

3.51

3.51 Water flows through the pipe contraction shown in Fig. P3.51. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe,  $D$ .

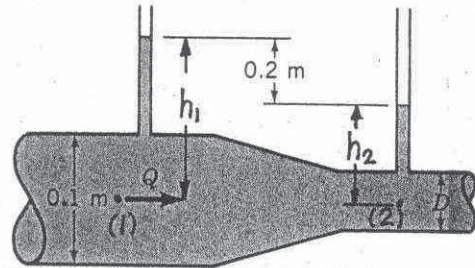


FIGURE P3.51

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{with } A_1 V_1 = A_2 V_2$$

Thus, with  $z_1 = z_2$  or  $V_2 = \frac{(\frac{\pi}{4} D_1^2)}{(\frac{\pi}{4} D_2^2)} V_1 = (\frac{0.1}{D})^2 V_1$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} = \frac{[(\frac{0.1}{D})^4 - 1] V_1^2}{2g}$$

but

$$p_1 = \gamma h_1 \text{ and } p_2 = \gamma h_2 \text{ so that } p_1 - p_2 = \gamma(h_1 - h_2) = 0.2 \gamma$$

Thus,

$$\frac{0.2 \gamma}{\gamma} = \frac{[(\frac{0.1}{D})^4 - 1] V_1^2}{2g} \quad \text{or } V_1 = \sqrt{\frac{0.2(2g)}{[(\frac{0.1}{D})^4 - 1]}}$$

and

$$Q = A_1 V_1 = \frac{\pi}{4} (0.1)^2 \sqrt{\frac{0.2(2(9.81))}{[(\frac{0.1}{D})^4 - 1]}}$$

or

$$Q = \frac{0.0156 D^2}{\sqrt{(0.1)^4 - D^4}} \frac{\text{m}^3}{\text{s}} \quad \text{when } D \sim \text{m}$$