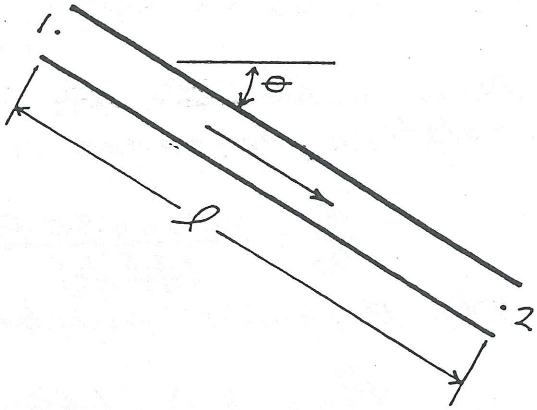
PROBLEM 8.45

GIVEN Water at 20°C flows through a 3-cm-inside-diameter pipe (plastic) at 0.001 m^3/s.

FIND Incline angle so static pressure is constant along pipe.



SOLUTION Apply the mechanical energy equation from 1 to 2.

$$\frac{p_1}{\rho}+\alpha_1\frac{V_1^2}{2}+gz_1+gh_p$$

$$=\frac{p_2}{\rho}+\alpha_2\frac{V_2^2}{2}+gz_2+gh_t+gh_L.$$

Now the problem statement gives $p_1=p_2$ and $gh_p=gh_t=0$ and the continuity equation gives $V_1=V_2$. Assuming turbulent flow, $\alpha_1\approx\alpha_2\approx 1.0$. The energy equation is

$$g(z_1-z_2)=gh_L$$

where

$$gh_L=f\frac{\ell}{D}\frac{V^2}{2} \quad \text{and} \quad (z_1-z_2)=\ell\sin\theta$$

so

$$g\ell\sin\theta=f\frac{\ell}{D}\frac{V^2}{2} \quad \text{or} \quad \sin\theta=\frac{fV^2}{2gD}.$$

Since plastic is a smooth pipe, the given Blasius equation gives

$$f=0.3164\,Re^{-1/4}=0.3164\left(\frac{VD}{\nu}\right)^{-1/4}$$

so

$$\sin\theta=0.1582\,\frac{V^{1.75}\,\nu^{0.25}}{g\,D^{1.25}}.$$

Using Table A.5, the numerical values give

$$V=\frac{Q}{A}=\frac{4Q}{\pi D^2}=\frac{4(0.001\,\frac{m^3}{s})}{\pi(0.03\,m)^2}=1.41\,m/s,$$

$$\sin\theta=0.1582\,\frac{(1.41\,\frac{m}{s})^{1.75}(1.0\times10^{-6}\,\frac{m^2}{s})^{0.25}}{(9.81\,\frac{m}{sec^2})(0.03)^{1.25}}=0.0774,$$

and

$\theta=4.44°$, inclined downward in the direction of flow.

PROBLEM 8.45 continued

Checking the Reynolds number gives

$$Re = \frac{VD}{\nu} = \frac{(1.41\frac{m}{s})(3.0cm)(\frac{m}{100cm})}{(1.0\times10^{-6}\frac{m^2}{s})} = 42300$$

so the flow is turbulent and in the proper range for the given Blasius equation.

(JIR, TLF)