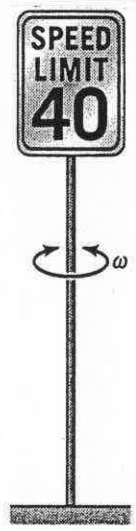
7.15

7.15 Under certain conditions, wind blowing past a rectangular speed limit sign can cause the sign to oscillate with a frequency ω. (See Fig. P7.19 and Video V9.9.) Assume that ω is a function of the sign width, b, sign height, h, wind velocity, V, air density, ρ, and an elastic constant, k, for the supporting pole. The constant, k, has dimensions of FL. Develop a suitable set of pi terms for this problem.



■ FIGURE P7.19

$$\omega = f(b, h, V, \rho, k)$$

$$\omega \doteq T^{-1} \quad b \doteq L \quad h \doteq L \quad V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2} \quad k \doteq FL$$

From the pi theorem $6-3=3$ pi terms required. Use b, V, and ρ as repeating variables. Thus,

$$\Pi_1 = \omega b^a V^b \rho^c$$

and

$$(T^{-1})(L)^a(LT^{-1})^b(FL^{-4}T^{2})^c \doteq F^0L^0T^0$$

so that

$$c = 0 \qquad \text{(for } F\text{)}$$
$$a + b - 4c = 0 \qquad \text{(for } L\text{)}$$
$$-1 - b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 1$, $b = -1$, $c = 0$, and therefore

$$\Pi_1 = \frac{\omega b}{V}$$

Check dimensions:

$$\frac{\omega b}{V} \doteq \frac{(T^{-1})(L)}{(LT^{-1})} \doteq L^0T^0 \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = h b^a V^b \rho^c$$

$$(L)(L)^a(LT^{-1})^b(FL^{-4}T^{2})^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$
$$-b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_2 = \frac{h}{b}$$

which is obviously dimensionless. (con't)