Chapter 6: Viscous Flow in Ducts

6.4 Turbulent Flow in Pipes and Channels using mean-velocity correlations.

1. *Smooth circular pipe*

Recall laminar flow exact solution:

$$f = \frac{8\tau_w}{\rho u_{ave}^2} = 64/\mathrm{Re}_d \qquad \mathrm{Re}_d = \frac{u_{ave}d}{\upsilon} \le 2000$$

A turbulent flow "approximate" solution can be obtained simply by computing u_{ave} based on log law.

$$\frac{u}{u^*} = \frac{1}{\kappa}\ln\frac{yu^*}{v} + B$$

Where:

$$u = u(y);\ \ \kappa = 0.41;\ \ B = 5;\ \ u^* = \sqrt{\tau_w/\rho};\ \ y = R - r$$

$$V = u_{ave} = \frac{Q}{A} = \frac{1}{\pi R^2}\int_0^R u^*\left[\frac{1}{\kappa}ln\frac{yu^*}{v} + B\right]2\pi r\ dr$$

$$= \frac{1}{2}u^*\left(\frac{2}{\kappa}ln\frac{Ru^*}{v} + 2B - \frac{3}{\kappa}\right)$$

$$u_{ave} = \frac{1}{\pi R^2}\int_0^R u^*\left[\kappa^{-1}\ln\frac{y u^*}{\nu} + B\right]2\pi r\,dr \qquad y = R - r \quad dy = -dr$$

$$= \frac{2u^*}{R^2}\int_R^0 \left[\kappa^{-1}\ln\frac{y u^*}{\nu} + B\right](y-R)\,dy$$

$$= \frac{2u^*}{R^2}\Bigg[\underbrace{\int_R^0\left[\kappa^{-1}\ln\frac{y u^*}{\nu} + B\right]y\,dy}_{①} - \underbrace{R\int_R^0\left[\kappa^{-1}\ln\frac{y u^*}{\nu} + B\right]dy}_{②}\Bigg]$$

$$x = \frac{y u^*}{\nu} \quad y = \frac{\nu}{u^*}x \quad dy = \frac{\nu}{u^*}dx \quad dx = \frac{u^*}{\nu}dy \quad y\,dy = \frac{\nu^2}{u^{*2}}x\,dx$$

$$① = \int_{\frac{Ru^*}{\nu}}^0 \left[\kappa^{-1}\ln x + B\right]\frac{\nu^2}{u^{*2}}x\,dx = \frac{\nu^2}{u^{*2}}\int_{\frac{Ru^*}{\nu}}^0\left(\frac{x}{\kappa}\ln x + Bx\right)dx$$

$$= \frac{\nu^2}{u^{*2}}\left[\kappa^{-1}\left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right) + \frac{Bx^2}{2}\right]_{\frac{Ru^*}{\nu}}^0$$

$$= \frac{\nu^2}{u^{*2}}\left[-\kappa^{-1}\frac{R^2u^{*2}}{2\nu^2}\ln\frac{Ru^*}{\nu} + \frac{R^2u^{*2}}{\kappa 4\nu^2} - \frac{BR^2u^{*2}}{2\nu^2}\right]$$

$$= -\frac{R^2}{2\kappa}\ln\frac{Ru^*}{\nu} + \frac{R^2}{4\kappa} - \frac{BR^2}{2}$$

$$② = -R\int_{\frac{Ru^*}{\nu}}^0\left[\kappa^{-1}\ln x + B\right]\frac{\nu}{u^*}dx = -\frac{R\nu}{u^*}\left[\kappa^{-1}(x\ln x - x) + Bx\right]_{\frac{Ru^*}{\nu}}^0$$

$$= \frac{R\nu}{u^*}\left[\kappa^{-1}\left(\frac{Ru^*}{\nu}\ln\frac{Ru^*}{\nu} - \frac{Ru^*}{\nu}\right) + B\frac{Ru^*}{\nu}\right]$$

$$= \frac{R^2}{\kappa}\ln\frac{Ru^*}{\nu} - \frac{R^2}{\kappa} + BR^2$$

$$\frac{2u^*}{R^2}\left[-\frac{R^2}{2\kappa}\ln\frac{Ru^*}{\nu} + \frac{R^2}{4\kappa} - \frac{BR^2}{2} + \frac{R^2}{\kappa}\ln\frac{Ru^*}{\nu} - \frac{R^2}{\kappa} + BR^2\right]$$

$$2u^*\left[\frac{1}{2\kappa}\ln\frac{Ru^*}{\nu} - \frac{3}{4\kappa} + B/2\right]$$

$$u^*\left[\kappa^{-1}\ln\frac{Ru^*}{\nu} - \frac{3}{2\kappa} + B\right]$$

$$\frac{1}{2}u^*\left[\frac{2}{\kappa}\ln\frac{Ru^*}{\nu} - \frac{3}{\kappa} + 2B\right] = u_{ave}$$

Or:

$$\frac{V}{u^*} = 2.44 \ln \frac{Ru^*}{\nu} + 1.34$$

$$\frac{V}{u^*} = \left(\frac{\rho V^2}{\tau_w}\right)^{1/2} = \left(\frac{8}{f}\right)^{1/2}$$

$$\frac{Ru^*}{\nu} = \frac{0.5Vd}{\nu}\frac{u^*}{V} = \frac{1}{2} Re_d \left(\frac{f}{8}\right)^{1/2}$$

$$f^{-1/2} = 1.99 \log[\mathrm{Re}_d\, f^{1/2}] - 1.02$$

$$= 2 \log[\mathrm{Re}_d\, f^{1/2}] - 0.8$$

EFD Adjusted constants.

f only drops by only a factor of 5 over $4\times10^3 \leq \mathrm{Re} \leq 10^8$

Since f equation is implicit, it is not easy to see dependency on ρ, μ, V, and D

$$f(pipe) = 0.316\,\mathrm{Re}_D^{-1/4}$$

$4000 < \mathrm{Re}_D < 10^5$
Blasius (1911) power law curve fit to data.

$$h_f = \frac{\Delta p}{\gamma} = f\,\frac{L}{D}\frac{V^2}{2g}$$

Turbulent Flow: $\Delta p = 0.158 L\rho^{3/4}\mu^{1/4}D^{-5/4}V^{7/4}$

Nearly quadratic (As expected)

Nearly linear

Only slightly with μ

Drops with pipe diameter.

$$= 0.241 L\rho^{3/4}\mu^{1/4}D^{-4.75}Q^{1.75}$$

Laminar flow: $\Delta p = 128\mu LQ/\pi D^4$

Δp (turbulent) decreases more sharply with D than Δp (laminar) for same Q; therefore, increase D for smaller Δp, although large D more expensive. 2D decreases Δp by 27 for same Q.

$$\frac{u_{\max}}{u^*} = \frac{u(r=0)}{u^*} = \frac{1}{\kappa}\ln\frac{Ru^*}{\upsilon} + B$$

Combine with $V = u_{ave}$

$$\frac{V}{u^*} = \frac{1}{\kappa}\ln\frac{Ru^*}{\upsilon} + B - \frac{3}{2\kappa}$$

$$\Rightarrow \frac{V}{u^*} = \frac{u_{\max}}{u^*} - \frac{3}{2\kappa} \Rightarrow V = u_{\max} - \frac{3u^*}{2\kappa} \Rightarrow \frac{u_{\max}}{V} = 1 + \frac{3u^*}{2\kappa V}$$

Also

$$\tau_w = \rho u^{*2} \ and \ \ f = \frac{\tau_w}{1/8\rho V^2} \Rightarrow f = \frac{\rho u^{*2}}{1/8\rho V^2} \Rightarrow \frac{u^*}{V} = \sqrt{f/8}$$

$$\Rightarrow \frac{u_{\max}}{V} = 1 + \frac{3u^*}{2\kappa V} = 1 + \frac{3}{2\kappa}\sqrt{f/8} = 1 + 1.3\sqrt{f}$$

Or:

For Turbulent Flow: $\frac{V}{u_{max}} = \left(1 + 1.3\sqrt{f}\right)^{-1}$

Re_D	4K	10^4	10^6	10^8
	.794	.814	.877	.909

Recall laminar flow:
V/u_{max} = 0.5

TABLE 10.1 EXPONENTS FOR POWER-LAW EQUATION AND RATIO OF MEAN TO MAXIMUM VELOCITY

Re →	4×10^3	2.3×10^4	1.1×10^5	1.1×10^6	3.2×10^6
m →	$\frac{1}{6.0}$	$\frac{1}{6.6}$	$\frac{1}{7.0}$	$\frac{1}{8.8}$	$\frac{1}{10.0}$
$\overline{V}/V_{\max}$ →	0.791	0.807	0.817	0.850	0.865

SOURCE: Schlichting (36). Used with permission of the McGraw-Hill Companies.

Power law fit to velocity profile:

$$\frac{\bar{u}}{\bar{u}_{\max}} = \left(1 - \frac{r}{r_o}\right)^m \qquad m = m(\mathrm{Re})$$

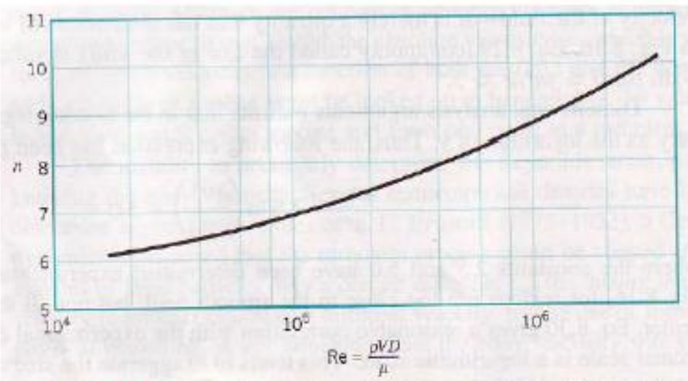


FIGURE 8.17 Exponent, n, for power-law velocity profiles. (Adapted from Ref. 1.)

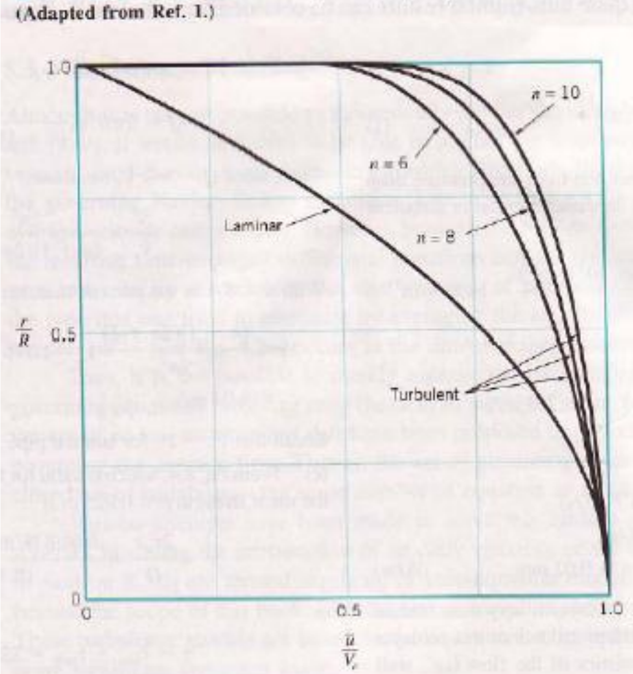


FIGURE 8.18 Typical laminar flow and turbulent flow velocity profiles.

2. *Turbulent Flow in Rough circular pipe*

Experiments: roughness height k forces log law outward on the abscissa by ln k^+ and downward on the ordinate by the amount $\Delta B(k^+)$ where $k^+ = \frac{ku^*}{\nu}$ with same slope $\frac{1}{\kappa}$ which causes B to be reduced by $\Delta B(k^+) \approx \frac{1}{\kappa}$ ln k^+.

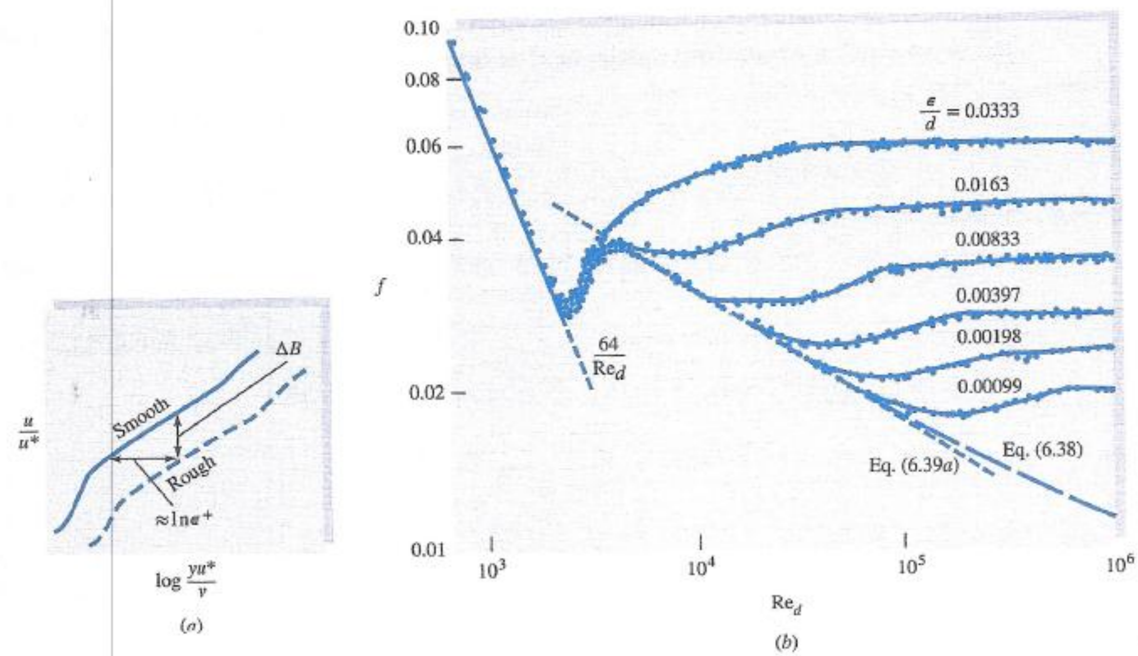


Fig. 6.12 Effect of wall roughness on turbulent pipe flow. (*a*) The logarithmic overlap velocity profile shifts down and to the right; (*b*) experiments with sand-grain roughness by Nikuradse [7] show a systematic increase of the turbulent friction factor with the roughness ratio.

Laminar flow unaffected, but for turbulent flow the effects of roughness initiate for lower $Re_d = Vd/\nu$ as k/d increases. For all k/d, the friction factor becomes constant (fully rough) at high Re_d:

1. $k^+ < 5$ hydraulically smooth
2. $5 < k^+ < 70$ transitional roughness (Re dependence)
3. $k^+ > 70$ fully rough (no Re dependence)

For fully rough flow:

$$\Delta B(k^+) \approx \frac{1}{\kappa} \ln k^+ - 3.5$$

And log law modified for roughness becomes:

$$u^+ = \frac{1}{\kappa} \ln y^+ + B - \Delta B(k^+) = \frac{1}{\kappa} \ln y/k + 8.5$$

i.e., independent viscosity/Re_d. Integration for $u_{ave} = V$ provides:

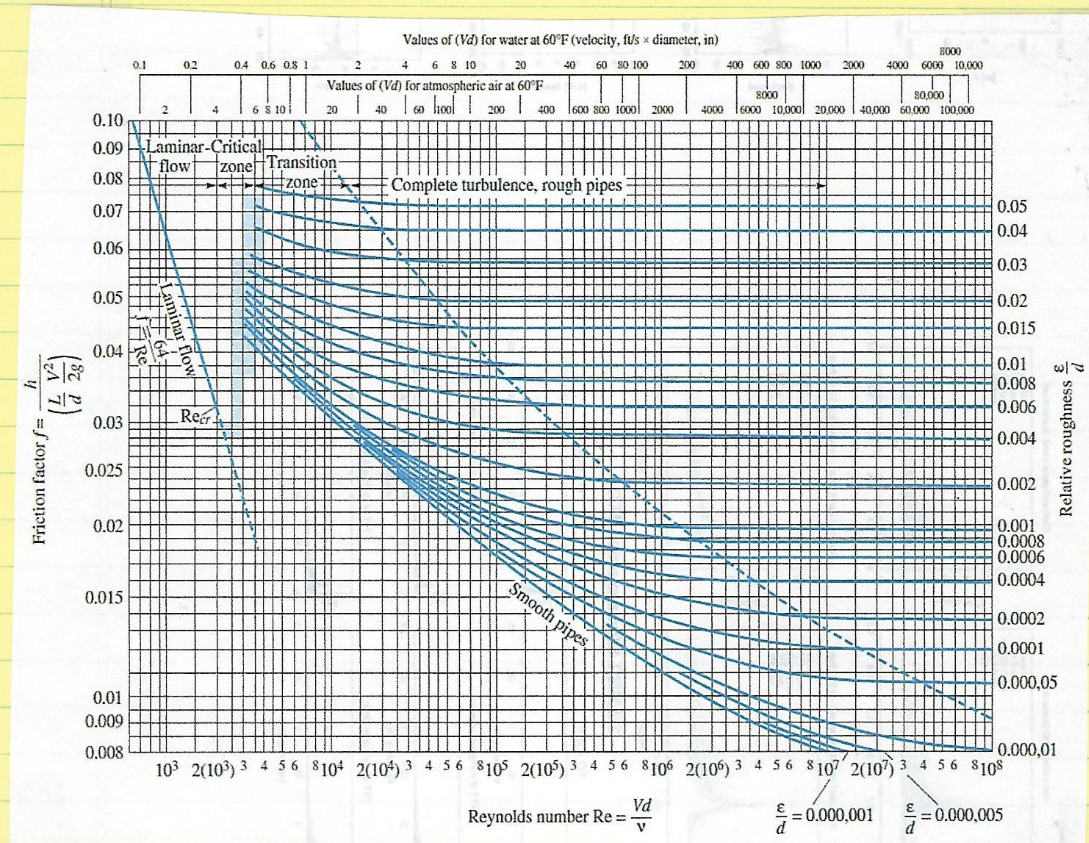
$$\frac{V}{u^*} = 2.44 \ln \frac{d}{k} + 3.2 \quad \text{or} \quad f^{-1/2} = -2 \log \frac{k/d}{3.7} \text{ (fully rough flow)}$$

There is no Re_d effect; therefore, head loss varies as V^2 and f increases 9 times as k/d increases by factor 5000. Combining smooth and fully rough friction factor formulas to include transitionally rough regime produces the Colebrook-White equation, i.e., Moody diagram:

$$f^{-\frac{1}{2}} = -2 \log \left[\frac{\frac{k}{d}}{3.7} + \frac{2.51}{Re_d f^{-\frac{1}{2}}} \right]$$ Moody diagram

$$\sim -1.8 \log \left[\frac{6.9}{Re_d} + \left(\frac{k/d}{3.7} \right)^{1.11} \right]$$ Approximate explicit formula

Moody accuracy ±15% for its full range and explicit within 2% Moody.


Values of (Vd) for water at 60°F (velocity, ft/s × diameter, in)
Values of (Vd) for atmospheric air at 60°F
Laminar flow
Critical zone
Transition zone
Complete turbulence, rough pipes
Laminar flow
f = 64/Re
Re_cr
Smooth pipes
Friction factor f = h / ((L/d)(V²/2g))
Relative roughness ε/d
Reynolds number Re = Vd/ν
ε/d = 0.000,001
ε/d = 0.000,005
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.000,05
0.000,01

There are basically four types of problems involved with uniform flow in a single pipe:

1. Given d, L, and V or Q, ρ, μ, and g, compute the head loss h_f (head loss problem).
2. Given d, L, h_f, ρ, μ, and g, compute the velocity V or flow rate Q (flow rate problem).
3. Given Q, L, h_f, ρ, μ, and g, compute the diameter d of the pipe (sizing problem).
4. Given Q, d, h_f, ρ, μ, and g, compute the pipe length L.

1. Determine the head loss.

The first problem of head loss is solved readily by obtaining f from the Moody diagram, using values of Re and k_s/D computed from the given data. The head loss h_f is then computed from the Darcy-Weisbach equation.

$$f = f(Re_D, k_s/D)$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = \Delta h \qquad \Delta h = (z_1 - z_2) + \left(\frac{p_1}{\gamma} - \frac{p_2}{\gamma}\right)$$

$$= \Delta\left(\frac{p}{\gamma} + z\right)$$

$$Re_D = Re_D(V, D)$$

2. Determine the flow rate.
   The second problem of flow rate is solved by trial, using a successive approximation procedure. This is because both Re and f(Re) depend on the unknown velocity, V. The solution is as follows:

   1) solve for V using an assumed value for f and the Darcy-Weisbach equation.

$$V = \underbrace{\left[\frac{2gh_f}{L/D}\right]^{1/2}}_{\text{known from given data.}} \cdot f^{-1/2}$$

note sign.

   2) using V compute Re
   3) obtain a new value for f = f(Re, k_s/D) and repeat as above until convergence

Or can use Re $f^{1/2} = \frac{D^{3/2}}{\nu}\left(\frac{2gh_f}{L}\right)^{1/2}$

scale on Moody Diagram

1) compute $Re\, f^{1/2}$ and k_s/D
2) read f
3) solve V from $h_f = f\frac{L}{D}\frac{V^2}{2g}$
4) Q = VA

3. Determine the size of the pipe.

   The third problem of pipe size is solved by trial, using a successive approximation procedure. This is because h_f, f, and Q all depend on the unknown diameter D. The solution procedure is as follows:

   1) solve for D using an assumed value for f and the Darcy-Weisbach equation along with the definition of Q

$$D = \underbrace{\left[\frac{8LQ^2}{\pi^2 g h_f}\right]^{1/5}}_{\text{known from given data.}} \cdot f^{1/5}$$

   2) using D compute Re and k_s/D

   3) obtain a new value of f = f(Re, k_s/D) and repeat as above until convergence

4. Determine the pipe length.

   The fourth problem of pipe length is solved by obtaining f from the Moody diagram, using values of Re and k_s/D computed from the given data. Then using given h_f, V, D, and calculated f to solve L from $L = \frac{2g}{V^2}\frac{Dh_f}{f}$.

10.5 Flow at Pipe Inlets and Losses From Fittings

For real pipe systems in addition to friction head loss there are additional losses called minor losses due to

1. entrance and exit effects
2. expansions and contractions
3. bends, elbows, tees, and other fittings
4. valves (open or partially closed)

can be large effect

For such complex geometries we must rely on experimental data to obtain a loss coefficient

$$K = \frac{h_m}{\dfrac{V^2}{2g}}$$

head loss due to minor losses

In general,

$$K = K(\text{geometry}, \underbrace{Re, \varepsilon/D}_{\text{dependence usually not known}})$$

Loss coefficient data is supplied by manufacturers and also listed in handbooks. The data are for turbulent flow conditions but seldom given in terms of Re.

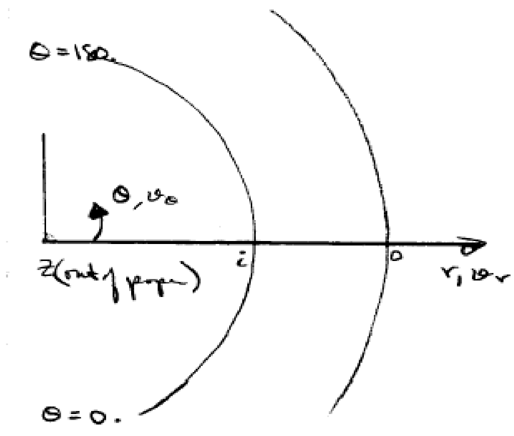
Modified Energy Equation to Include Minor Losses:

$$\frac{p_1}{\gamma} + z_1 + \frac{1}{2g}\alpha_1 V_1^2 + h_p = \frac{p_2}{\gamma} + z_2 + \frac{1}{2g}\alpha_2 V_2^2 + h_t + h_f + \sum \boxed{h_m}$$

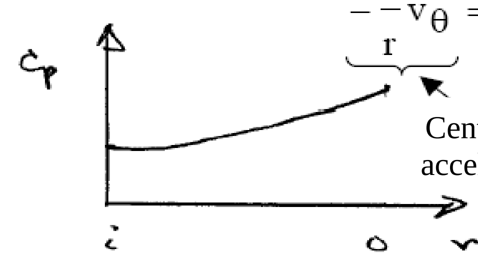
$$h_m = K\frac{V^2}{2g}$$

Note: Σh_m does not include pipe friction and e.g. in elbows and tees, this must be added to h_f.

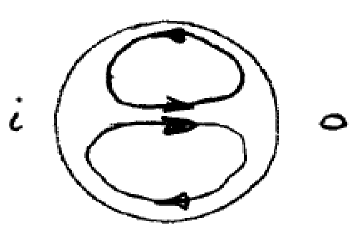
1. Flow in a bend:



$$\underbrace{-\frac{1}{r}v_\theta^2}_{\text{Centrifugal acceleration}} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$

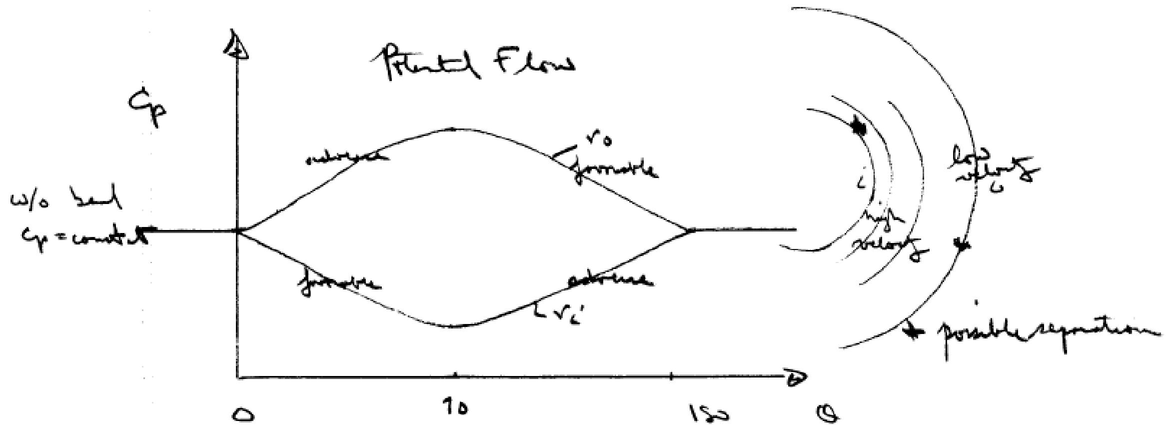


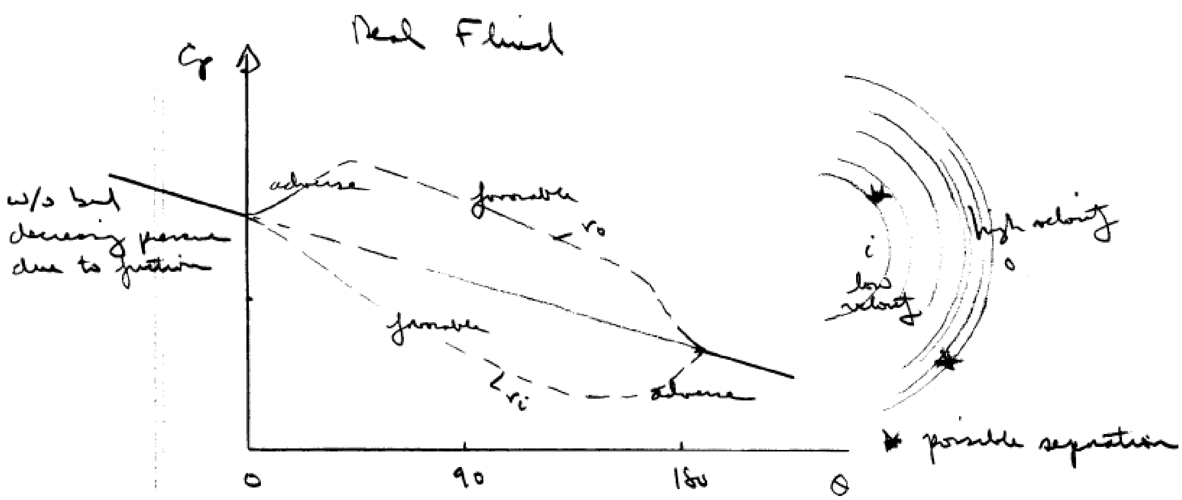
i.e. $\frac{\partial p}{\partial r} > 0$ which is an adverse pressure gradient in r direction. The slower moving fluid near wall responds first and a swirling flow pattern results.



This swirling flow represents an energy loss which must be added to the h_L.

Also, flow separation can result due to adverse longitudinal pressure gradients which will result in additional losses.





This shows potential flow is not a good approximate in internal flows (except possibly near entrance)

2. Valves: enormous losses

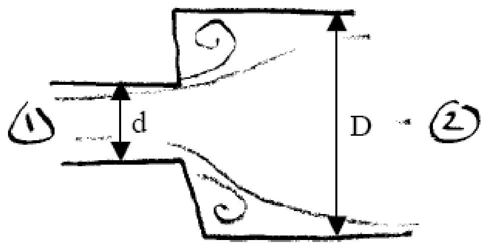
3. Entrances: depends on rounding of entrance

4. Exit (to a large reservoir): K = 1
   i.e., all velocity head is lost

5. Contractions and Expansions
   $\underbrace{\text{sudden}}$ or gradual

   theory for expansion:

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$



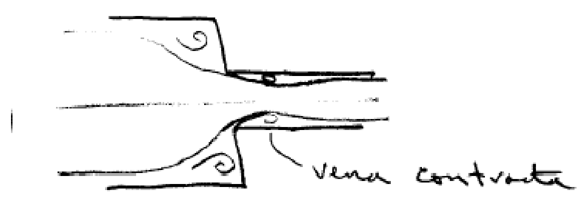
from continuity, momentum, and energy
(assuming $p = p_1$ in separation pockets)

$$\Rightarrow \quad K_{SE} = \left(1 - \frac{d^2}{D^2}\right)^2 = \frac{h_m}{V_1^2 / 2g}$$

no theory for contraction:

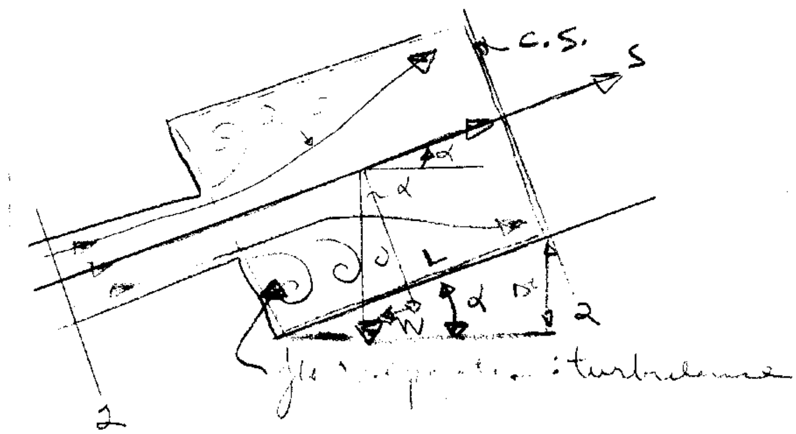
$$K_{SC} = .42\left(1 - \frac{d^2}{D^2}\right)$$

from experiment



Abrupt Expansion

Consider the flow from a small pipe to a larger pipe. Would like to know $h_L = h_L(V_1,V_2)$. Analytic solution to exact problem is extremely difficult due to the occurrence of flow separations and turbulence. However, if the assumption is made that the pressure in the separation region remains approximately constant and at the value at the point of separation, i.e., p_1, an approximate solution for h_L is possible:



Apply Energy Eq from 1-2 ($\alpha_1 = \alpha_2 = 1$)

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_L$$

Momentum eq. For CV shown (shear stress neglected)

$$\sum F_s = p_1 A_2 - p_2 A_2 - \underbrace{W \sin\alpha} = \sum \rho u \underline{V} \cdot \underline{A}$$

$$= \rho V_1(-V_1 A_1) + \rho V_2 (V_2 A_2)$$

$$= \rho V_2^2 A_2 - \rho V_1^2 A_1$$

$$\underbrace{\gamma A_2 L}_{W} \underbrace{\frac{\Delta z}{L}}_{\sin\alpha}$$

$\div \gamma A_2$
$$\underbrace{\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - (z_2 - z_1)}_{\text{from energy equation}} = \frac{V_2^2}{g} - \frac{V_1^2}{g}\frac{A_1}{A_2}$$

$\Downarrow$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_L = \frac{V_2^2}{g} - \frac{V_1^2}{g}\frac{A_1}{A_2}$$

$$h_L = \frac{V_2^2}{2g} + \frac{V_1^2}{2g}\left(1 - \frac{2A_1}{A_2}\right)$$

$$h_L = \frac{1}{2g}\left[V_2^2 + V_1^2 \underbrace{- 2V_1^2\frac{A_1}{A_2}}_{-2V_1V_2}\right]$$

continuity eq.
$V_1A_1 = V_2A_2$
$$\frac{A_1}{A_2} = \frac{V_2}{V_1}$$

$$\boxed{h_L = \frac{1}{2g}\left[V_2 - V_1\right]^2}$$

If $V_2 << V_1$, i.e., if $A_2 \to \infty$ $\left(V_2 = \frac{A_1}{A_2}V_1\right)$

$$\boxed{h_L = \frac{1}{2g}V_1^2}$$

And $K_L = \frac{h_L}{(V_1^2/2g)} \to 1$

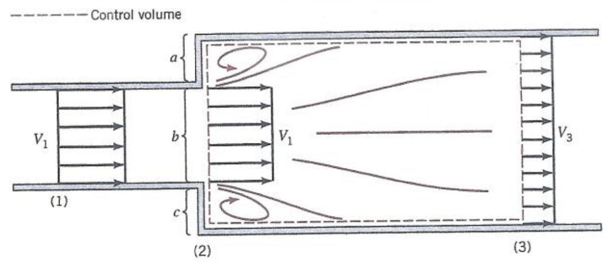


Figure 8.27 Control volume used to calculate the loss coefficient for a sudden expansion.

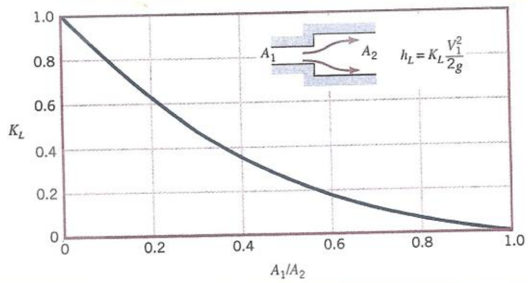


Figure 8.26 Loss coefficient for a sudden expansion (Ref. 10).

In many ways, the flow in a sudden expansion is similar to exit flow. As is indicated in Fig. 8.27, the fluid leaves the smaller pipe and initially forms a jet-type structure as it enters the larger pipe. Within a few diameters downstream of the expansion, the jet becomes dispersed across the pipe, and fully developed flow becomes established again. In this process [between sections (2) and (3)] a portion of the kinetic energy of the fluid is dissipated as a result of viscous effects. A square-edged exit is the limiting case with $A_1/A_2 = 0$.

A sudden expansion is one of the few components (perhaps the only one) for which the loss coefficient can be obtained by means of a simple analysis. To do this we consider the continuity and momentum equations for the control volume shown in Fig. 8.27 and the energy equation applied between (2) and (3). We assume that the flow is uniform at sections (1), (2), and (3) and the pressure is constant across the left side of the control volume ($p_a = p_b = p_c = p_1$). The resulting three governing equations (mass, momentum, and energy) are

$$A_1 V_1 = A_3 V_3$$

$$p_1 A_3 - p_3 A_3 = \rho A_3 V_3 (V_3 - V_1)$$

The loss coefficient for a sudden expansion can be theoretically calculated.

and

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + h_L$$

These can be combined to give the loss coefficient, $K_L = h_L/(V_1^2/2g)$, as

$$K_L = \left(1 - \frac{A_1}{A_2}\right)^2$$

where we have used the fact that $A_2 = A_3$. This result, plotted in Fig. 8.26, is in good agreement with experimental data. As with so many minor loss situations, it is not the viscous effects directly

(i.e., the wall shear stress) that cause the loss. Rather, it is the dissipation of kinetic energy (anot type of viscous effect) as the fluid decelerates inefficiently.

Gradual Expansion: diffuser

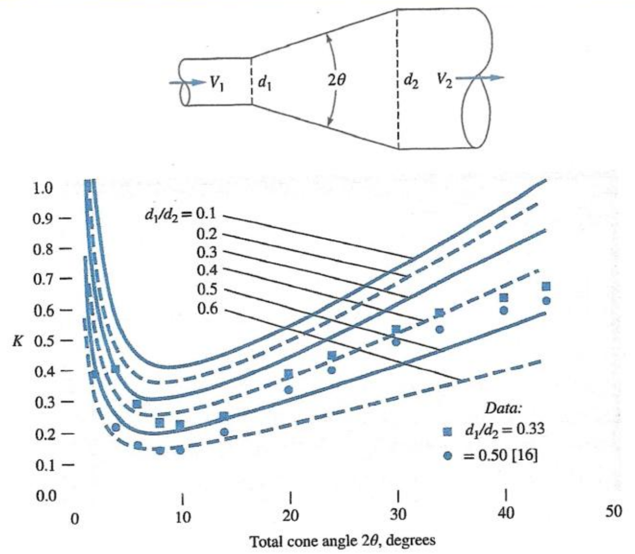


Fig. 6.23 Flow losses in a gradual conical expansion region, as calculated from Gibson's suggestion [15, 50], Eq. (6.80), for a smooth wall.

Velocity drops and pressure rises: efficient diffuser reduces required pump power

h_L can be large due to flow separation. If inlet boundary layer thin or fully developed flow somewhat reduce. $h_m = h_m(\Delta p_{nonideal}, \text{friction})$

$$K_m = \frac{h_m}{V_1^2/2g} = \underbrace{2.61 \sin\theta}_{\text{drop coeffit}} \left(1 - \frac{d^2}{D^2}\right) + f_{ave}\frac{L}{d_{ave}} \qquad 2\theta \le 45^\circ$$

drop coeffit (for $2\theta > 45^\circ$ replace by 1 so same abrupt expansion)

Good agreement data

Minimum loss $5^\circ < 2\theta < 15^\circ$

$2\theta < 5^\circ$ too long and friction large

$2\theta > 15^\circ$ large separation

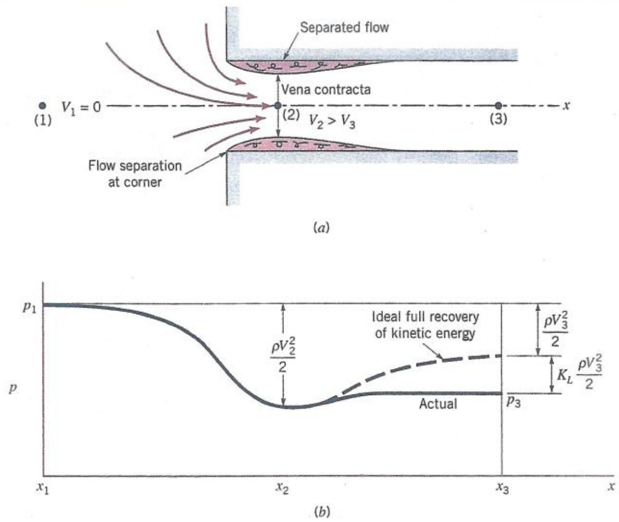


Figure 8.23 Flow pattern and pressure distribution for a sharp-edged entrance.

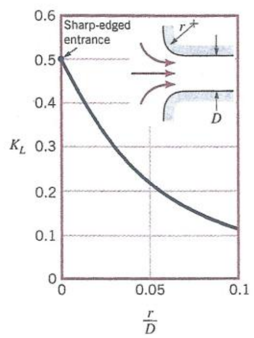


Figure 8.24 Entrance loss coefficient as a function of rounding of the inlet edge (Data from Ref. 9).

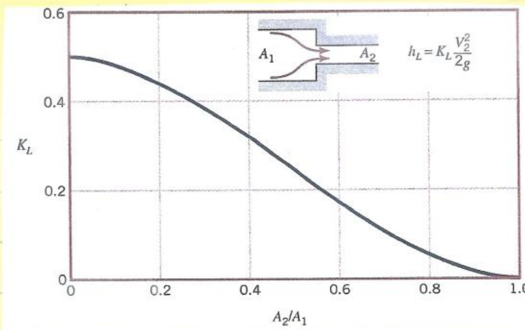


Figure 8.25 Loss coefficient for a sudden contraction (Ref. 10).

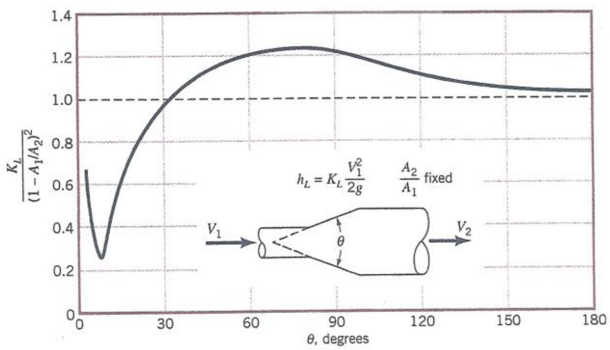


Figure 8.28 Loss coefficient for a typical conical diffuser (Ref. 5).

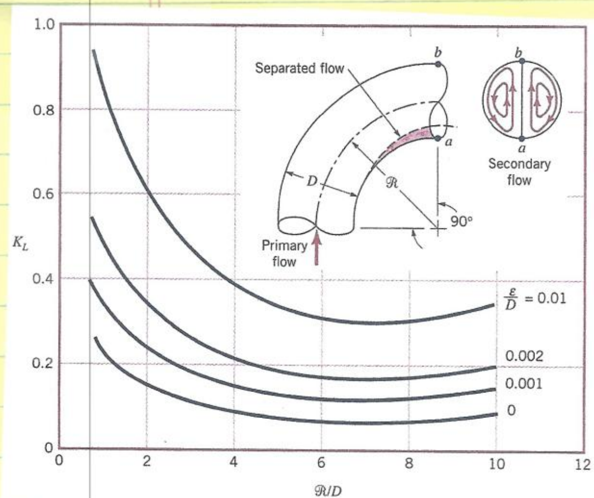


Figure 8.29 Character of the flow in a 90° bend and the associated loss coefficient (Ref. 5).

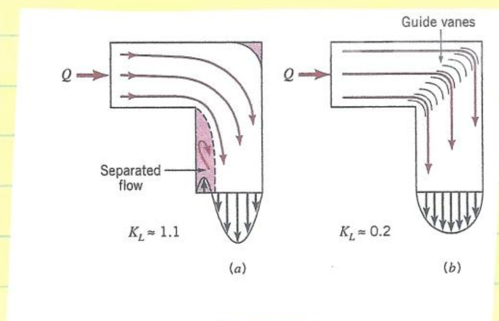


Figure 8.30 Character of the flow in a 90° mitered bend and the associated loss coefficient: (a) without guide vanes, (b) with guide vanes.

FIGURE 10.10
Flow characteristics at a pipe inlet (not to scale).

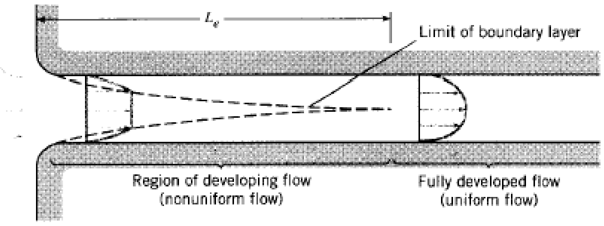
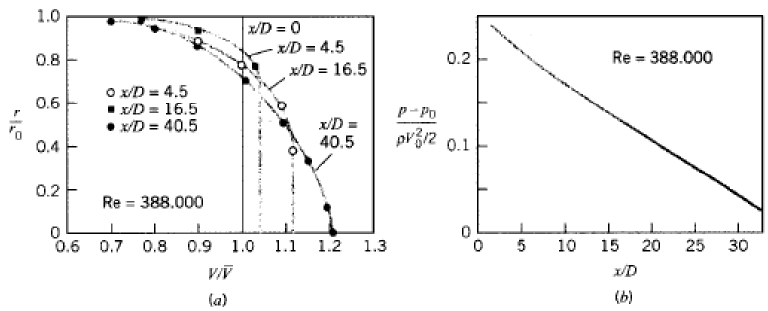
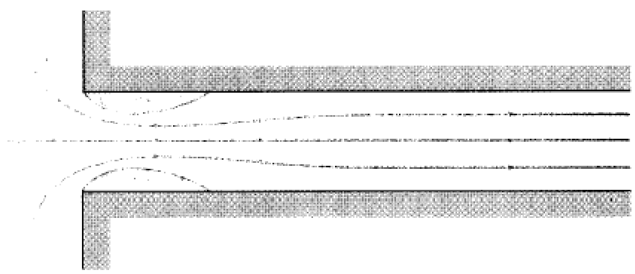


FIGURE 10.11
Distribution of velocity and pressure in the inlet region of a pipe [Barbin and Jones (3)].
(a) Velocity distribution.
(b) Pressure distribution.



Turbulent flow



K = .5

FIGURE 10.12
Flow at a sharp-edged inlet.

FIGURE 10.13
Flow pattern in an elbow.

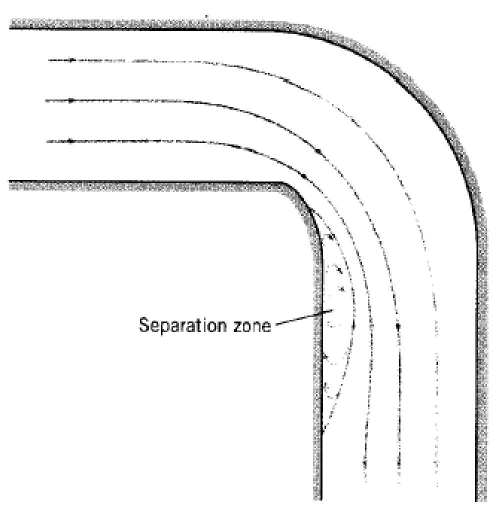


TABLE 10.2 LOSS COEFFICIENTS FOR VARIOUS TRANSITIONS AND FITTINGS

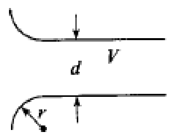
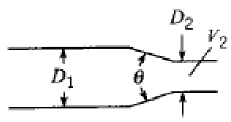
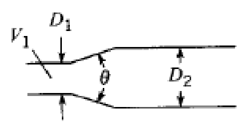
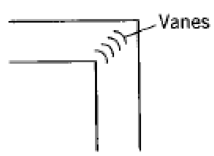
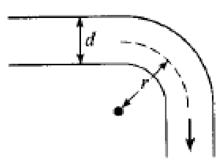
Description	Sketch	Additional Data		K	Source
Pipe entrance		r/d		K_e	(2)*
		0.0		0.50	
		0.1		0.12	
$h_L = K_e V^2/2g$		>0.2		0.03	
Contraction		D_2/D_1	K_C $\theta = 60°$	K_C $\theta = 180°$	(2)
		0.0	0.08	0.50	
		0.20	0.08	0.49	
		0.40	0.07	0.42	
		0.60	0.06	0.27	
		0.80	0.06	0.20	
$h_L = K_C V_2^2/2g$		0.90	0.06	0.10	
Expansion		D_1/D_2	K_E $\theta = 20°$	K_E $\theta = 180°$	(2)
		0.0		1.00	
		0.20	0.30	0.87	
		0.40	0.25	0.70	
		0.60	0.15	0.41	
$h_L = K_E V_1^2/2g$		0.80	0.10	0.15	
90° miter bend		Without vanes		$K_b = 1.1$	(37)
		With vanes		$K_b = 0.2$	(37)
90° smooth bend		r/d			(5) and (19)
		1		$K_b = 0.35$	
		2		0.19	
		4		0.16	
		6		0.21	
		8		0.28	
		10		0.32	
Threaded pipe fittings	Globe valve—wide open	$K_v = 10.0$			(37)
	Angle valve—wide open	$K_v = 5.0$			
	Gate valve—wide open	$K_v = 0.2$			
	Gate valve—half open	$K_v = 5.6$			
	Return bend	$K_b = 2.2$			
	Tee				
	straight-through flow	$K_t = 0.4$			
	side-outlet flow	$K_t = 1.8$			
	90° elbow	$K_b = 0.9$			
	45° elbow	$K_b = 0.4$			

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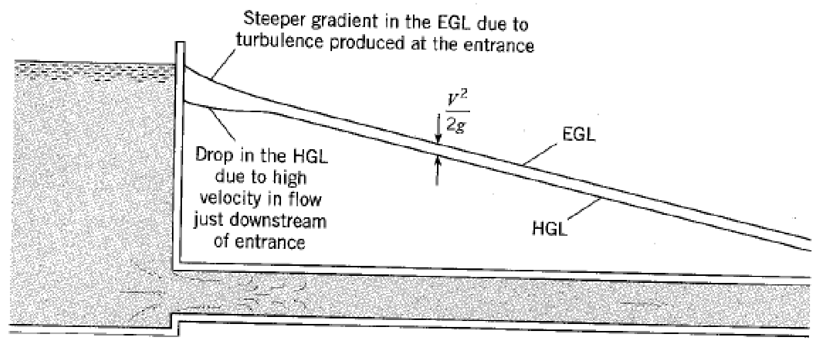
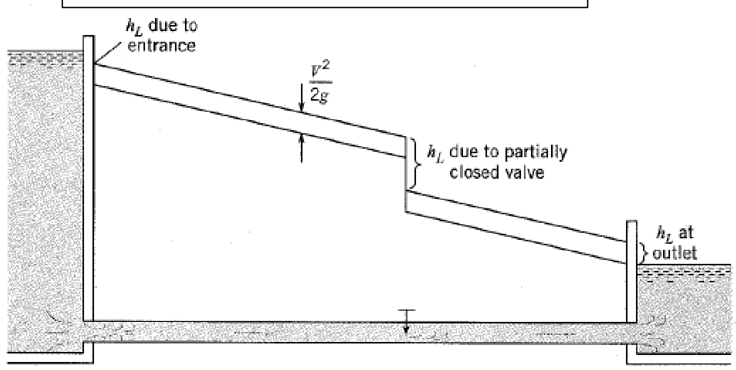
FIGURE 10.14
EGL and HGL at a sharp-edged pipe entrance.



$$HGL = \frac{P}{\gamma} + z$$

$$EGL = \frac{P}{\gamma} + z + \frac{V^2}{2g} = HGL + \frac{V^2}{2g}$$



IGURE 10.15
ead losses in a pipe.

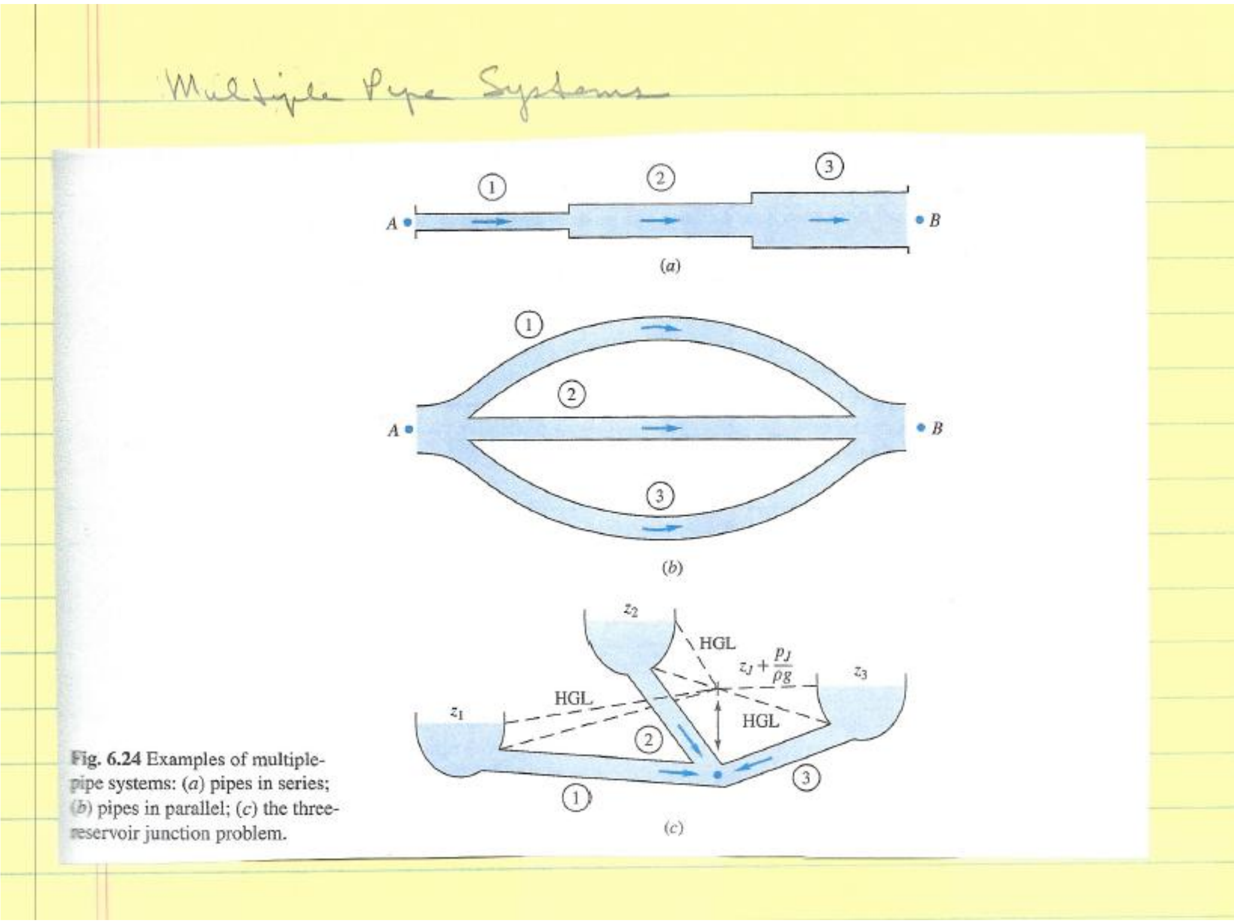
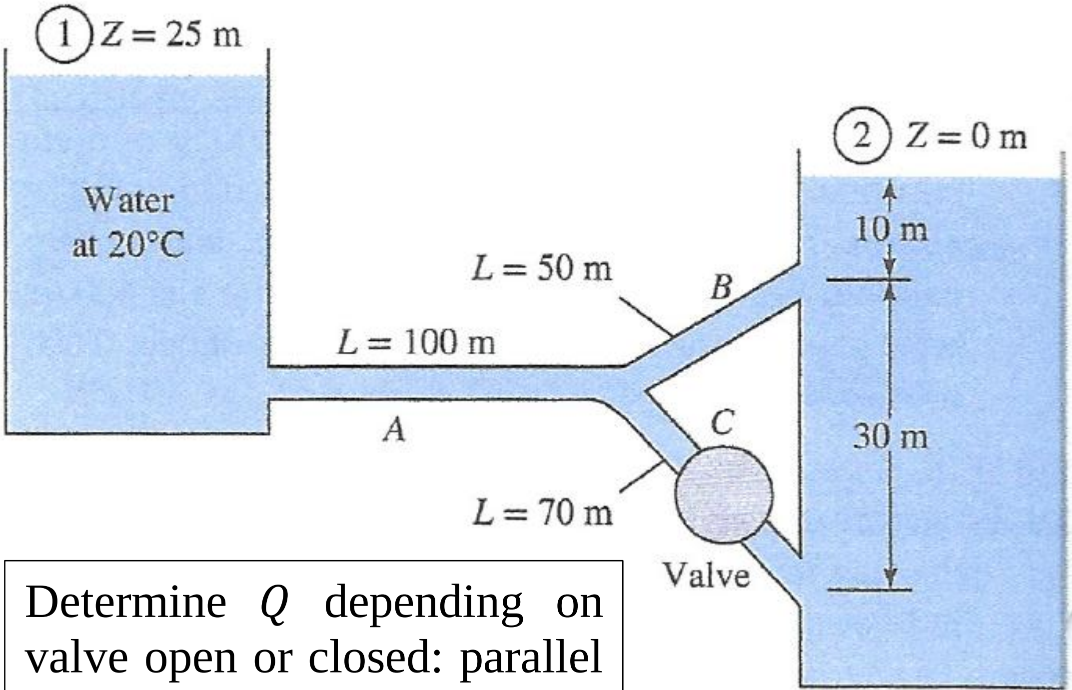
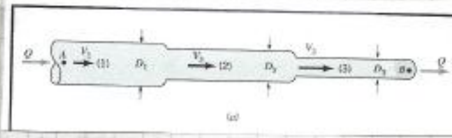


Fig. 6.24 Examples of multiple-pipe systems: (a) pipes in series; (b) pipes in parallel; (c) the three-reservoir junction problem.



Determine Q depending on valve open or closed: parallel pipes between two reservoirs with minor losses.



Pipes in Series

$$Q_1 = Q_2 = Q_3$$

$$V_1 D_1^2 = V_2 D_2^2 = V_3 D_3^2 \Rightarrow \frac{V_2^2}{2g} = \frac{V_1^2 D_1^4}{2g D_2^4}$$

$$\Delta h_{AB} = \Delta h_1 + \Delta h_2 + \Delta h_3 \qquad \frac{V_3^2}{2g} = \frac{V_1^2 D_1^4}{2g D_3^4}$$

$$\Delta h_{AB} = \frac{V_1^2}{2g}\left(\frac{f_1 L_1}{D_1} + \Sigma K_1\right) + \frac{V_2^2}{2g}\left(\frac{f_2 L_2}{D_2} + \Sigma K_2\right) + \frac{V_3^2}{2g}\left(\frac{f_3 L_3}{D_3} + \Sigma K_3\right)$$

$$= \frac{V_1^2}{2g}\left(\frac{f_1 L_1}{D_1} + \Sigma K_1\right) + \frac{V_1^2}{2g}\frac{D_1^4}{D_2^4}\left(\frac{f_2 L_2}{D_2} + \Sigma K_2\right) + \frac{V_1^2 D_1^4}{2g D_3^4}\left(\frac{f_3 L_3}{D_3} + \Sigma K_3\right)$$

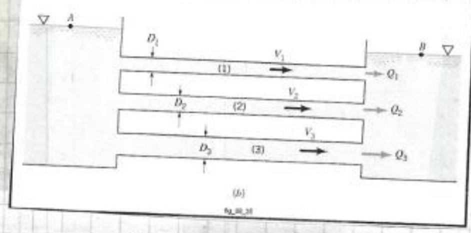
$$= \frac{V_1^2}{2g}\left[\alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3 + \alpha_0\right]$$

$$\alpha_1 = L_1/D_1 \qquad \alpha_2 = \frac{L_2}{D_2}\frac{D_1^4}{D_2^4} \qquad \alpha_3 = \frac{L_3}{D_3}\frac{D_1^4}{D_3^4}$$

$$\alpha_0 = \Sigma K_1 + \frac{D_1^4}{D_2^4}\Sigma K_2 + \frac{D_1^4}{D_3^4}\Sigma K_3$$

Q given : evaluate Δh_{AB}

Δh_{AB} given: iterate as per single pipe method for either Q or D



Pipes in Parallel

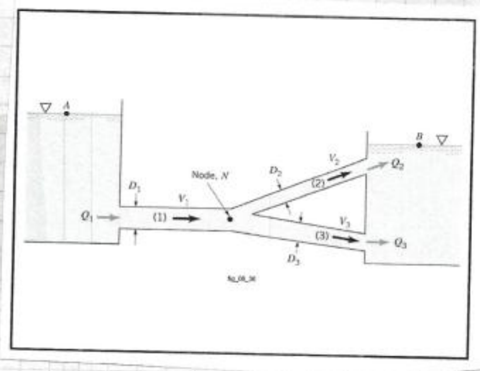
$$Q = Q_1 + Q_2 + Q_3$$

$$\Delta h_{AB} = \Delta h_1 = \Delta h_2 = \Delta h_3$$

Q given evaluate RHS

Δh_{AB} given: solve Q_i each pipe & $Q = \Sigma Q_i$

$\sim D_i$



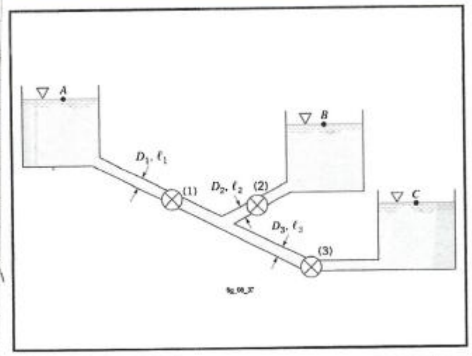
Multiple Pipe Loop

$$Q_1 = Q_2 + Q_3$$

$$\Delta h_2 = \Delta h_3 \Rightarrow h_{L_2} = h_{L_3}$$

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_2}$$

$$= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_3}$$



Three Reservoir Pipe Junction

$$Q_1 + Q_2 + Q_3 = 0$$ at J one or two < 0

$$h_J = z_J + \frac{p_J}{\gamma}$$

$$\Delta h_1 = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = z_1 - h_J$$

$$\Delta h_2 = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} = z_2 - h_J$$

$$\Delta h_3 = f_3 \frac{L_3}{D_3} \frac{V_3^2}{2g} = z_3 - h_J$$

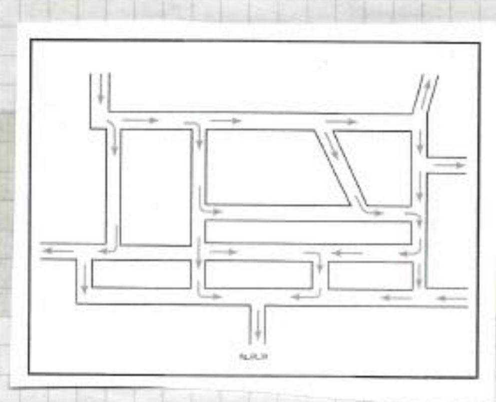
assuming $p_{A,B,C} = 0$ at reservoir surface at J Δh given $z_i - h_J$

guess h_J: solve V_i evaluate $\Sigma Q_i = 0$ iterate

$$\Delta h_1 = \left(\cancel{p_1/\gamma} + z_1\right) - \left(\frac{p_j}{\gamma} + z_j\right) = h_{f1} = z_1 - h_j$$

$$\Delta h_2 = \left(\cancel{p_2/\gamma} + z_2\right) - h_j = h_{f2} = z_2 - h_j$$

$$\Delta h_3 = \left(\cancel{p_3/\gamma} + z_3\right) - h_j = h_{f3} = z_3 - h_j$$



Piping Network

1. $\Sigma Q_i = 0$ all junctions

2. h_J = constant all junctions

3. $\Delta h = f \frac{L}{D} \frac{V^2}{2g}$ $\quad h_m = \frac{V^2}{2g} K$

Prof Hardy Cross (1936)

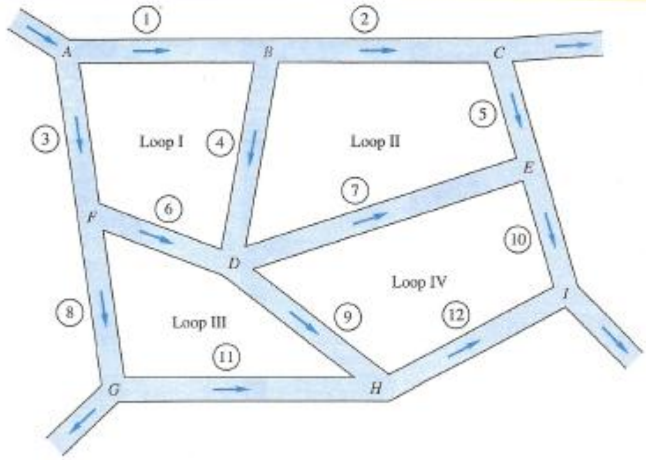


Fig. 6.25 Schematic of a piping network.

Pipe Networks

The ultimate case of a multipipe system is the *piping network* illustrated in Fig. 6.25. This might represent a water supply system for an apartment or subdivision or even a city. This network is quite complex algebraically but follows the same basic rules:

1. The net flow into any junction must be zero.
2. The net pressure change around any closed loop must be zero. In other words, the HGL at each junction must have one and only one elevation.
3. All pressure changes must satisfy the Moody and minor-loss friction correlations.

By supplying these rules to each junction and independent loop in the network, one obtains a set of simultaneous equations for the flow rates in each pipe leg and the HGL (or pressure) at each junction. Solution may then be obtained by numerical iteration, as first developed in a hand calculation technique by Prof. Hardy Cross in 1936 [17]. Computer solution of pipe network problems is now quite common and is covered in at least one specialized text [18]. Network analysis is quite useful for real water distribution systems if well calibrated with the actual system head loss data.