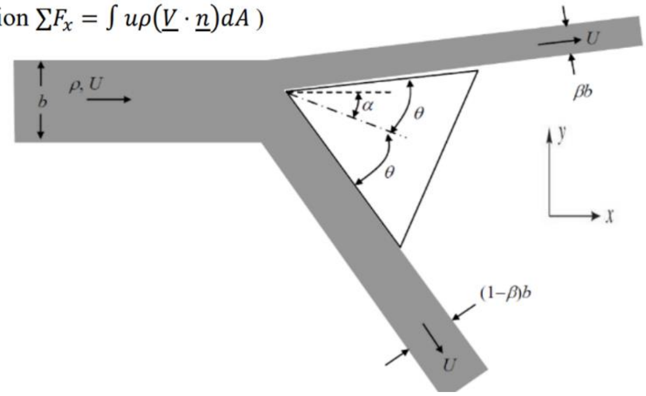
Name: ------------------ **Final Exam** **Time: 120 minutes**

ME:5160 **Fall 2024**

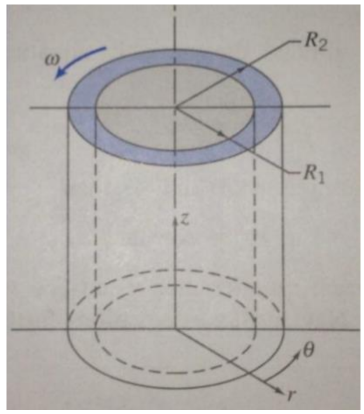
The exam is closed book and closed notes.

1. A wedge with a vertex angle of $2\theta = 60°$ is inserted at an angle of attack $\alpha = 20°$ into a jet of water of density $\rho = 1000\ kg/m^3$, width $b = 30\ cm$, and velocity $U = 0.5$ m/s, as shown in the figure below. After impinging on the wedge, the single incident jet is divided into two jets, both of which leave the back edge of the wedge with velocity U. The widths of the two departing jets are βb and $(1-\beta)b$, as indicated in the figure ($0 \le \beta \le 1$). Assume that the flow is planar with a depth $d = 50$ cm normal to the sketch, gravity may be neglected, and the pressure in the surrounding air is everywhere atmospheric. **(a)** Calculate the drag and lift forces acting on the wedge for $\beta = 0.3$. **(b)** Find the value of β for which the lift is zero when all other parameters are fixed at the above values.

(Hint : Momentum Equation $\sum F_x = \int u\rho(\underline{V} \cdot \underline{n})dA$)



2. A Newtonian fluid of density ρ and viscosity μ fills the annular gap between concentric cylinders as shown blow. The inner cylinder of radius R_1 is stationary, and the outer cylinder of radius R_2 rotates steadily at a constant angular speed ω. The flow between the cylinders is laminar and incompressible. Assume steady, purely circulating motion, 2D flow, circumferentially symmetric pressure, and neglect gravity. Simplify the continuity and momentum equations and use appropriate boundary conditions to determine the velocity distribution in the gap.



The equations of motion of an incompressible Newtonian fluid with constant density and viscosity in cylindrical coordinates (r, θ, z) with velocity components (v_r, v_θ, v_z):

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta)+\frac{\partial}{\partial z}(v_z)=0$$

r-momentum:

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\theta^2}{r}\right)=\rho g_r-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]$$

θ-momentum:

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+v_z\frac{\partial v_\theta}{\partial z}+\frac{v_r v_\theta}{r}\right)=\rho g_\theta-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]$$

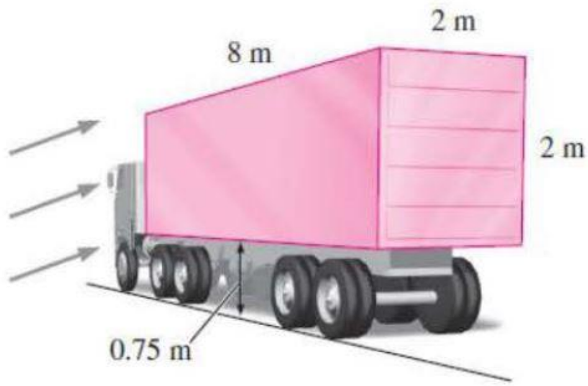
z-momentum:

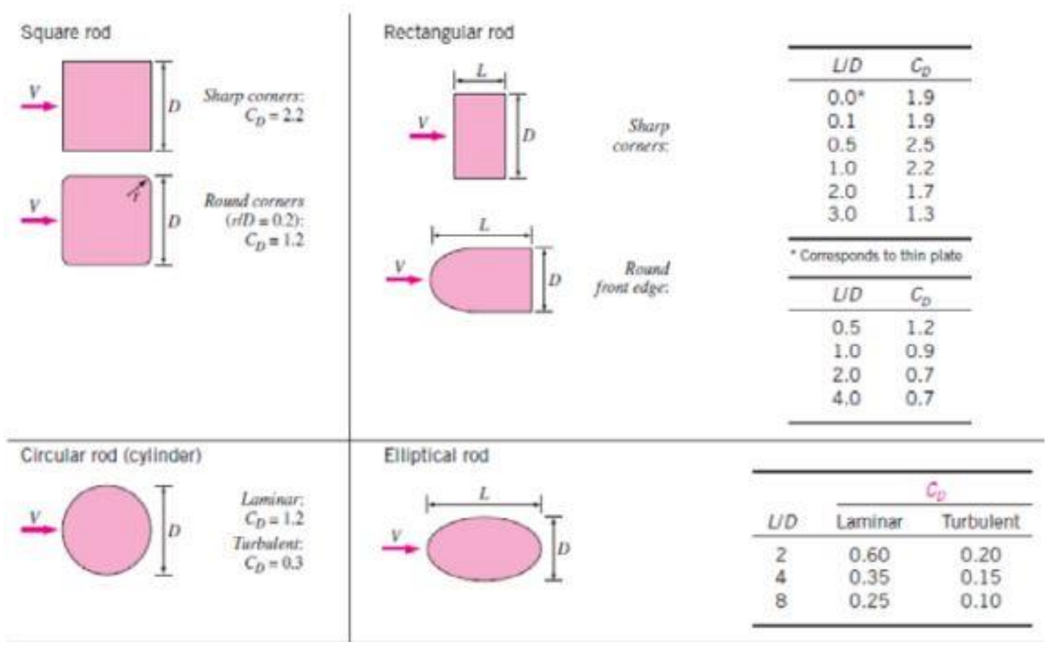
$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=\rho g_z-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$$

---

3. During major windstorms, high vehicles such as RVs and semis may be thrown off the road, especially when they are empty and in open areas. Consider a 5000-kg semi that is 8m long, 2m high, and 2m wide. The distance between the bottom of the truck and the road is 0.75m. The truck is exposed to winds from its side surface. (a) Determine the wind velocity that will tip the truck over to its side. Take the air density to be 1.1 kg/m^3 and assume the weight to be uniformly distributed. (b) If the wind arrives with a 45° angle on the side of the truck, does the tip velocity increase or decrease (suppose same C_D)? Why? (c) If you are to repeat the experiment for a 2 times smaller model using Reynolds similarity and the new semi weights 2000-kg, determine the new tipping velocity (consider that the distance between the bottom of the truck and the road is 0.5m).

Hint: $C_D = \frac{D}{0.5\rho A V^2}$ (use square rod) $Re_{full\ scale} = \frac{\rho U(2m)}{\mu}$



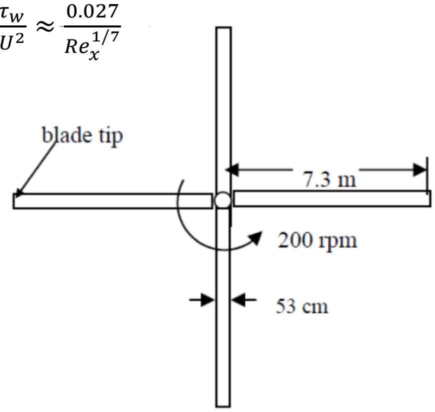


---

4. A helicopter rotor rotates at ω =20.94 rad/s in air (ρ=1.2 kg/m^3 and μ=1.8E-5 kg/m-s). Each blade has a chord length of 53 cm and extends a distance of 7.3 m from the center of the rotor hub. Assume that the blades can be modeled as very thin flat plates at a zero angle of attack. (a) At what radial distance from the hub center is the flow at the blade trailing edge turbulent (Re_{crit} = 5E5). (b) Find the boundary layer thickness at the blade tip trailing edge (c) At what rotor angular velocity does the wall shear stress at the blade tip trailing edge become 80 N/m^2?

Hint : $Re = \frac{\rho U C}{\mu}$ where $U = r\omega\,,\ C{:}\,Chord\ Length$

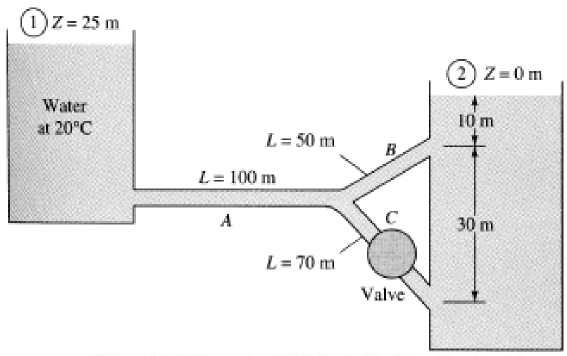
Turbulent BL : $\frac{\delta}{x} \approx \frac{0.16}{Re^{1/7}}\,,\ c_f = \frac{2\tau_w}{\rho U^2} \approx \frac{0.027}{Re_x^{1/7}}$

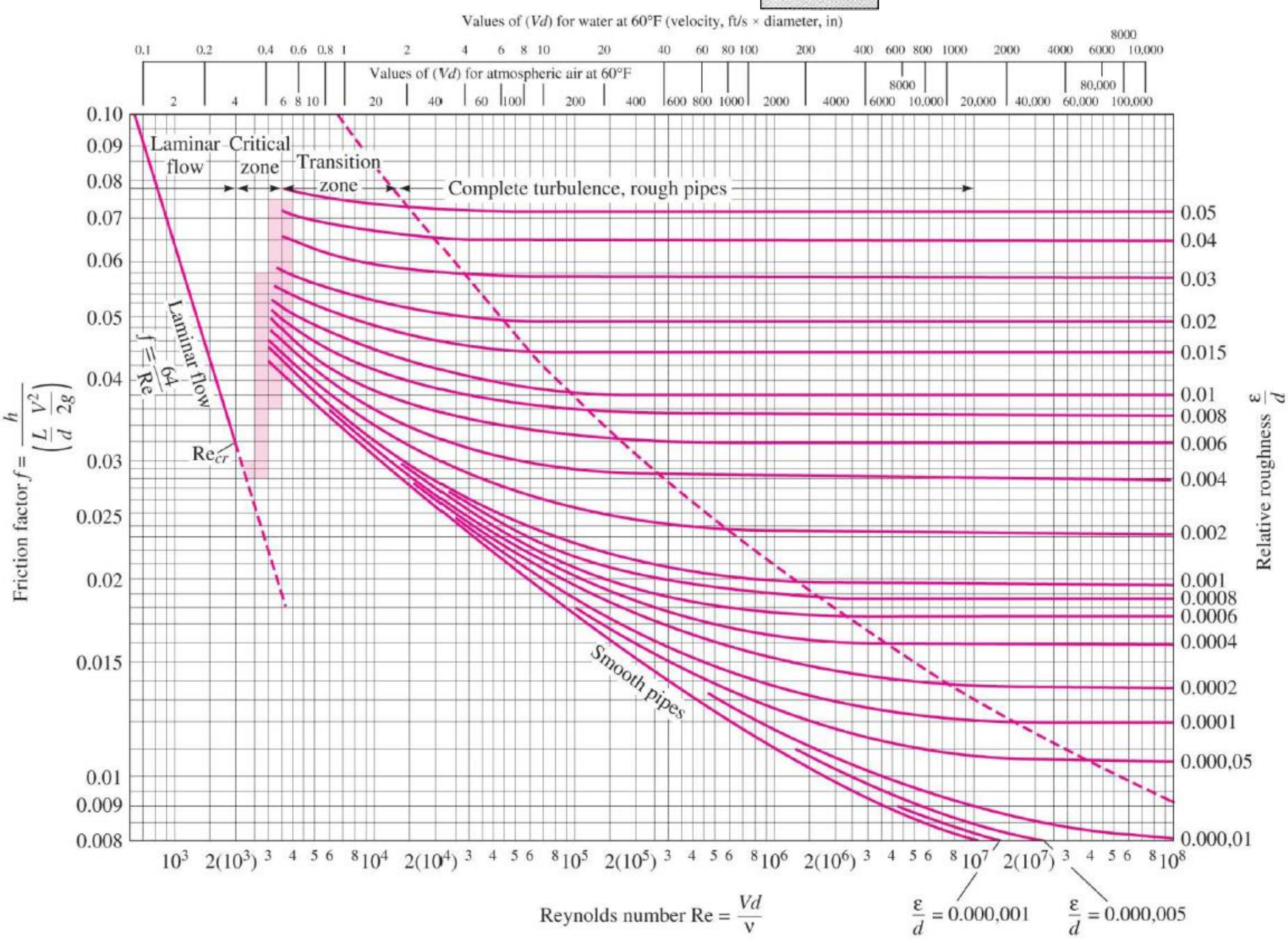


---

5. In the figure below, all pipes are 8 *cm*-diameter cast iron (ε = 0.26 *mm*). The fluid is water at 20˚C (ρ = 998 kg/m^3, μ = 0.001 *kg/ms*). Minor loss coefficients are: K_1 = 0.5 for the sharp entrance at A; K_2 = 0.9 for the line-type junction from A to B; K_3 = 1.3 for the branch-type junction from A to C; K_4 = 1.0 for the submerged exits in B and C; K_{valve} = 0.5. **(a)** Determine velocity in pipe A (V_A) if valve C is closed (use f = 0.02 as initial guess). **(b)** If valve C is open, set up the system of equations for the pipe network as a function of the variables V_A, V_B, V_C, f_A, f_B, and f_C. **(c)** Calculate V_A if V_C = 1.57 *m/s* and friction factors are the same in all pipes and equal to the one found in part (a).

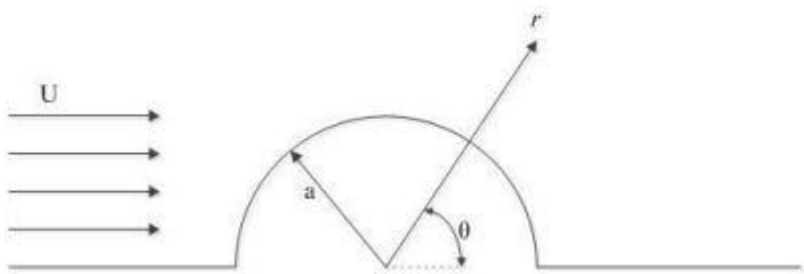
Hint: Energy equation $\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_1 = \left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_2 + h_f$





---

6. A practice facility for the Cal football team is to be covered by a semi-circular cylindrical dome, as shown in the sketch. The dome with radius *a* is placed normal to the cross flow of velocity *U*. Assume that the dome is very long (into the page) so you can ignore end effects.



a. The stream function and velocity potential for this flow $0 \le \theta \le \pi$ are given by

$$\Psi = Ur\left(1 - \frac{a^2}{r^2}\right)\sin\theta$$

$$\Phi = Ur\left(1 + \frac{a^2}{r^2}\right)\cos\theta$$

Use potential flow theory to find the tangential velocity along the surface of the dome $(r = a)$ as a function of the angle θ.

Hint: $u_r = \frac{\partial \Phi}{\partial r}, \quad u_\theta = \frac{1}{r}\frac{\partial \Phi}{\partial \theta}$

b. Choose a reference condition to be $p = p_0$ at z=0 (on the ground) far from the cylindrical dome. Use the Bernoulli equation to find the pressure field on the surface of the dome $(r = a)$ as a function of θ. Do NOT neglect hydrostatic variations.

c. If the vertical force on the dome per unit length (upwards) is given by $F_{lift} = \frac{5}{3}\rho U^2 a$, find the mass per unit length (m_c) required to keep the cylindrical dome on the ground when the wind blows at 30m/s, with air density ρ=1.2kg/m^3, and *a*=50m.

---

1. Solution

(a) Continuity

$$Q_{in} = Q_{out_1} + Q_{out_2}$$

$$bdU = \beta bdU + (1-\beta)bdU = bdU(\beta + 1 - \beta) \text{ OK, continuity satisfied}$$

$$\dot{m}_{in} = \rho bdU = (1000)(0.3)(0.5)(0.5) = 75\ kg/s$$

$$\dot{m}_{out_1} = \beta \dot{m}_{in} = (0.3)(75) = 22.5\ kg/s$$

$$\dot{m}_{out_2} = (1-\beta)\dot{m}_{in} = (1-0.3)(75) = 52.5\ kg/s$$

+3

x - Momentum:

$$\sum F_x = \dot{m}_{out_1} u_{out_1} + \dot{m}_{out_2} u_{out_2} - \dot{m}_{in} u_{in}$$

$$-D = \dot{m}_{out_1}[U\cos(\theta-\alpha)] + \dot{m}_{out_2}[U\cos(\theta+\alpha)] - \dot{m}_{in} U$$

$$D = U\left[\dot{m}_{in} - \dot{m}_{out_1}\cos(\theta-\alpha) - \dot{m}_{out_2}\cos(\theta+\alpha)\right]$$

$$D = (0.5)[(75) - (22.5)(\cos 10°) - (52.5)(\cos 50°)] = 9.55\ N$$

+2

y - Momentum:

$$\sum F_y = \dot{m}_{out_1} v_{out_1} + \dot{m}_{out_2} v_{out_2} - \dot{m}_{in} v_{in}$$

$$-L = \dot{m}_{out_1}[U\sin(\theta-\alpha)] + \dot{m}_{out_2}[-U\sin(\theta+\alpha)] - \dot{m}_{in}(0)$$

$$L = U\left[\dot{m}_{out_2}\sin(\theta+\alpha) - \dot{m}_{out_1}\sin(\theta-\alpha)\right]$$

$$L = (0.5)[(52.5)(\sin 50°) - (22.5)(\sin 10°)] = 18.16\ N$$

+2

(b)

$$L = U\left[\dot{m}_{out_2}\sin(\theta+\alpha) - \dot{m}_{out_1}\sin(\theta-\alpha)\right] = 0$$

$$\dot{m}_{out_2}\sin(\theta+\alpha) = \dot{m}_{out_1}\sin(\theta-\alpha)$$

$$[(1-\beta)\dot{m}_{in}]\sin(\theta+\alpha) = [\beta\dot{m}_{in}]\sin(\theta-\alpha)$$

$$(1-\beta)\sin(50°) = \beta\sin(10°)$$

+2

$$\frac{1-\beta}{\beta} = \frac{\sin(10°)}{\sin(50°)} \approx 0.23$$

$$\frac{1}{\beta} - 1 = 0.23$$

$$\beta = \frac{1}{1+0.23} = 0.81$$

+1

---

2. Solution

ASSUMPTIONS: **(1)**

1. Steady flow ($\frac{\partial}{\partial t} = 0$)
2. Purely circumferential flow (v_r=v_z=0)
3. 2D flow ($\frac{\partial}{\partial z} = 0$)
4. circumferentially symmetric pressure ($\frac{\partial p}{\partial \theta} = 0$)
5. Negligible gravity

ANALYSIS:

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

$$0(2) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + 0(2) = 0$$

$$\frac{\partial v_\theta}{\partial \theta} = 0 \quad (6)$$

(2)

θ-momentum:

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = \rho g_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]$$

$$\rho(0(1) + 0(2) + 0(6) + 0(2,3) + 0(2)) = 0(5) - 0(4) + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + 0(6) + 0(3) + 0(2)\right]$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) = 0$$

(2)

Integrate:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta) = C_1$$

$$\frac{\partial}{\partial r}(rv_\theta) = C_1 r$$

(1)

Integrate again:

$$rv_\theta = C_1\frac{r^2}{2} + C_2$$

$$v_\theta = C_1\frac{r}{2} + \frac{C_2}{r}$$

(1)

B.C.:

1) At $r = R_1$, $v_\theta = 0$ **(0.75)**

2) At $r = R_2$, $v_\theta = R_2\omega$ **(0.75)**

Replace:

$$0 = C_1 \frac{R_1}{2} + \frac{C_2}{R_1}$$

$$R_2\omega = C_1 \frac{R_2}{2} + \frac{C_2}{R_2}$$

And find:

$$C_1 = \frac{2\omega}{1 - \left(\frac{R_1}{R_2}\right)^2}$$ **(0.25)**

$$C_2 = \frac{-\omega {R_1}^2}{1 - \left(\frac{R_1}{R_2}\right)^2}$$ **(0.25)**

Substitute into the expression for v_θ:

$$v_\theta = \frac{\omega R_1}{1 - \left(\frac{R_1}{R_2}\right)^2}\left(\frac{r}{R_1} - \frac{R_1}{r}\right)$$ **(1)**

Solution 3:

(a) From table: L/D=2/2 → $C_D = 2.2$ **(2)**

When the truck is first tipped, the wheels on the wind-loaded side of the truck will be off the ground, and thus all reaction forces from the ground will act on wheels on the other side. Taking the moment about an axis passing through these wheels and setting it equal to zero gives the required frag force and consequently the wind speed:

$$\sum M = 0 \rightarrow M_{drag} - M_{weight} = 0$$ **(1)**

$$M_{drag} = F_D \times arm_{drag}$$ **(1)**

$$F_D = 0.5\rho A V^2 C_D = 0.5(1.1)(8)(2)V^2(2.2) = 19.36V^2$$ **(1)**

$$arm_{drag} = \frac{2}{2} + 0.75 = 1.75m$$

$$M_{weight} = mg \times arm_{weight}$$ **(1)**

$$arm_{weight} = \frac{2}{2} = 1$$

$$\rightarrow 19.36V^2(1.75) - 49050(1) = 0$$

$$\rightarrow V = 38.05m/s$$ **(1)**

(b)

It the wind arrives with a 45° angle, the normal area decreases, reducing F_D and increasing the tipping velocity. **(1)**

(c)

Reynolds similarity:

$$\frac{\rho V L}{\mu} = \frac{\rho V_m L_m}{\mu}$$ **(1)**

$$V_m = \frac{VL}{L_m} = \frac{2V}{1} = 2V$$

$$M_{drag} = F_D \times arm_{drag}$$

$$F_D = 0.5\rho A V^2 C_D = 0.5(1.1)(4)(1)(2V)^2(2.2) = 19.36V^2$$

$$arm_{drag} = \frac{1}{2} + 0.5 = 1m$$

$$M_{weight} = mg \times arm_{weight}$$ **(1)**

$$arm_{weight} = \frac{1}{2} = 0.5$$

$$\rightarrow 19.36V^2(1) - 19620(0.5) = 0$$

$$\rightarrow V = 22.51 m/s \quad \textbf{(1)}$$

Solution 4:

a)

$$U = r\omega$$

$$Re = \frac{\rho U c}{\mu} = \frac{\rho r \omega c}{\mu} \quad \textbf{(1)}$$

$$Re_{crit} = \frac{\rho r_{crit} \omega c}{\mu}$$

$$r_{crit} = \frac{Re_{crit}\mu}{\rho \omega c} = \frac{(5E5)(1.8E-5)}{(1.2)(20.94)(0.53)} = 0.68\ m \quad \textbf{(1)}$$

b) At the tip trailing edge:

$$Re = \frac{\rho r_{tip} \omega c}{\mu} = \frac{(1.2)(7.3)(20.94)(0.53)}{(1.8E-5)} = 5{,}401{,}124 \quad \textbf{(1)}$$

$$\frac{\delta}{x} \approx \frac{0.16}{Re^{1/7}} \quad \textbf{(1)}$$

$$\delta \approx \frac{0.16(0.53)}{(5401124)^{\frac{1}{7}}} = 0.00926\ m = 9.26\ mm \quad \textbf{(2)}$$

c)

$$\tau_w = 0.0135\frac{\rho U^2}{Re_x^{1/7}} = \frac{0.0135\mu^{1/7}\rho^{6/7}U^{13/7}}{x^{1/7}} \quad \textbf{(2)}$$

Re-arrange to find *U*:

$$U = r_{tip}\omega = \left(\frac{\tau_w\, x^{1/7}}{0.0135\mu^{1/7}\rho^{6/7}}\right)^{7/13} \quad \textbf{(1)}$$

$$\omega = \frac{1}{(7.3)}\left(\frac{(80.0)(0.53)^{1/7}}{0.0135\,(1.8E-5)^{1/7}(1.2)^{6/7}}\right)^{7/13} = 29.88\ rad/s \quad \textbf{(1)}$$

Solution 5:

$$\frac{\varepsilon}{d} = \frac{0.26}{80} = 0.00325$$

$$p_1 = p_2; \quad V_1 = V_2 = 0$$

(a) If valve C is closed, we have a straight series path through A and B, with the same flow rate Q, velocity V, and friction factor f in each pipe.

Energy equation:

$$\left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_1 = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_2 + h_f$$

$$z_1 - z_2 = h_{fA} + \sum h_{mA} + h_{fB} + \sum h_{mB} = \frac{V^2}{2g}\left(f\frac{L_A}{d} + K_1 + K_2 + f\frac{L_B}{d} + K_4\right)$$

$$(25) = \frac{V^2}{(2)(9.81)}\left(f\frac{(100)}{(0.08)} + (0.5) + (0.9) + f\frac{(50)}{(0.08)} + (1.0)\right)$$

$$V = \sqrt{\frac{490.5}{1875f + 2.4}}; \qquad Re = \frac{\rho V d}{\mu}$$ +4

Use initial guess and determine velocity and Re:

$$f = 0.02; \quad V = 3.5\frac{m}{s}; \quad Re = 280000$$

Iterate using the Moody chart:

$$f = 0.026; \quad V = 3.1\frac{m}{s}; \quad Re = 247000$$

$$f = 0.027; \quad V = 3.0\frac{m}{s}; \quad Re = 24300 \quad (converged)$$

$$V_A = V = 3.0\frac{m}{s}$$ +2.5

(b) If the valve C is open, we have parallel flow through B and C with

$$Q_A = Q_B + Q_C; \qquad V_A = V_B + V_C$$

The total head loss is the same for paths A-B and A-C:

$$z_1 - z_2 = h_{fA} + \sum h_{mA-B} + h_{fB} + \sum h_{mB} = \frac{V_A^2}{2g}\left(f_A\frac{L_A}{d} + K_1 + K_2\right) + \frac{V_B^2}{2g}\left(f_B\frac{L_B}{d} + K_4\right)$$

$$z_1 - z_2 = h_{fA} + \sum h_{mA-C} + h_{fC} + \sum h_{mC} = \frac{V_A^2}{2g}\left(f_A \frac{L_A}{d} + K_1 + K_3\right) + \frac{V_C^2}{2g}\left(f_C \frac{L_C}{d} + K_4 + K_{valve}\right)$$

$$\begin{cases} z_1 - z_2 = \frac{V_A^2}{2g}\left(f_A \frac{L_A}{d} + K_1 + K_2\right) + \frac{V_B^2}{2g}\left(f_B \frac{L_B}{d} + K_4\right) \\ z_1 - z_2 = \frac{V_A^2}{2g}\left(f_A \frac{L_A}{d} + K_1 + K_3\right) + \frac{V_C^2}{2g}\left(f_C \frac{L_C}{d} + K_4 + K_{valve}\right) \\ V_A = V_B + V_C \end{cases}$$

$$\begin{cases} (25) = \frac{V_A^2}{(2)(9.81)}\left(f_A \frac{(100)}{(0.08)} + (0.5) + (0.9)\right) + \frac{V_B^2}{(2)(9.81)}\left(f_B \frac{(50)}{(0.08)} + (1.0)\right) \\ (25) = \frac{V_A^2}{(2)(9.81)}\left(f_A \frac{(100)}{(0.08)} + (0.5) + (1.3)\right) + \frac{V_C^2}{(2)(9.81)}\left(f_C \frac{(70)}{(0.08)} + (1.0) + (0.5)\right) \\ V_A = V_B + V_C \end{cases}$$

$$\begin{cases} 490.5 = V_A^2(1250 f_A + 1.4) + V_B^2(625 f_B + 1) \\ 490.5 = V_A^2(1250 f_A + 1.8) + V_C^2(875 f_C + 1.5) \\ V_A = V_B + V_C \end{cases}$$

+2

(c) Use:

$$f_A = f_B = f_C = f = 0.027; \quad V_C = 1.57\frac{m}{s}$$

Select two equations (for example, 2nd and 3rd) for calculating solution:

$$\begin{cases} 490.5 = 35.6V_A^2 + 61.9 \\ V_A = V_B + 1.57 \end{cases}$$

$$V_A = \sqrt{\frac{490.5 - 61.9}{35.6}} = 3.5\ \frac{m}{s}$$

$$V_B = V_A - 1.57 = 1.9\frac{m}{s}$$

+1.5

Solution 6

a. Find the tangential velocity along the surface of the dome

$$u_\theta = \frac{1}{r}\frac{\partial \Phi}{\partial \theta} = \frac{1}{r}\frac{\partial}{\partial \theta}\left(Ur\left(1+\frac{a^2}{r^2}\right)\cos\theta\right) \quad \textbf{(2)}$$

$$u_\theta = \frac{1}{r}\left(-Ur\left(1+\frac{a^2}{r^2}\right)\sin\theta\right)$$

At $r=a$, we have

$$u_\theta = -2U\sin\theta \quad \textbf{(1)}$$

b. Apply Bernoulli between any 2 points

$$p_0 + \frac{1}{2}\rho {V_0}^2 + \gamma z_0 = p_s + \frac{1}{2}\rho {V_s}^2 + \gamma z_s \quad \textbf{(3)}$$

s in on the surface.

$$p_0 + \frac{1}{2}\rho U^2 + 0 = p_s + \frac{1}{2}\rho(-2U\sin\theta)^2 + \gamma a\sin\theta$$

$$p_s = p_0 + \frac{1}{2}\rho U^2 - 2\rho(U\sin\theta)^2 - \gamma a\sin\theta \quad \textbf{(2)}$$

c.

$$\sum F_z = 0 \quad \textbf{(1)}$$

$$F_{lift} - F_w = 0$$

$$\frac{5}{3}\rho U^2 a = m_c g$$

$$m_c = \frac{5}{3}(1.2)(30)^2(50)\left(\frac{1}{9.8}\right)$$

$$m_c = 9170 kg/m \quad \textbf{(1)}$$