Chapter 7: Properties of Turbulent Free Shear Flow (Chap. 11 Bernard)

Part 1: Introduction

In many instances flows evolve without solid boundaries: wakes, jets, and mixing layers.

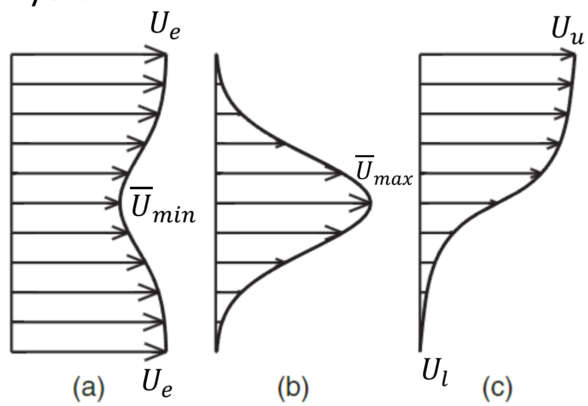

Figure 11.1 Basic characteristics of the mean flow in (a) wakes, (b) jets, and (c) mixing layers.

Wake flow: mean velocity deficit due to body gradually recovers.

Jet flow: high speed fluid at speed greater than surrounding expands into larger domain.

Mixing layer: mean velocity monotonically falls from high to low free stream over lateral distance that increases with downstream distance.

Thin Flow Approximation

$\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$ ⇒ BL type similarity solutions

$U =$ velocity scale

$l =$ length scale in crossflow direction (y) and to satisfy thin flow approximation:

$$\frac{l(x)}{x} \ll 1$$

For wake flow:

$$U = U_e - \overline{U}_{min}$$

Where U_e represents the velocity of the external flow.

For jet flow:

$$U = \overline{U}_{max}$$

For both wakes/jets, length scale $l =$ distance from the location of U_{min}/U_{max} to a point within a fixed percentage of free stream velocity.

For mixing layers:

$$U = U_u - U_l$$

And the length scale l is taken as $1/2$ the distance between where mean velocity within fixed percentage outer flow.

In all cases, $l(x)$ grows with x, but $l(x)/x \ll 1$. For wakes $l(x)/x \to 0$ as $x \to \infty$, whereas for jets and mixing layers $\lim_{x\to\infty} l(x)/x \approx 0.06$, which is small enough that the thin flow approximation is valid. When the rate of spreading of the shear layer is small, the flow is approximately parallel.

Recall equation for steady mean momentum in crossflow direction (Chapter 3):

$$\overline{U_j}\frac{\partial \overline{U_2}}{\partial x_j} + \frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_2} + \frac{\partial}{\partial x_j}\overline{u_2 u_j} = \nu \frac{\partial}{\partial x_j}\left(\frac{\partial \overline{U_2}}{\partial x_j}\right) \qquad \boxed{U_i = \overline{U}_i + u_i}$$

y BL equation:

$$\frac{\partial \overline{v^2}}{\partial y} = -\frac{1}{\rho}\frac{\partial \overline{P}}{\partial y}$$

Integrating across the thin shear layer gives:

$$\frac{\overline{P}}{\rho} + \overline{v^2} = \frac{P_e}{\rho} \quad (1)$$

$\frac{\partial P_e}{\partial x} = 0$ in free shear flows, but $\neq 0$ in BL flows, except flat plate BL.

Where $P_e =$ pressure outer flow. Differentiating Eq. (1) with respect to x gives

$$\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x} + \frac{\partial \overline{v^2}}{\partial x} = 0 \quad (2)$$

Recall equation for steady mean momentum in streamwise direction (Chapter 3):

$$\overline{U_j}\frac{\partial \overline{U_1}}{\partial x_j} + \frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_1} + \frac{\partial}{\partial x_j}\overline{u_1 u_j} = \nu \frac{\partial}{\partial x_j}\left(\frac{\partial \overline{U_1}}{\partial x_j}\right) \quad (3)$$

And substituting Eq. (2) into (3) yields:

$$\overline{U}\frac{\partial \overline{U}}{\partial x} + \overline{V}\frac{\partial \overline{U}}{\partial y} + \frac{\partial}{\partial x}\left(\overline{u^2} - \overline{v^2}\right) + \frac{\partial \overline{uv}}{\partial y} = \nu\left(\frac{\partial^2 \overline{U}}{\partial x^2} + \frac{\partial^2 \overline{U}}{\partial y^2}\right)$$

Dominant terms are like BL theory:

$$\overline{U}\frac{\partial \overline{U}}{\partial x} + \overline{V}\frac{\partial \overline{U}}{\partial y} + \frac{\partial \overline{uv}}{\partial y} = 0 \quad (4)$$

Eq. (4) represents the basis for subsequent analyses of thin free shear flows.

For wakes and jets:

$$(\overline{U}-U_e)\underbrace{\left(\frac{\partial \overline{U}}{\partial x}+\frac{\partial \overline{V}}{\partial y}\right)}_{\boxed{\text{continuity}}}=0 \quad (5)$$

And since $U_e=$ constant,

$$\overline{U}\frac{\partial U_e}{\partial x}+\overline{V}\frac{\partial U_e}{\partial y}=0 \quad (6)$$

Adding Eq. (4) and (5) and subtracting Eq. (6) gives

$$\frac{\partial}{\partial x}\left[\overline{U}(\overline{U}-U_e)\right]+\frac{\partial}{\partial y}\left[\overline{V}(\overline{U}-U_e)\right]+\frac{\partial}{\partial y}\overline{uv}=0 \quad (7)$$

$\overline{U}-U_e$ and $\overline{uv}\rightarrow 0$ for $|y|\rightarrow\infty$. Thus, Eq, (7) can be integrated across the flow:

$$\frac{d}{dx}\int_{-\infty}^{\infty}\overline{U}(\overline{U}-U_e)dy=0$$

Thus, total mean flux of momentum per unit length in spanwise direction is:

$$M=\rho\int_{-\infty}^{\infty}\overline{U}(\overline{U}-U_e)dy=\text{constant}\neq f(x)$$

For wake $M=$ momentum deficit due to body drag

For jet $M=$ initial value exiting nozzle $\rho U_j^2 A$ or h per unit width, neglecting losses developing region.

Laminar Jet Asymptotic Laws:

Same solutions as for turbulent flow (solutions are indistinguishable except $\nu = \nu_t$)

	u_{max}	W_{max}	$\dot{m}$	δ
AXI	x^{-1}		x	x
AXI with W (rotationally symmetric)	x^{-1}	x^{-2}	x	x
2D	$x^{-1/3}$		$x^{1/3}$	$x^{2/3}$
	$x^{-1/2}$			x

2D turbulent jet

Wake Asymptotic Laws:

	$(u_1)_{max}$	δ	
2D	$x^{-1/2}$	$x^{1/2}$	Same turbulent
AXI	x^{-1}	$x^{1/2}$	but different for axi
	$x^{-2/3}$	$x^{1/3}$	