Flow Classification

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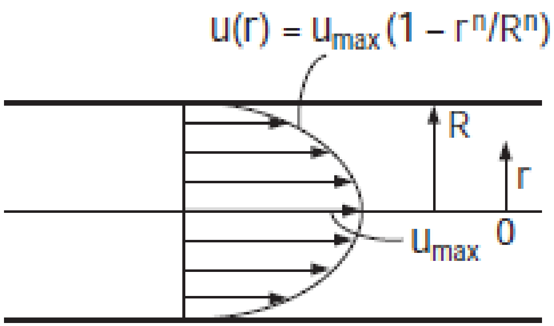
1. One-, Two-, and Three-dimensional Flow

- A flow is said to be one-, two-, or three-dimensional if the flow velocity varies in one, two, or three primary dimensions, respectively. For example,

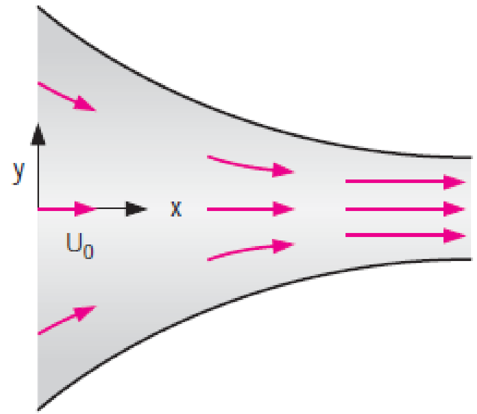
  1D: $\underline{V} = u(y)\hat{\boldsymbol{\imath}}$ or $\underline{V} = u_r\hat{\boldsymbol{e}}_{\boldsymbol{r}}$

  2D: $\underline{V} = u(x,y)\hat{\boldsymbol{\imath}} + v(x,y)\hat{\boldsymbol{\jmath}}$ or $\underline{V} = u_r\hat{\boldsymbol{e}}_{\boldsymbol{r}} + u_\theta\hat{\boldsymbol{e}}_{\boldsymbol{\theta}}$

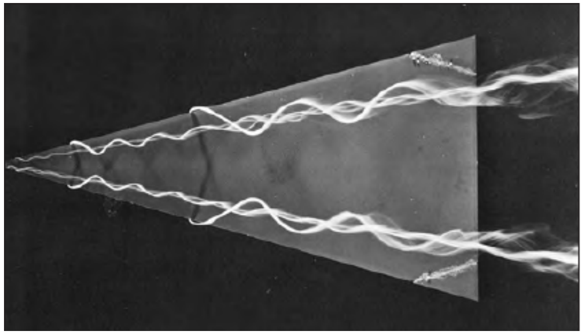
  3D: $\underline{V} = u(x,y,z)\hat{\boldsymbol{\imath}} + v(x,y,z)\hat{\boldsymbol{\jmath}} + w(x,y,z)\widehat{\boldsymbol{k}}$ or $\underline{V} = u_r\hat{\boldsymbol{e}}_{\boldsymbol{r}} + u_\theta\hat{\boldsymbol{e}}_{\boldsymbol{\theta}} + u_z\hat{\boldsymbol{e}}_{\boldsymbol{z}}$



1D flow: Fully-developed laminar pipe flow



2D flow: Converging duct flow



3D flow: Visualization of flow over a 15° delta wing at a 20° angle of attack at a Reynolds number of 20,000.

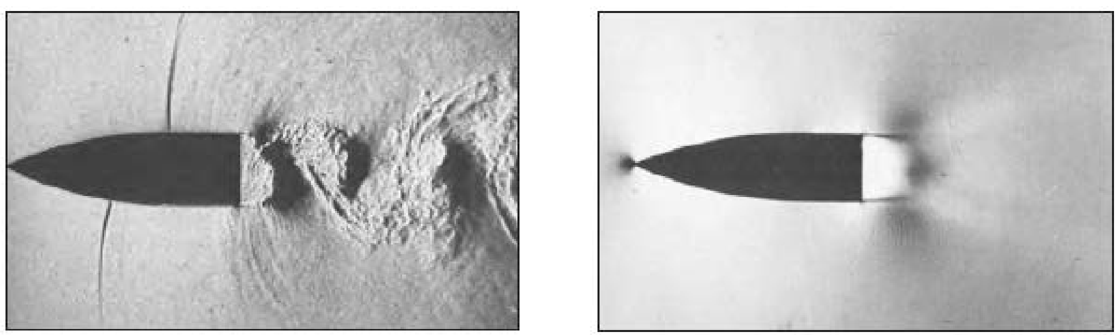
2. Steady vs. Unsteady Flow

- The term **steady** implies no change at a point with time; the opposite of steady is **unsteady** (Note: Uniform flow: No change with location over a specific region).

  $\underline{V} = \underline{V}(\underline{x})$ Steady flow

  $\underline{V} = \underline{V}(\underline{x}, t)$ Unsteady flow

- During steady flow, fluid properties can change from point to point, but at any fixed point they remain constant.



(a) (b)

Oscillating wake of a blunt-based airfoil at Mach number 0.6. Photo (a) is an instantaneous image (i.e., unsteady), while Photo (b) is a long-exposure (time-averaged, i.e., steady) image.

3. Incompressible vs. Compressible Flow

- A flow is said to be **incompressible** if the density remains constant throughout. Therefore, the volume of every portion of fluid remains unchanged over the course of its motion when the flow (or the fluid) is incompressible.

$$\frac{D\rho}{Dt} = 0 \qquad \Rightarrow \qquad \text{imcompressilbe flow}$$

- When analyzing rockets, spacecraft, and other systems that involve high-speed gas flows, the flow speed is often expressed in terms of the dimensionless **Mach number** defined as

$$\mathrm{Ma} = \frac{V}{c} = \frac{\text{Speed of flow}}{\text{Speed of sound}}$$

o Ma < 0.3	Incompressible
o Ma > 0.3	Compressible
o Ma = 1	Sonic (commercial air craft Ma ~ 0.8)
o Ma > 1	Super-sonic

- Ma is the most important non-dimensional parameter for compressible flows (See Chapter 7 Dimensional Analysis).

U.S. Navy F/A-18 approaching the sound barrier. The white cloud forms as a result of the supersonic expansion fans dropping the air temperature below the dew point.

4. Viscous vs. Inviscid Flows

- Flows in which the frictional effects are significant are called **viscous flows**: The flow viscosity $\mu \neq 0$ and "Real-Flow Theory" which requires complex analysis, often with no choice.
- However, in many flows of practical interest, there are regions (typically regions not close to solid surfaces) where viscous forces are negligibly small compared to inertial or pressure forces. Neglecting the viscous terms (i.e., $\mu = 0$) in such **inviscid flow regions** greatly simplifies the analysis without much loss in accuracy. (However, must decide when this is a good approximation; D' Alembert paradox: $C_D = 0$ for a body in steady motion!)

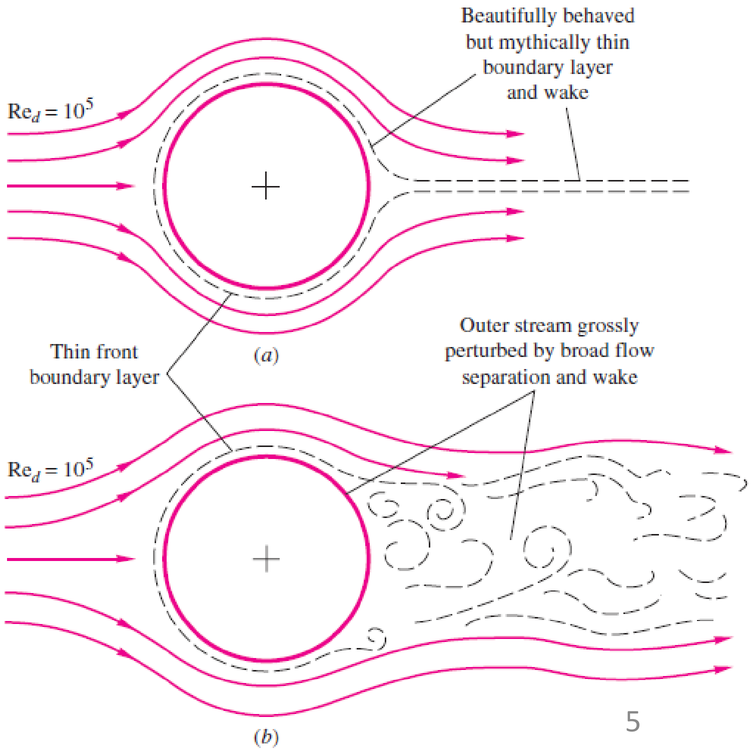


Illustration of the strong interaction between viscous and inviscid regions in the rear of blunt-body flow: (a) idealized and definitely false picture of the blunt-body flow; (b) actual picture of blunt-body flow.

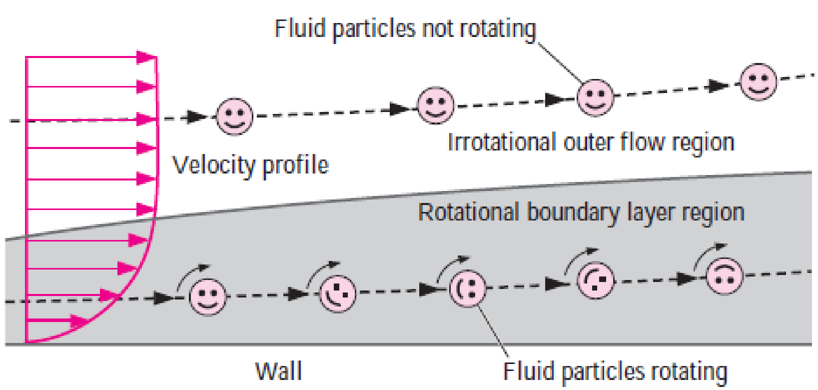
5. Rotational vs. Irrotational Flow

- Vorticity:

$$\underline{\Omega} = \nabla \times \underline{V} = 0 \qquad \text{Rotational flow}$$

$$\underline{\Omega} = 0 \qquad \text{Irrotational flow}$$

- If the vorticity at a point in a flow field is nonzero, the fluid particle that happens to occupy that point in space is rotating; the flow in that region is called **rotational.**
- Likewise, if the vorticity in a region of the flow is zero (or negligibly small), fluid particles there are not rotating; the flow in that region is called **irrotational.**
- Generation of vorticity usually is the result of viscosity, ∴ viscous flows are always rotational, whereas inviscid flows are usually irrotational.
- Inviscid, irrotational, incompressible flow is referred to as ideal-flow theory.



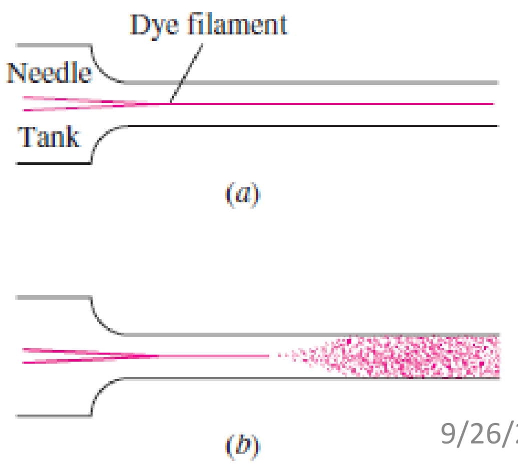
The difference between rotational and irrotational flow: fluid elements in a rotational region of the flow rotate, but those in an irrotational region of the flow do not.

6. Laminar vs. Turbulent Viscous Flows

- **Laminar flow**: Smooth orderly motion composed of thin sheets (i.e., laminas) gliding smoothly over each other
- **Turbulent flow**: Disorderly high frequency fluctuations superimposed on main motion. Fluctuations are visible as eddies which continuously mix, i.e., combine and disintegrate (average size is referred to as the scale of turbulence).
- Reynolds decomposition

$$u = \underbrace{\bar{u}}_{\substack{\text{mean}\\\text{motion}}} + \underbrace{u'}_{\substack{\text{turbulent}\\\text{fluctuation}}}$$

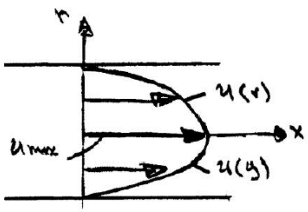
Usually u' is 0.01 ~ 0.1 $\bar{u}$, but its influence is as if μ increase by 100 – 10,000 times or more.



Reynold's sketches of pipe flow transition: (a) low-speed, laminar flow; (b) high-speed, turbulent flow.

6. Laminar vs. Turbulent Viscous Flows – Contd.

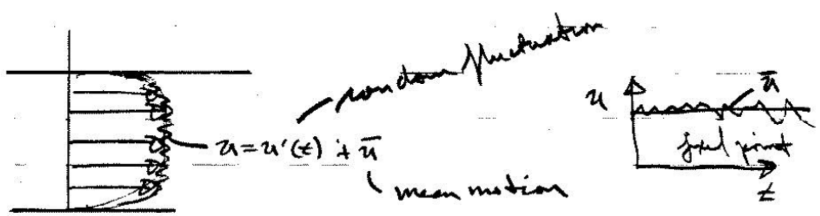
Example: Pipe Flow (Chapter 8 = Flow in Conduits)
Laminar flow:



$$u(r) = \frac{R^2 - r^2}{4\mu}\left(-\frac{dp}{dx}\right)$$

u(y),velocity profile in a paraboloid

Turbulent flow: fuller profile due to turbulent mixing extremely complex fluid motion that defies closed form analysis.



Turbulent flow is the most important area of motion fluid dynamics research.

The most important nondimensional number for describing fluid motion is the Reynolds number (Chapter 8)

$$Re = \frac{VD\rho}{\mu} = \frac{VD}{\nu}$$

V = characteristic velocity
D = characteristic length

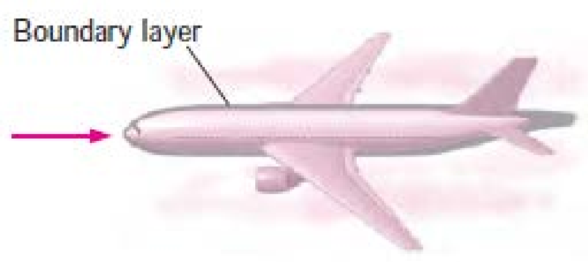
For pipe flow
- $V = \overline{V}$ = average velocity
- D = pipe diameter

Re < 2300	laminar flow
Re > 2300	turbulent flow

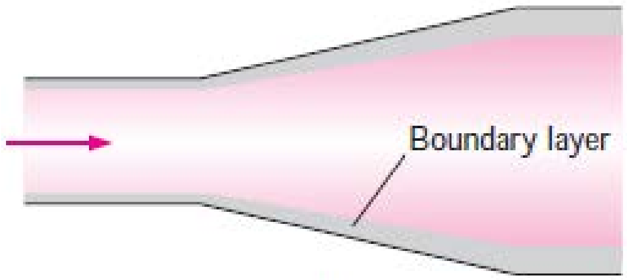
Also depends on roughness, free-stream turbulence, etc.

7. Internal vs. External Flows

- **Internal flows** = completely wall bounded; Usually requires viscous analysis, except near entrance (Chapter 8)
- **External flows** = unbounded; i.e., at some distance from body or wall flow is uniform (Chapter 9, Surface Resistance)
- Internal flows are dominated by the influence of viscosity throughout the flow field, whereas in external flows the viscous effects are limited to boundary layers near solid surfaces and to wake regions downstream of bodies.



(a)



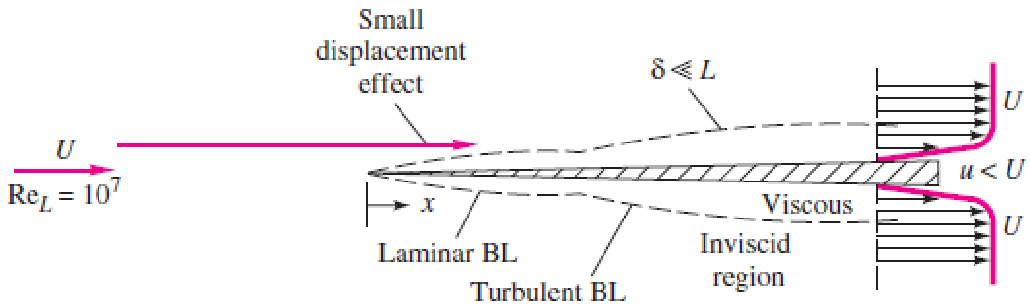
(b)

(a) External flow: boundary layer developing along the fuselage of an airplane and into the wake, and (b) Internal flow: boundary layer growing on the wall of a diffuser.

7. Internal vs. External Flows – Contd.

- Flow Field Regions (high Re flows):

- External Flow exhibits flow-field regions such that both inviscid and viscous analysis can be used depending on the body shape and Re.
- Important features:
  1) **low Re:** viscous effects important throughout entire fluid domain: creeping motion
  2) **high Re flow about streamlined body:** viscous effects confined to narrow region: boundary layer and wake
  3) **high Re flow about bluff bodies**: in regions of adverse pressure gradient flow is susceptible to separation and viscous-inviscid interaction is important

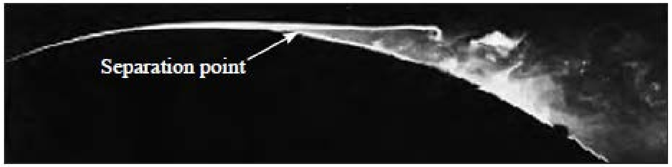


Comparisons of flow past a sharp flat plate at a high Reynolds number. The boundary layer (BL) divides the flow into two regions: the viscous region in which the frictional effects are significant, and the inviscid region in which the frictional effects are negligible.

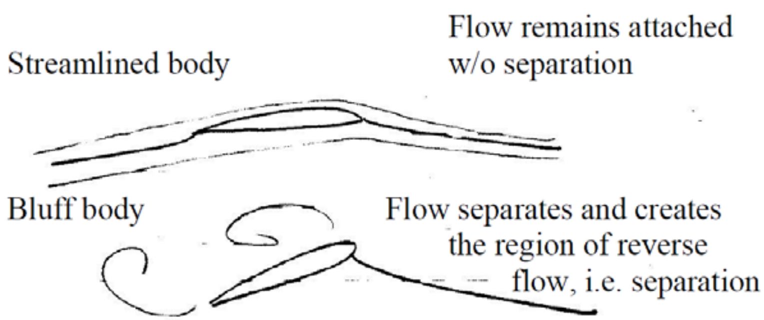
8. Separated vs. Unseparated Flow

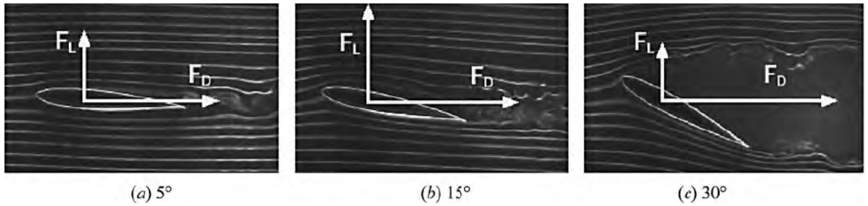
- When a fluid is forced to flow over a curved surface, such as the back side of a cylinder at sufficiently high velocity, the boundary layer can no longer remain attached to the surface, and at some point it separates from the surface—a process called flow separation.

- When a fluid separates from a body, it forms a **separated region** between the body and the fluid stream. The region of flow trailing the body where the effects of the body on velocity are felt is called the **wake**.



Flow separation during flow over a curved surface





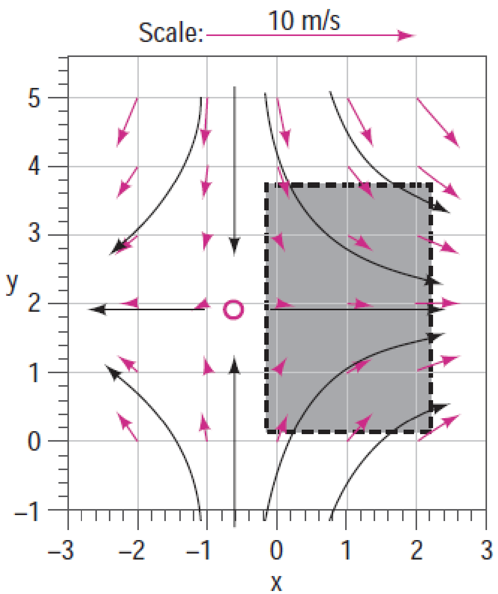
The occurrence of separation is not limited to blunt bodies. At large angles of attack (usually larger than 15°), flow may separate completely from the top surface of an airfoil, reducing lift drastically and causing the airfoil to stall.

Example 1: Stagnation Point Flow

A steady, incompressible, two-dimensional velocity field is given by

$$\vec{V} = (u, v) = (0.5 + 0.8x)\,\vec{i} + (1.5 - 0.8y)\,\vec{j} \qquad (1)$$

where the *x*- and *y*-coordinates are in meters and the magnitude of velocity is in m/s. A **stagnation point** is defined as *a point in the flow field where the velocity is identically zero*. (*a*) Determine if there are any stagnation points in this flow field and, if so, where? (*b*) Sketch velocity vectors at several locations in the domain between $x = -2$ m to 2 m and $y = 0$ m to 5 m; qualitatively describe the flow field.



Velocity vectors for the given velocity field. The solid black curves represent the approximate shapes of some streamlines. The stagnation point is indicated by the circle. The shaded region represents a portion of the flow field that can approximate flow into an inlet.

- At stagnation point, the velocity magnitude becomes zero,

$$V = |\underline{V}| = \sqrt{u^2 + v^2} = \sqrt{(0.5 + 0.8x)^2 + (1.5 - 0.8y)^2} = 0$$

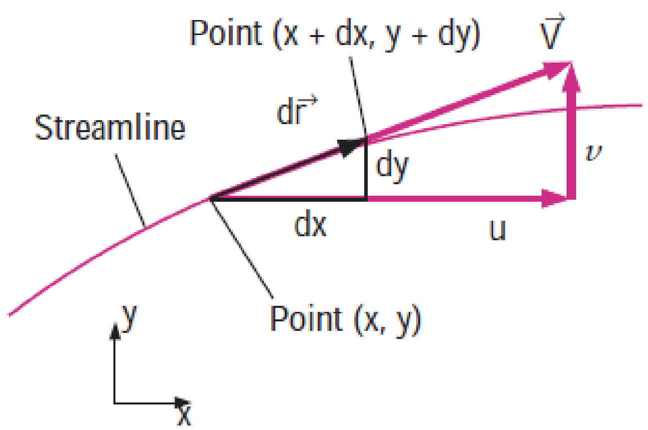
or

$$u = 0.5 + 0.8x = 0 \quad \rightarrow \quad x = -0.625 \text{ m}$$
$$v = 1.5 - 0.8y = 0 \quad \rightarrow \quad y = 1.875 \text{ m}$$

- The flow can be described as stagnation point flow in which flow enters from the top and bottom and spreads out to the right and left about a horizontal line of symmetry at $y = 1.875$ m.

Streamlines

- A streamline is a curve that is everywhere tangent to the instantaneous local velocity vector.
- Streamlines are useful as indicators of the instantaneous direction of fluid motion throughout the flow field. For example, regions of recirculating flow and separation of a fluid off a solid wall are easily identified by the streamline patter.
- Streamlines cannot be directly observed experimentally except in steady flow fields, in which they are coincident with pathlines and streaklines.



For two-dimensional flow in the xy-plane, arc length $d\vec{r} = (dx, dy)$ along a streamline is everywhere tangent to the local instantaneous velocity vector $\vec{V} = (u, v)$.

- Equation for a streamline:

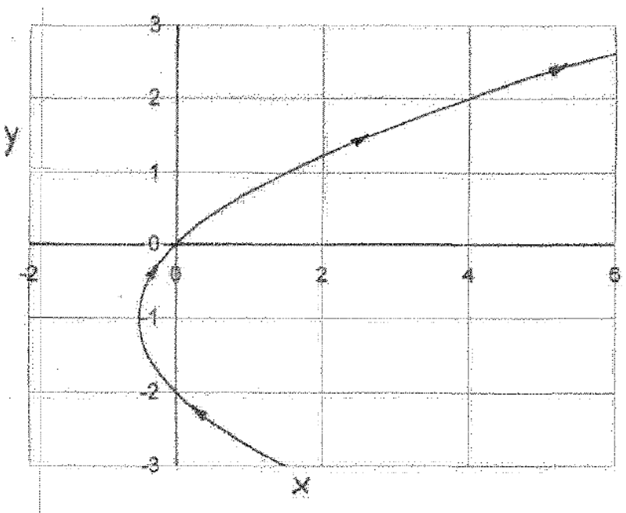
$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

- Streamline in the xy-plane:

$$\left(\frac{dy}{dx}\right)_{\substack{\text{along} \\ \text{a streamline}}} = \frac{v}{u}$$

Example 2: Streamline

4.4 A two-dimensional velocity field is given by $u = 1 + y$ and $v = 1$. Determine the equation of the streamline that passes through the origin. On a graph, plot this streamline.



Note that the direction of flow is upward since $v > 0$.

We solve the streamline equation by separation of variables,

$$\int u dy = \int v dx$$

or, for the given velocity field $u = 1 + y$ and $v = 1$,

$$\int (1 + y) dy = \int (1) dx$$

Thus,

$$y + \frac{1}{2} y^2 = x + C$$

where, C is a constant of integration. For the streamline that goes through x=y=0, $C = 0$. Hence,

$$x = y + \frac{1}{2} y^2$$