Name: ------------------ **Quiz 4** **Time: 20 minutes**

ME:5160 **Fall 2025**

The exam is closed book and closed notes.

Water flows through a circular nozzle, exits into the air as a jet, and strikes a plate, as shown in below figure. The force required to hold the plate steady is 70 N. Assuming steady, one-dimensional flow and Turbulent pipe flow kinetic correction factor α = 1. Consider the head loss of nozzle. (Use $K_c = 0.7$) estimate (*a*) the velocities at sections (1) and (2) and (*b*) the mercury manometer reading *h*.

[ρ_{water} = 998 kg/m^3, ρ_{Hg} = 13567.78 kg/m^3]

Continuity equation: $-\frac{d}{dt}\int_{CV}\rho d\forall = \int_{CS}\rho \underline{V_R}\cdot\underline{n}dA$

Momentum equation: $\sum F = \frac{d}{dt}\int_{CV}\rho\underline{V}d\forall + \int_{CS}\rho\underline{V}\underline{V_R}\cdot\underline{n}dA$

$$\left(\frac{p}{\rho g}+\frac{\alpha V^2}{2g}+z\right)_1 = \left(\frac{p}{\rho g}+\frac{\alpha V^2}{2g}+z\right)_2 + h_{loss}$$

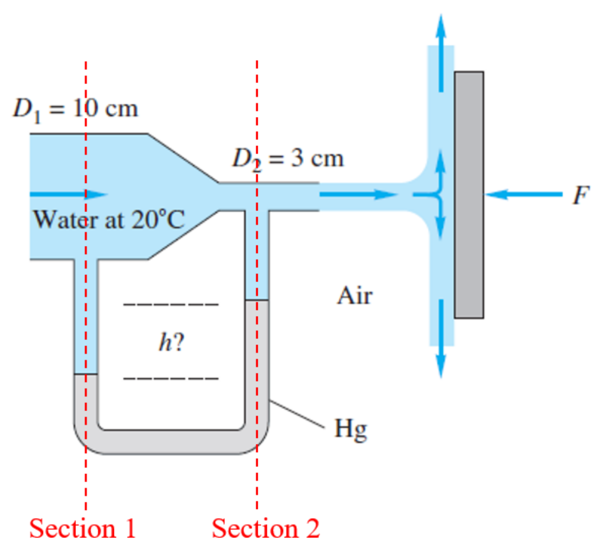
Turbulent pipe flow kinetic correction factor α = 1

$$\left[h_{loss} = K_c\frac{V_2^2}{2g}\right]$$

Hint: Calculate V_2 using linear momentum equation.

Calculate V_1 using continuity with V_2.

Calculate pressure difference between section 1 and 2 considering head loss, then calculate h using both water and mercury density.



Solution:

(a) Momentum equation in x direction:

$$\sum \underline{F} = \frac{d}{dt}\int_{CV} \rho \underline{V_R} d\forall + \int_{CS} \rho \underline{V_R} \left(\underline{V_R} \cdot \underline{n}\right) dA$$

$$\sum F_x - \dot{m}_{in} u_{in} = -\rho A_2 V_2^2 \qquad \textbf{(+3.5)}$$

$$70N = -(998)\frac{\pi}{4}(0.03^2)(V_2^2) \quad \therefore V_2 = 9.96\, m/s \qquad \textbf{(+1)}$$

Continuity:

$$V_1 A_1 = V_2 A_2, \qquad V_1 = \frac{V_2 A_2}{A_1} = \frac{(9.96)\frac{\pi}{4}0.03^2}{\frac{\pi}{4}0.1^2} \qquad \therefore V_1 = 0.9\, m/s \qquad \textbf{(+1)}$$

(b) Calculate pressure difference using Bernoulli (Energy) equation

$$p_1 - p_2 = \frac{1}{2}\rho(V_2^2 - V_1^2) + \rho g h_{loss} \qquad \textbf{(+2.5)}$$

$$p_1 - p_2 = \frac{1}{2}\rho(V_2^2 - V_1^2) + \rho g K_c \frac{V_2^2}{2g}$$

$$= \frac{1}{2}(998)(9.96^2 - 0.9^2) + (998)(9.81)\left(0.7\frac{9.96^2}{2 \times 9.81}\right) = 83748.53\, Pa \qquad \textbf{(+1)}$$

$$\therefore \Delta P = 83748.53 Pa$$

$$\Delta P = \Delta(\rho) g h$$

$$h = \frac{\Delta P}{\Delta(\rho) g} = \frac{83748.53}{(13567.78 - 998)9.81} \approx 0.679m \qquad \textbf{(+1)}$$