$\Gamma = \oint_S \underline{V} \cdot \underline{ds} = \int_A \nabla \times \underline{V} \cdot \underline{dA}$ (Stokes's theorem - relates line and area integrals) line integral for tangential velocity component $= \int_A \underline{\omega} \cdot \underline{n}\ dA$ = flux (surface integral) of normal vorticity component.

Kutta-Joukowski Theorem: lift (L) per unit span for an arbitrary 2D cylinder in uniform stream U with density ρ is L = ρUΓ, with direction of L perpendicular to U.

The **Kutta–Joukowski theorem** is a fundamental theorem in aerodynamics used for the calculation of lift of an airfoil (and any two-dimensional body including circular cylinders) translating in a uniform fluid at a constant speed so large that the flow seen in the body-fixed frame is steady and unseparated. The theorem applies to two-dimensional inviscid flow around an airfoil section (or any shape of infinite span).

There isn't a contradiction between the Kutta-Joukowski theorem and irrotational flow; rather, the theorem is derived using the assumption of irrotational (potential) flow. The perceived conflict arises from Kelvin's theorem, which states that an initially irrotational fluid remains irrotational, thus creating no mechanism for generating the circulation needed by the Kutta-Joukowski theorem to produce lift. This is resolved by the Kutta condition and the concept of the starting vortex in viscous flow, which explains how circulation is established from an inviscid, irrotational start-up.

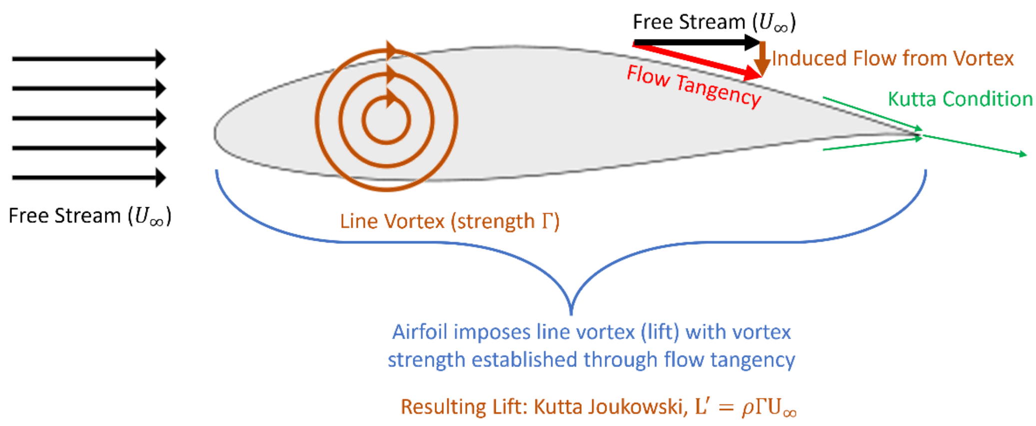

Figure: Basic components of the lift model relevant to airfoil theory.

Irrotational Flow and the Kutta-Joukowski Theorem

- **Irrotational flow**, also known as potential flow, is a theoretical model where fluid particles do not rotate.
- The Kutta-Joukowski theorem relates the lift force on an airfoil to the flow's circulation (a measure of rotationality) and the freestream velocity.
- **The theorem requires circulation to exist**: to produce lift; otherwise, the lift would be zero (D'Alembert's paradox).

The Problem: Kelvin's Circulation Theorem

- **Kelvin's circulation theorem**: states that in an ideal (inviscid), incompressible fluid, the circulation around a closed contour remains constant over time.
- If a fluid starts as irrotational, it should always remain irrotational according to this theorem.
- This presents a fundamental problem: where does the circulation come from if the flow starts irrotational? There's no apparent mechanism in a truly inviscid, irrotational flow to create the necessary circulation for lift.

The Resolution: Viscosity and the Kutta Condition

- The issue is resolved by considering viscosity, even for an "inviscid" flow.
- When an airfoil starts moving, a starting vortex forms and detaches from the trailing edge.
- This starting vortex has a strength and direction of circulation that is opposite to the circulation that forms around the airfoil.
- The Kutta condition states that the flow must leave the sharp trailing edge smoothly, without infinite velocities.
- By satisfying the Kutta condition and the generation of the starting vortex, the necessary circulation around the airfoil is established.

In Summary

- The contradiction is not between the theorem and irrotational flow but between the theoretical implication (no initial mechanism for circulation in inviscid flow) and the observed reality (lift and circulation are generated).
- Viscosity is essential for creating this initial circulation, which is explained by the formation of the starting vortex and the Kutta condition.
- Therefore, the Kutta-Joukowski theorem, while derived for ideal flow, is applied to real-world viscous flows by recognizing that the initial conditions (viscosity and the start-up vortex) provide the circulation.