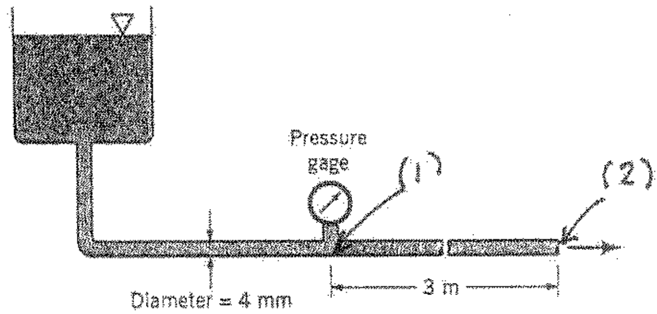
6.89

6.89 A simple flow system to be used for steady flow tests consists of a constant head tank connected to a length of 4-mm-diameter tubing as shown in Fig. P6.103. The liquid has a viscosity of 0.015 N · s/m², a density of 1200 kg/m³, and discharges into the atmosphere with a mean velocity of 2 m/s. (a) Verify that the flow will be laminar. (b) The flow is fully developed in the last 3 m of the tube. What is the pressure at the pressure gage? (c) What is the magnitude of the wall shearing stress, τ_{rz}, in the fully developed region?



■ FIGURE P6.103

(a) Check Reynolds number to determine if flow is laminar:

$$Re = \frac{\rho V (2R)}{\mu} = \frac{(1200 \frac{kg}{m^3})(2 \frac{m}{s})(0.004 m)}{0.015 \frac{N \cdot s}{m^2}} = 640$$

Since the Reynolds number is well below 2100 the flow is laminar.

(b) For laminar flow,

$$V = \frac{R^2}{8\mu}\frac{\Delta p}{\ell} \qquad (Eq.\ 6.152)$$

Since $\Delta p = p_1 - p_2 = p_1 - 0$ (see figure)

$$p_1 = \frac{8\mu V \ell}{R^2} = \frac{8(0.015 \frac{N \cdot s}{m^2})(2 \frac{m}{s})(3 m)}{(\frac{0.004}{2} m)^2} = \underline{\underline{180\ kPa}}$$

(c)

$$\tau_{rz} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) \qquad (Eq.\ 6.126f)$$

For fully developed pipe flow, $v_r = 0$, so that

$$\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$$

Also, $$v_z = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad (Eq.\ 6.154)$$

and with $v_{max} = 2V$, where V is the mean velocity

$$\tau_{rz} = 2V\mu\left(-\frac{2r}{R^2}\right)$$

Thus, at the wall, $r = R$,

$$\left|(\tau_{rz})_{wall}\right| = \left|-\frac{4V\mu}{R}\right| = \left|-\frac{4(2\frac{m}{s})(0.015 \frac{N \cdot s}{m^2})}{(\frac{0.004}{2} m)}\right| = \underline{\underline{60.0 \frac{N}{m^2}}}$$