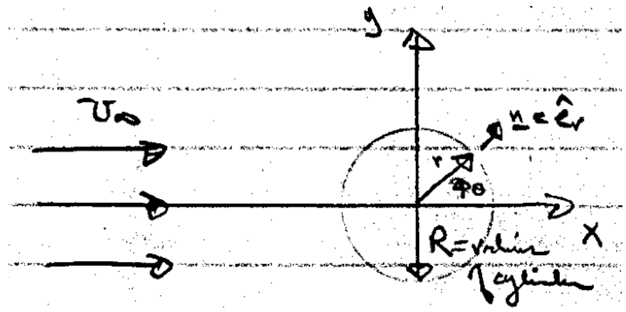
Chapter 8.3 Superposition of Plane Flow Solutions



Circular cylinder

In the previous we derived the following equation for the doublet:

$$\psi_{Doublet=} = -\frac{\lambda y}{x^2 + y^2} = -\frac{\lambda \sin\theta}{r}$$

When this doublet is superposed with a uniform flow parallel to the x- axis, we get:

$$\psi = U_\infty r \sin\theta - \frac{\lambda \sin\theta}{r} = U_\infty \left(1 - \frac{\lambda}{U_\infty}\frac{1}{r^2}\right) r \sin\theta$$

Where λ =doublet strength which is determined from the kinematic body boundary condition that the body surface must be a stream surface. Recall that for inviscid flow it is no longer possible to satisfy the no slip condition as a result of the neglect of viscous terms in the GDEs.

The inviscid flow body surface boundary condition is that the body surface is a steam surface, i.e.,

$$\frac{DF}{Dt} = 0 \rightarrow \frac{\partial F}{\partial t} + \underline{V} \cdot \nabla F = 0 \rightarrow \underline{V} \cdot \underline{n} = -\frac{1}{|\nabla F|}\frac{\partial F}{\partial t} = 0$$

Where F = r – R is the surface function and for steady flow $\frac{\partial F}{\partial t} = 0$.

Therefore, on r = R $\underline{V} \cdot \underline{n} = 0$, i.e., $v_r = 0|_{r=R}$

$$\underline{V} = v_r\hat{e}_r + v_\theta\hat{e}_\theta, \quad \underline{n} = \frac{\nabla F}{|\nabla F|} = \frac{\frac{\partial F}{\partial r}\hat{e}_r + \frac{\partial F}{\partial \theta}\hat{e}_\theta}{\sqrt{F_r^2 + F_\theta^2}} = \hat{e}_r$$

$$\underline{V} \cdot \underline{n} = v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = U_\infty\left(1 - \frac{\lambda}{U_\infty r^2}\right)\cos\theta = 0$$

$$\Rightarrow \lambda = U_\infty R^2$$

If we replace the constant $\frac{\lambda}{U_\infty}$ by a new constant R^2, the above equation becomes:

$$\psi = U_\infty\left(1 - \frac{R^2}{r^2}\right)r\sin\theta$$

The radial velocity is zero on all points on the circle r = R. That is, there can be no velocity normal to the circle r = R. Thus, this circle itself is a streamline.

The tangential component of velocity for flow over the circular cylinder is

$$v_\theta = -\frac{\partial \psi}{\partial r} = -U_\infty \left(1 + \frac{R^2}{r^2}\right) sin\,\theta$$

On the surface of the cylinder r=R, we get the following expression for the tangential and radial components of velocity:

$$v_\theta = -2U_\infty\, sin\,\theta$$

$$v_r = 0$$

The pressure is obtained from Bernoulli's equation:

$$\frac{p}{\rho} + \frac{1}{2}({v_r}^2 + {v_\theta}^2) = \frac{p_\infty}{\rho} + \frac{1}{2} U_\infty^2$$

After some rearrangement we get the following non-dimensional form:

$$C_p(r,\theta) = \frac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2} = 1 - \frac{{v_r}^2 + {v_\theta}^2}{U_\infty^2}$$

At the surface, the only velocity component that is non-zero is the tangential component of velocity. Using $v_\theta = -2U_\infty \sin\theta$, we get at the cylinder surface the following expression for the pressure coefficient:

$$C_p = 1 - 4\sin^2\theta$$

DRAG ON IMMERSED BODIES

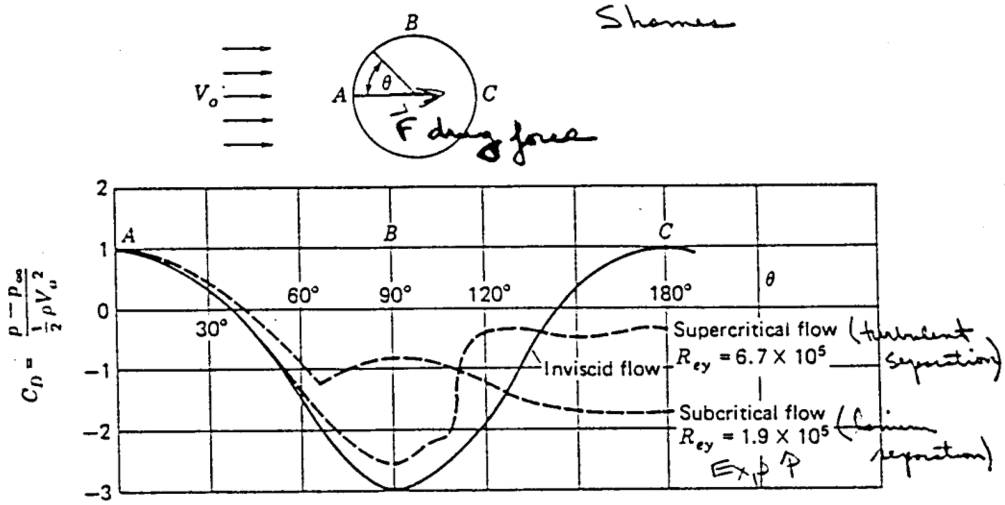


Figure 10.23 Pressure distributions around a cylinder for subcritical, supercritical, and inviscid flows.

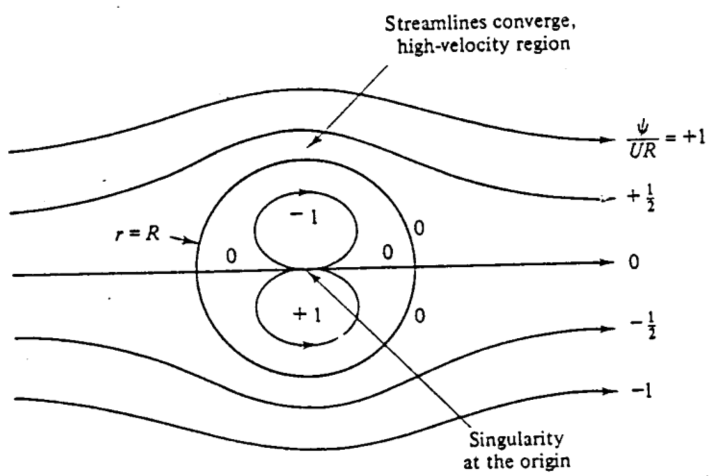


Fig. E4.7

From the pressure coefficient we can calculate the fluid force on the cylinder:

$$\underline{F} = -\int_A (p - p_\infty)\underline{n}dA = -\frac{1}{2}\rho U_\infty^2 \int_A C_p(R,\theta)\underline{n}dA$$

$dA = (Rd\theta)b$ b=span length

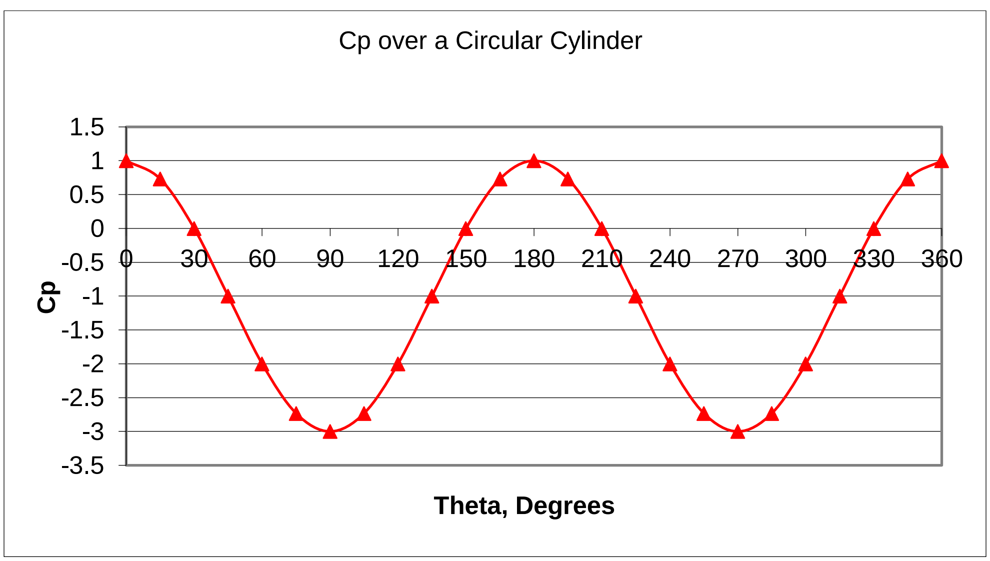
$$\underline{F} = -\frac{1}{2}\rho U_\infty^2 bR \int_0^{2\pi} (1 - 4\sin^2\theta)(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})d\theta$$

$$C_L = \frac{Lift}{\frac{1}{2}\rho U_\infty^2 bR} = \frac{\underline{F}\cdot\hat{\jmath}}{\frac{1}{2}\rho U_\infty^2 bR} = -\int_0^{2\pi}(1 - 4\sin^2\theta)\sin\theta\, d\theta = 0$$

Due to symmetry of flow about x axis

$$C_D = \frac{Drag}{\frac{1}{2}\rho U_\infty^2 bR} = \frac{\underline{F}\cdot\hat{\imath}}{\frac{1}{2}\rho U_\infty^2 bR} = -\int_0^{2\pi}(1 - 4\sin^2\theta)\cos\theta\, d\theta = 0$$

dÁlembert paradox: symmetry of flow about y axis



Circular cylinder with circulation

The stream function associated with the flow over a circular cylinder, with a point vortex of strength Γ placed at the cylinder center is:

$$\psi = U_\infty r \sin\theta - \frac{\lambda \sin\theta}{r} - \frac{\Gamma}{2\pi} \ln r$$

Recall the vortex strength K is related to the circulation, i.e., K= $\frac{\Gamma}{2\pi}$

From V·n=0 at r=R: $\lambda = U_\infty R^2$

Therefore, $\psi = U_\infty r \sin\theta - \frac{U_\infty R^2 \sin\theta}{r} - \frac{\Gamma}{2\pi} \ln r$

The radial and tangential velocity is given by:

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U_\infty\left(1 - \frac{R^2}{r^2}\right)\cos\theta$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -U_\infty\left(1 + \frac{R^2}{r^2}\right)\sin\theta + \frac{\Gamma}{2\pi r}$$

On the surface of the cylinder (r=R):

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U_\infty\left(1 - \frac{R^2}{R^2}\right)\cos\theta = 0$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -2U_\infty \sin\theta + \frac{\Gamma}{2\pi R}$$

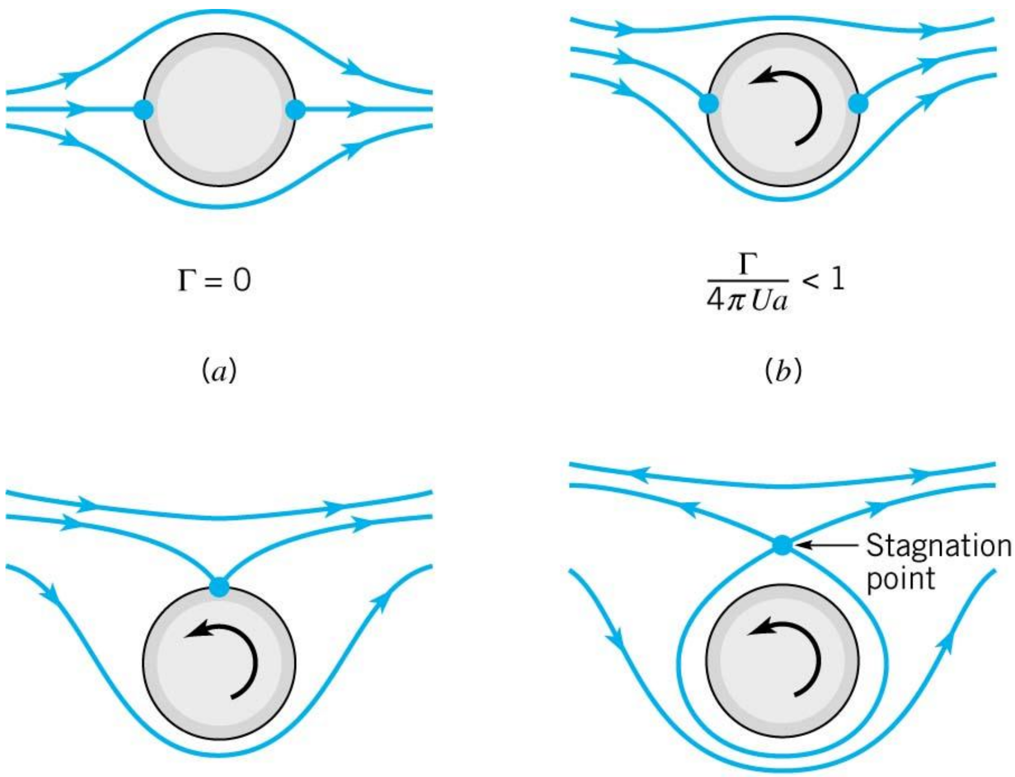
Consider the flow pattern as a function of Γ. The stagnation points on the cylinder are given by:

$$v_\theta = -2U_\infty \sin\theta + \frac{\Gamma}{2\pi R} = 0$$

$$\sin\theta = \frac{\Gamma}{4\pi U_\infty R} = \frac{K}{2U_\infty R} = \beta/2$$

The location of stagnation point is function of Γ:

$\beta = \frac{K}{U_\infty R} = \frac{\Gamma}{2\pi U_\infty R}$	θ_s (stagnation point)
0 ($\sin\theta = 0$)	0,180
1($\sin\theta = 0.5$)	30,150
2 ($\sin\theta = 1$)	90
>2($\sin\theta > 1$)	Is not on the circle but where $v_r = v_\theta = 0$



$\Gamma = 0$ (a)

$\frac{\Gamma}{4\pi U a} < 1$ (b)

$\frac{\Gamma}{4\pi U a} = 1$ (c)

$\frac{\Gamma}{4\pi U a} > 1$ (d)

For flow patterns like above (except a), we should expect to have lift force in –y direction.

Summary of stream and potential function for elementary 2-D flows

In Cartesian coordinates:

$$u = \psi_y = \varphi_x$$

$$v = -\psi_x = \varphi_y$$

In polar coordinates:

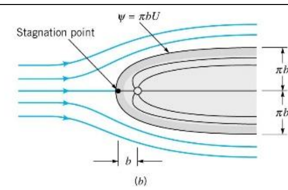
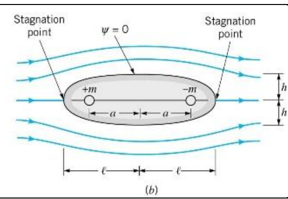
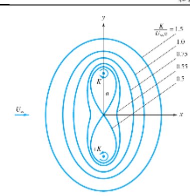
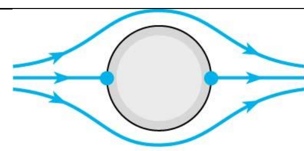
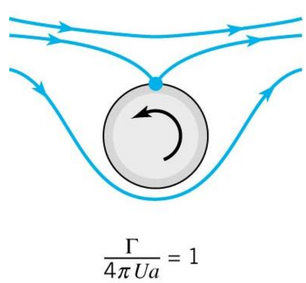
$$v_r = \frac{\partial \varphi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

$$v_\theta = \frac{1}{r}\frac{\partial \varphi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

Flow	ϕ	ψ
Uniform Flow	$U_\infty x$	$U_\infty y$
Source (m>0) Sink (m<0)	$m \ln r$	$m\theta$
Doublet	$\frac{\lambda \cos\theta}{r}$	$-\frac{\lambda \sin\theta}{r}$
Vortex	$K\theta$	$-K \ln r$
90 Corner flow	$1/2A(x^2 - y^2)$	Axy
Solid-Body rotation	Doesn't exist	$\frac{1}{2}\omega r^2$

These elementary solutions can be combined in such a way that the resulting solution can be interpreted to have physical significance; that is, represent the potential flow solution for various geometries. Also, methods for arbitrary geometries combine uniform stream with distribution of the elementary solution on the body surface.

Some combination of elementary solutions to produce body geometries of practical importance

Body name	Elemental combination	Flow Patterns
Rankine Half Body	Uniform stream + source	
Rankine Oval	Uniform stream + source + sink	
Kelvin Oval	Uniform stream + vortex point	
Circular Cylinder without circulation	Uniform stream + doublet	
Circular Cylinder with circulation	Uniform stream + doublet + vortex	

Keep in mind that this is the potential flow solution and may not well represent the real flow especially in region of adverse p_x.

The Kutta-Joukowski Lift Theorem

$$p/\rho + \frac{1}{2}\left(v_r^2 + v_\theta^2\right) = \frac{p_\infty}{\rho} + \frac{1}{2}U_\infty^2 \qquad p = p(r,\theta)$$

on $r = R$: $v_r = 0$ $\quad v_\theta = -2U_\infty \sin\theta + \frac{\Gamma}{2\pi R}$

$$p_s - p_\infty = -\frac{\rho}{2}v_\theta^2 + \frac{\rho}{2}U_\infty^2$$
$$= \rho/2\left[U_\infty^2 - \left(-2U_\infty \sin\theta + \frac{\Gamma}{2\pi R}\right)^2\right]$$
$$4U_\infty^2 \sin^2\theta - 4U_\infty \sin\theta\frac{\Gamma}{2\pi R} + \frac{\Gamma^2}{4\pi^2R^2}$$
$$= \frac{1}{2}\rho U_\infty^2\left[1 - 4\sin^2\theta + 2\sin\theta\frac{\Gamma}{U_\infty \pi R} - \frac{\Gamma^2}{4\pi^2R^2U_\infty^2}\right]$$

$B = \frac{K}{U_\infty R} = \frac{\Gamma}{2\pi U_\infty R}$

$$= \frac{1}{2}\rho U_\infty^2\left[\underbrace{1 - 4\sin^2\theta}_{\text{w/o }\Gamma} \underbrace{+ 4B\sin\theta - B^2}_{\text{due }\Gamma = 2\pi U_\infty R B}\right]$$

$$\underline{F} = -\int (p_s - p_\infty)\,\underline{n}\,dA \qquad \underline{n} = \hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$dA = b\,r\,d\theta \qquad b = \text{span length}$

$$p_s - p_\infty = \frac{1}{2}\rho U_\infty^2(1 - B^2) + 2\rho U_\infty^2 B\sin\theta - 2\rho U_\infty^2 \sin^2\theta$$
$$= A + B\sin\theta + C\sin^2\theta$$

$$\text{Drag} = F_x = -\int_0^{2\pi}\left(A + B\sin\theta + C\sin^2\theta\right)\cos\theta\, bR\, d\theta$$

$\int \cos ax\,dx = a^{-1}\sin ax$ $\qquad$ for $a = 1$

$\int \sin ax \cos ax\,dx = \frac{1}{2}a^{-1}\sin^2 ax$ $\qquad$ limits 0 to 2π

$\int \sin^2 ax \cos ax\,dx = \frac{\sin^3 ax}{3a}$ $\qquad$ all $= 0$

Drag = 0 d'Alembert paradox

not affected Γ

$$\text{Lift} = F_y = -\int_0^{2\pi} \left(A + B\sin\theta + C\sin^2\theta\right)\sin\theta \, bR\,d\theta$$

$$\int \sin ax\,dx = -a^{-1}\cos ax = -\cos x = -(1-1) = 0$$

$$\int \sin^2 ax\,dx = \tfrac{1}{2}x - \tfrac{1}{4a}\sin 2ax = \pi$$

$$\int \sin^3 ax\,dx = -\tfrac{1}{3a}(\cos ax)(\sin^2 ax + 2) = 0$$

$$\text{Lift} = -B\,bR\pi = -(2\rho U_\infty^2 \beta)\,bR\pi$$

$$L/b = -2\rho U_\infty^2 R\pi \frac{\Gamma}{2\pi U_\infty R}$$

$$L/b = -\rho U_\infty \Gamma$$ lift per unit span

According to inviscid theory, the lift per unit depth of any cylinder of any shape immersed in a uniform stream equals $\rho u_\infty \Gamma$, where Γ is the total net circulation contained within the body shape. The direction of the lift is 90° from the stream direction, rotating opposite to the circulation.

The Kutta condition requires smooth tangential flow at airfoil trailing edge, which determines Γ and therefore Lift.

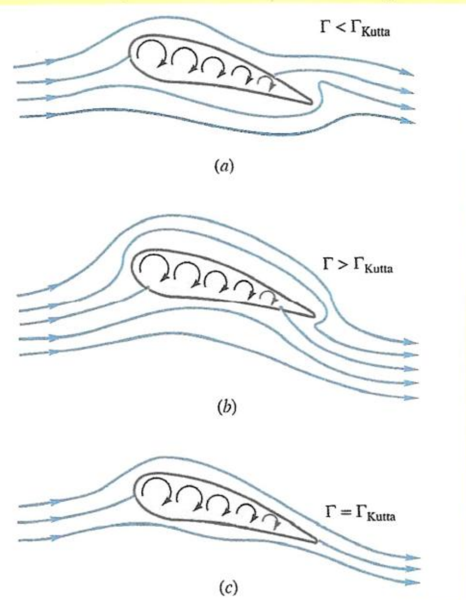


Fig. 8.22 The Kutta condition properly simulates the flow about an airfoil; (*a*) too little circulation, stagnation point on rear upper surface; (*b*) too much, stagnation point on rear lower surface; (*c*) just right, Kutta condition requires smooth flow at trailing edge.

Lift and Drag for Rotating Cylinder:

$L/b = \rho U_\infty \Gamma$ (clockwise rotation); therefore,

$$C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 (2Rb)} = \frac{\Gamma}{U_\infty R}$$

Note: $\Gamma = 2\pi K$ and $v_{\theta_s} = \frac{K}{R}$

$$\Rightarrow C_L = \frac{2\pi}{U_\infty} v_{\theta_s}$$

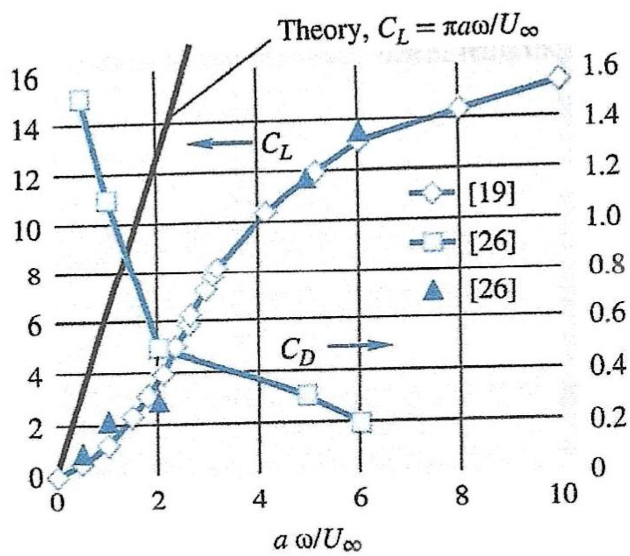


Fig. 8.15 Drag and lift of a rotating cylinder of large aspect ratio at $Re_D = 3800$, after Tokumaru and Dimotakis [19] and Sengupta et al. [26].

Theoretical and experimental lift and drag of a rotating cylinder

Experiments have been performed that simulate the previous flow by rotating a circular cylinder in a uniform stream. In this case $v_\theta = R\omega$ which is due to no slip boundary condition.

- Lift is quite high but not as large as theory (due to viscous effect i.e. flow separation)

- Much larger typical airfoil same length; thus, have practical application

- Note drag force is also fairly high

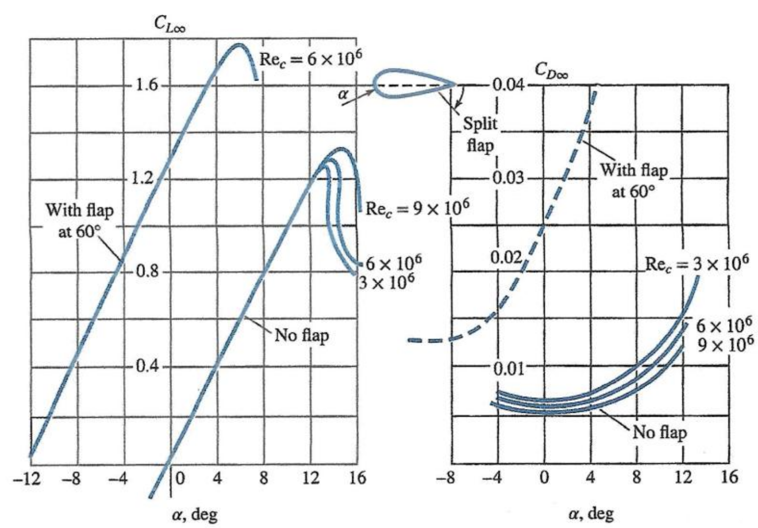
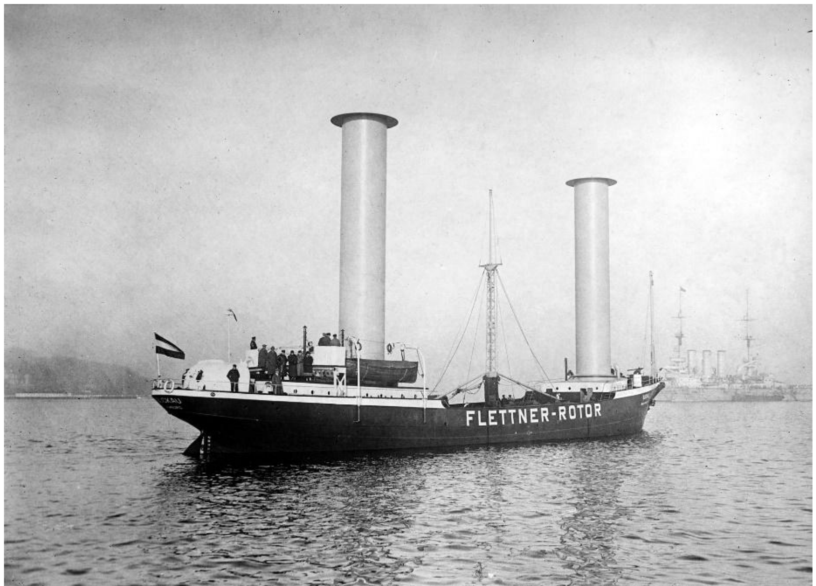


Fig. 7.25 Lift and drag of a symmetric NACA 0009 airfoil of infinite span, including effect of a split-flap deflection. Note that roughness can increase C_D from 100 to 300 percent.

Flettner (1924) used rotating cylinder to produce forward motion.


FLETTNER-ROTOR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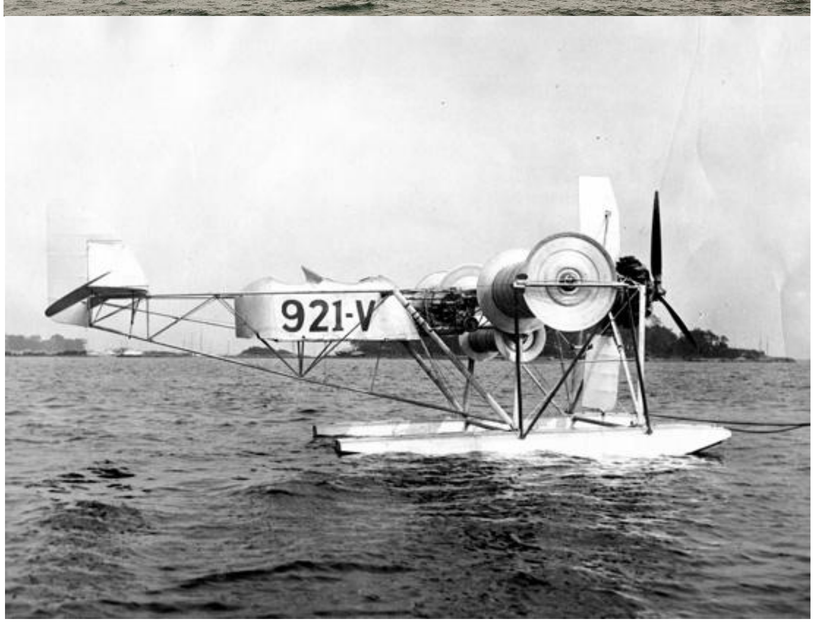
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Spindle Rotors Take the Place of Wings

by LAWRENCE E. ANDREWS

Using spindle shaped slotted rotors, the inventor expects to eliminate many of the difficulties formerly experienced with cylindrical rotors

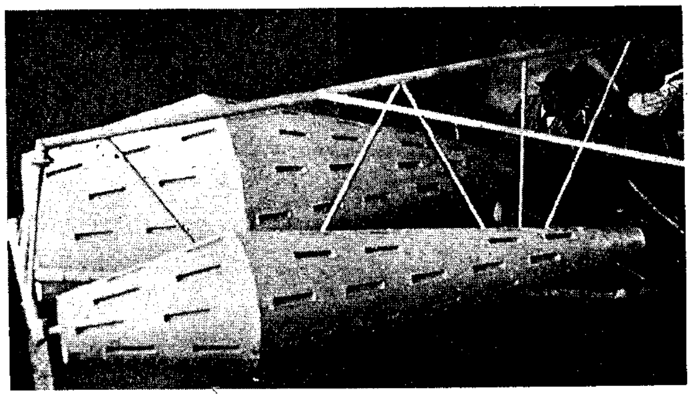
ROTATING conical spindles instead of wings will provide the lifting surface for a new flight machine to be launched at Roosevelt Field this spring. While it is not the first machine projected with lifting rotors, it is the first using slotted, conical surfaces.

It is the invention of John G. Guest, while actual construction is being carried out by L. C. Popper, construction and designing engineer of New York city. A rotor-wing airplane was made a few years ago and was tried out on Long Island but its cylindrical type of rotor set up such an air disturbance that its control was seriously hampered.

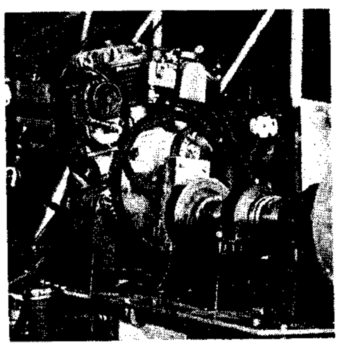
This new ship makes use of the same general principle, but its mechanical execution is decidedly different. Laboratory tests have shown that it has a lifting power of 900 per cent greater than an equally surfaced conventional plane. In addition, it has the ability to land or take-off in very short distances —greatly like the Autogiro.

There are four spindles, two on each side in place of wings. The lift is produced by the rotation of the spindles turning in the slip stream of the forward propeller, the rotation distorting and deflecting the air-stream downwards. Slots are hollowed out in the spindles which offer no resistance to the wind but are caught by the wind as they turn under. The slots serve to reduce the drag which disrupted control with former ships of this type.

There are three motors in the machine. One, a 90 horse power Cirrus engine provides power to the tractor propeller. Two others, with two cylinders each, provide power to the spindles. A universal throttle connects with the pilot's seat. By speeding up or slowing down the rotor motors, lateral control is accomplished. Elevator and rudder controls govern longitudinal direction in the usual manner.



A close view of the curious rotors showing their shape and the slots cut into their surfaces. Note that the forward rotors are larger than those in the rear.



A detailed view of one of the small 2-cylinder engines employed for driving the forward spindles, together with the main bearings and gearing. End of spindle at right.

With full weight of pilot and fuel, the machine weighs 1,734 pounds. The cruising range is about 340 miles. It measures 23½ feet from tip to tip of spindles and is 18 feet long. The size compares favorably with that of the small training ship.

The spindles and their assembly weigh more than the wings in the ordinary airplane, but the gross weight is well under the figures set for light airplanes powered with the 90 horse-power Cirrus motors.

A New York manufacturing concern is financing the arrangements for the research and development work on the plane. They plan to manufacture the odd looking craft after the preliminary field and flight tests are made.

This machine is an excellent example of many similar attempts now being made toward producing a direct lift wingless ship. There is undoubtedly a great field for wingless ships of this same general type and inventors will make no mistake in experimenting along these lines.

From experiments made to date, it is evident that the weight of a machine can be supported with a smaller expenditure of power than where wings are employed. Very little power is taken by the rotors, and this fact alone justifies the additional complication.

Whether it will pay to employ auxiliary wings for safety in case of engine failure, it is difficult to say, but in such a case the use of a parachute is an alternative.

airplane carrier.

Recently the navy has placed landing-lights around the edges of the decks to facilitate night flying. Ability to fly at night is now a requirement, and much time is spent to keep the flyers in practice.

A plane about to land on the deck of the carrier.

licenses.

10,780 airplanes were registered, including 3,227 unlicensed, having identification numbers only.

The licensed pilots included 532 women of which 433 were private and 42 were transport licenses.

New York has the greatest number of aircraft of all kinds, 1,227, with California second. On the other hand, California has the greatest number of licensed pilots leading with 3,327, and New York second.

Gliders were also listed. There were 1,270 gliders of which 89 were licensed. Licensed glider pilots numbered 267.

The report is interesting in that there is a decided increase in every item over those released for July of 1931.

One very important improvement on wing construction, and one that has proved very practicable in service, is the "slotted wing" invented by Handley-Page. This device very materially increases the speed range of a ship by varying the lift, and by allowing higher angles of attack than possible with a plain wing.

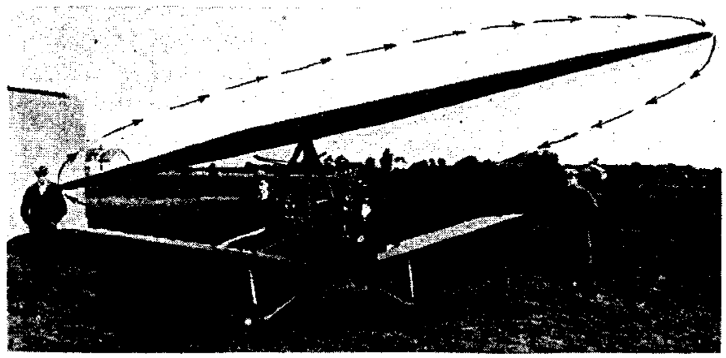
Essentially, this invention consists of a metal guide placed along the entering edge of the wing. This so controls the flow of air over the top surface of the wing that the air does not break away from the wing or "burble" until very high angles of attack are attained. The slots are controlled automatically or manually, depending upon conditions.

However, it has been discovered that the wing "slots" are much more effective if an aileron is installed along the whole length of the trailing edge. At high angles, this hinged rear flap is depressed, and by this means even a greater lift is obtained at low speeds than with the slots alone. The writer has witnessed a ship of this type taking off and landing easily inside of a 100 foot circle. In a 15 m.p.h. breeze it hovered directly over one spot.

Handley-Page also instituted another innovation in wing construction which departs entirely from the standard wing. Essentially, it consists of a series of short streamline blades built into a unit, much after the manner of a Venetian blind or lattice. Each of these streamline blades is placed progressively at a flatter angle as we approach the trailing edge of the structure, and in this way, the whole area of the wing is utilized effectively.

Next, in the development of wingless wings, is the cellule construction of the "Vacuplane," described in the November issue of POPULAR AVIATION. This, it will be remembered, consisted of a short stubby cell carried over the fuselage, the upper surface of the cell consisting of rods or slats. It is claimed that this arrangement so greatly decreases the pressure on the top of the cell that a very much greater speed range is obtained.

Helicopters, of some sort or other, have always been with us. Few of them have shown much indication of success until the coming of the Autogiro, which in general, belongs to the helicopter family. Helicopters, or machines equipped with lifting propellers, look nice on paper but they have more inherent defects than wings. True, they have the advantage of landing and taking off at near zero speed, but they are mechanically complicated.



The Stauffer "Gyroplane," showing the single blade rotor which turns only on landing or take-off.

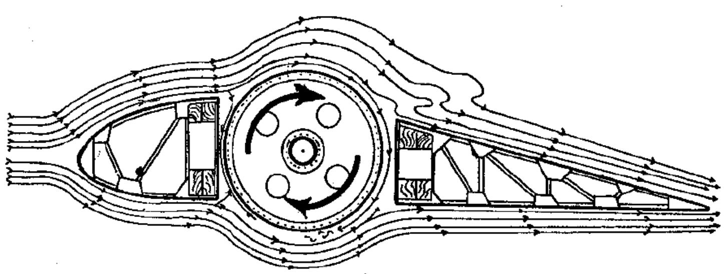
We only look at their one advantage, that is of slow landing, but fail to see at the same time that their top speed is limited. When we simmer the whole thing down into a nutshell, the speed range is not much greater, and usually less than an airplane.

In the point of low landing speed, a helicopter or lifting screw type has little advantage over an equally standard loaded wing, and still less advantage over a slotted wing type. The Autogiro, for example, has a top speed of about 100 m.p.h. but with the same top speed and loading, an airplane can land nearly as slowly.

Now, a helicopter type known as the Gyroplane, has recently been developed. It is apparently based upon a more logical principle than those that have gone before it. This is a combination of an airplane and helicopter, with the rotor used as an auxiliary to the wing.

When taking off, flying at slow horizonal speed, or in landing, the lifting propeller revolves and assists the wings. However, when the plane is to fly at high speed, the lifting propeller or rotor is stopped so that flight is now maintained by the wings alone.

Thus, if the wings are of the high speed type, this gives a tremendous speed range. It has a good gliding angle with a dead engine. This ship will probably range from a low speed of 15 m.p.h. to a high speed of about 145 m.p.h.



The Thompson Rotor-wing with the rotor imbedded in a deep wing section. Lines show air distribution.

And now we get down to the so-called "rotor" or cylinder type of lift, which as you probably know, consists of a large diameter rotating cylinder projecting out on both sides of the fuselage. When the rotors are not turning, the air-stream splits equally around the cylinders and there is no lifting force exerted.

However, when the cylinders revolve the air-stream is twisted about in such a way that the pressure is higher on one side of the cylinder than on the other, thus producing the lift. Very little power is required to produce the rotation, and the cylinders can either be driven directly by the engine or else through the action of the wind-stream. A small amount of cylinder surface produces a remarkable amount of lift.

Now, this plain rotor is entirely ineffective when the engine stops, hence the machine will drop suddenly as soon as the engine cuts out. To avoid this difficulty, it is safest to combine the rotor with a wing in such a way that the wing will always be available alone for dead engine landings or high speed operation.

One experimenter, Mr. Ray Thompson, who has recently come into our notice, has designed a new application of the rotor and wing. He has done quite a bit of experimenting with large models and has obtained quite remarkable results. This general class of lifting device, in my opinion, is the first step in the complete elimination of wings—far more practical than any possible helicopter arrangement. We hereby quote from a letter by Mr. Thompson on the subject:

"The rotor wing model had a span of 38 inches and a length of 42 inches, with a wing area of 360 square inches. With the rotors turning, it carried a load of 9.5 pounds to a height of 18 feet, the rotor being driven by an electric motor. This model had no propeller for pulling it forward, but was

(Continued on page 58)