PROBLEM 5.37

GIVEN Fig. P5.37, 10°C liquid water, $V_1 = 20\ m/s$, $A_1 = 1.0\ m^2$, $A_2 = 0.25\ m^2$, $p_1 = p_{atm} + 30\ kPa$, $p_2 = p_{atm}$. Neglect gravity.

FIND F_x and F_y.

SOLUTION Apply the linear momentum equation in the x-direction to the control volume shown on the right.

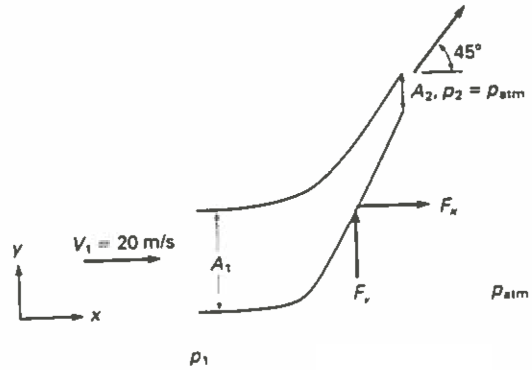
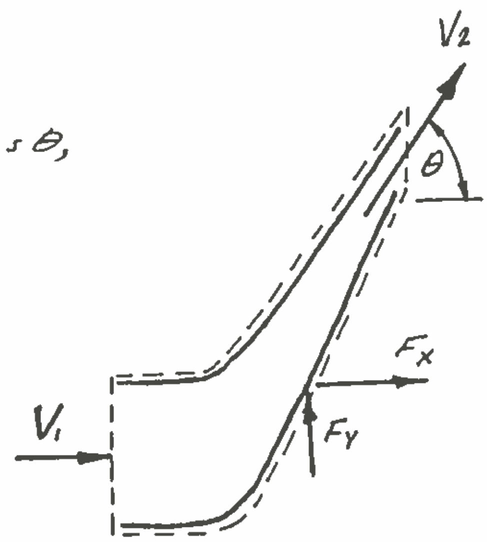


Fig. P 5.37

$$\dot{M}_{x,out} - \dot{M}_{x,in} = \sum F_x$$

where



$$\dot{M}_{x,out} = \dot{m} V_2 \cos\theta_2 = \rho A_1 V_1 V_2 \cos\theta,$$

$$\dot{M}_{x,in} = \dot{m} V_1 = \rho A_1 V_1^2,$$

and

$$\sum F_x = F_x + (p_1 - p_{atm}) A_1$$

where p_1 is an absolute pressure. The x-direction linear momentum equation is

$$F_x = \rho A_1 V_1 (V_2 \cos\theta - V_1) - (p_1 - p_{atm}) A_1 .$$

Assuming constant fluid density, the continuity equation gives

$$\rho A_1 V_1 = \rho A_2 V_2 \cos\theta , \qquad V_2 = \frac{A_1 V_1}{A_2 \cos\theta},$$

or

$$V_2 = \frac{(1.0 m^2)(20 m/s)}{(0.25 m^2) \cos 45°} = 113\ m/s.$$

Where, $\rho = 1000\ kg/m^3$ and

$$F_x = \left(1000 \frac{kg}{m^3}\right)(1.0\,m^2)(20\,m/s)\left[\left(113\,\frac{m}{s}\right)\cos 45° - 20\,\frac{m}{s}\right]\left(\frac{N\cdot s^2}{kg\cdot m}\right)$$

$$-\left(30\times 10^3\,\frac{N}{m^2}\right)(1.0\,m^2)$$

$$\boxed{F_x = 1.17\times 10^6\,N = 1170\,kN.}$$

We now apply the linear momentum equation in the y-direction to the control volume.

$$\dot{M}_{Y,out} - \dot{M}_{Y,in} = \sum F_Y$$

where

$$\dot{M}_{Y,out} = \dot{m}\,V_2 \sin\theta = \rho A_1 V_1 V_2 \sin\theta,$$

$$\dot{M}_{Y,in} = \dot{m}\,(0) = 0, \qquad \text{and} \qquad \sum F_Y = F_Y.$$

The y-direction linear momentum equation is

$$F_Y = \rho A_1 V_1 V_2 \sin\theta.$$

The numerical values give

$$F_Y = \left(1000\,\frac{kg}{m^3}\right)(1.0\,m^2)\left(20\,\frac{m}{s}\right)\left(113\,\frac{m}{s}\right)\sin 45°\left(\frac{N\cdot s^2}{kg\cdot m}\right)$$

$$\boxed{F_Y = 1.60\times 10^6\,N = 1600\,kN.}$$