7.13

7.13 The drag, $\mathfrak{D}$, on a washer shaped plate placed normal to a stream of fluid can be expressed as

$$\mathfrak{D} = f(d_1, d_2, V, \mu, \rho)$$

where d_1 is the outer diameter, d_2 the inner diameter, V the fluid velocity, μ the fluid viscosity, and ρ the fluid density. Some experiments are to be performed in a wind tunnel to determine the drag. What dimensionless parameters would you use to organize these data?

$\mathfrak{D} \doteq F \quad d_1 \doteq L \quad d_2 \doteq L \quad V \doteq LT^{-1} \quad \mu \doteq FL^{-2}T \quad \rho \doteq FL^{-4}T^2$

From the pi theorem, $6-3=3$ pi terms required. Use d_1, V, and ρ as repeating variables. Thus,

$$\Pi_1 = \mathfrak{D}\, d_1^a V^b \rho^c$$

and

$$(F)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

so that

$$1 + c = 0 \qquad \text{(for } F)$$
$$a + b - 4c = 0 \qquad \text{(for } L)$$
$$-b + 2c = 0 \qquad \text{(for } T)$$

It follows that $a = -2$, $b = -2$, $c = -1$, and therefore

$$\Pi_1 = \frac{\mathfrak{D}}{d_1^2 V^2 \rho}$$

Check dimensions using MLT system:

$$\frac{\mathfrak{D}}{d_1^2V^2\rho} \doteq \frac{MLT^{-2}}{(L)^2(LT^{-1})^2(ML^{-3})} \doteq M^0L^0T^0 \quad \therefore \text{ OK}$$

For Π_2:

$$\Pi_2 = d_2\, d_1^a V^b \rho^c$$

$$(L)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F)$$
$$1 + a + b - 4c = 0 \qquad \text{(for } L)$$
$$-b + 2c = 0 \qquad \text{(for } T)$$

(con't)

7.13 (con't)

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_2 = \frac{d_2}{d_1}$$

which is obviously dimensionless.

For Π_3:

$$\Pi_3 = \mu\, d_1^{a} V^{b} \rho^{c}$$

$$(FL^{-2}T)(L)^{a}(LT^{-1})^{b}(FL^{-4}T^{2})^{c} \doteq F^0L^0T^0$$

$$1+c=0 \qquad \text{(for } F\text{)}$$
$$-2+a+b-4c=0 \qquad \text{(for } L\text{)}$$
$$1-b+2c=0 \qquad \text{(for } T\text{)}$$

It follows that $a=-1$, $b=-1$, $c=-1$, and therefore

$$\Pi_3 = \frac{\mu}{d_1 V \rho}$$

Check dimensions using MLT system:

$$\frac{\mu}{d_1 V\rho} \doteq \frac{ML^{-1}T^{-1}}{(L)(LT^{-1})(ML^{-3})} \doteq M^0L^0T^0 \qquad \therefore \text{OK}$$

Thus,

$$\frac{\mathcal{D}}{d_1^2 V^2 \rho} = \phi\left(\frac{d_2}{d_1}, \frac{\mu}{d_1 V\rho}\right) \qquad (1)$$

Since $\dfrac{\rho V d_1}{\mu}$ is a standard dimensionless parameter (Reynolds number), Eq. (1) would more commonly be expressed as

$$\underline{\underline{\frac{\mathcal{D}}{d_1^2 V^2 \rho} = \phi\left(\frac{d_2}{d_1}, \frac{\rho V d_1}{\mu}\right)}} \qquad (2)$$

As far as dimensional analysis is concerned, Eqs. (1) and (2) are equivalent.