7.77 7.66

7.77 7.66 The drag on a sphere moving in a fluid is known to be a function of the sphere diameter, the velocity, and the fluid viscosity and density. Laboratory tests on a 4-in.-diameter sphere were performed in a water tunnel and some model data are plotted in Fig. P7.77. For these tests the viscosity of the water was 2.3×10^{-5} lb·s/ft² and the water density was 1.94 slugs/ft³. Estimate the drag on an 8-ft diameter balloon moving in air at a velocity of 3 ft/s. Assume the air to have a viscosity of 3.7×10^{-7} lb·s/ft² and a density of 2.38×10^{-3} slugs/ft³.

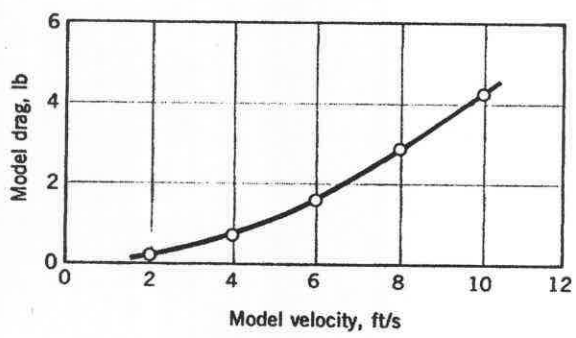

FIGURE P7.77

$$\mathcal{D} = f(d, V, \rho, \mu)$$

where: $\mathcal{D} \sim$ drag $\doteq F$, $d \sim$ sphere diameter $\doteq L$, $V \sim$ velocity $\doteq LT^{-1}$, $\rho \sim$ fluid density $\doteq FL^{-4}T^2$, $\mu \sim$ fluid viscosity $\doteq FL^{-2}T$.

From the pi theorem, 5-3 = 2 pi terms required, and a dimensional analysis yields

$$\frac{\mathcal{D}}{\rho V^2 d^2} = \phi\left(\frac{\rho V d}{\mu}\right)$$

Thus, Reynolds number similarity is required so that

$$\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho V d}{\mu}$$

or

$$V_m = \frac{\mu_m}{\mu}\frac{\rho}{\rho_m}\frac{d}{d_m}V$$

$$= \frac{\left(2.3 \times 10^{-5}\ \frac{lb \cdot s}{ft^2}\right)}{\left(3.7 \times 10^{-7}\ \frac{lb \cdot s}{ft^2}\right)} \frac{\left(2.38 \times 10^{-3}\ \frac{slugs}{ft^3}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)} \frac{(8\ ft)}{\left(\frac{4}{12}\ ft\right)}\left(3\ \frac{ft}{s}\right)$$

$$= 5.49\ \frac{ft}{s}$$

From the graph, for $V_m = 5.49$ ft/s, $\mathcal{D}_m = 1.30$ lb. Since

$$\frac{\mathcal{D}}{\rho V^2 d^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 d_m^2}$$

or

$$\mathcal{D} = \frac{\rho}{\rho_m}\frac{V^2}{V_m^2}\frac{d^2}{d_m^2}\mathcal{D}_m$$

so that

$$\mathcal{D} = \frac{\left(2.38 \times 10^{-3}\ \frac{slugs}{ft^3}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)} \frac{\left(3\ \frac{ft}{s}\right)^2}{\left(5.49\ \frac{ft}{s}\right)^2} \frac{(8\ ft)^2}{\left(\frac{4}{12}\ ft\right)^2}(1.30\ lb) = \underline{\underline{0.274\ lb}}$$