

Solution to Navier-Stokes Equation with 6DoF Solid Body Motions

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[ME:6260 Viscous flow \(Spring 2024\)](#)

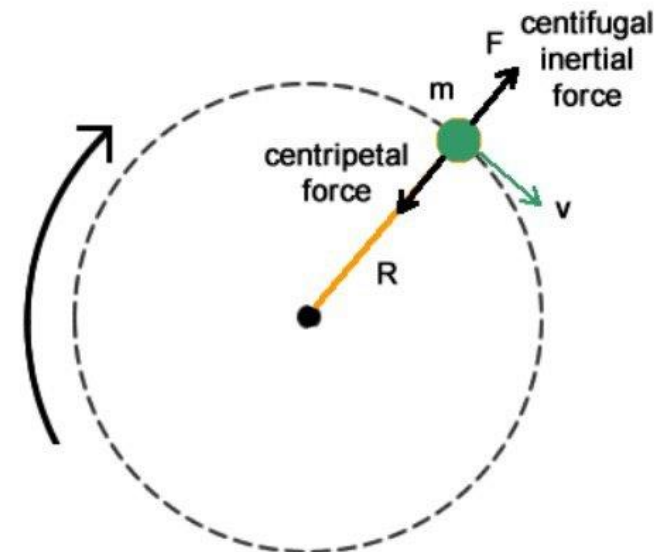
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Outline

- ❑ **Inertial and Non-inertial Frames of Reference**
- ❑ **Navier-Stokes Equation in Different Frames of Reference**
 - Absolute inertial
 - Relative inertial
 - Non-inertial
- ❑ **6DoF Solid Body Motions**
- ❑ **Flow Solvers with Solid Body Motions in CFDShip-Iowa**
 - V6.2, non-inertial frame of reference
 - V4.5/5.5, inertial frame of reference

Inertial and Non-inertial Frames of Reference

- ❑ A **frame of reference** is a coordinate system relative to which motion is described or observed.
- ❑ An **inertial frame of reference** is one that moves at a constant velocity or is at rest. The law of inertia holds.
- ❑ A **non-inertial frame of reference** is one that undergoes acceleration because of an external forces. The law of inertia does not hold.
- ❑ **Fictitious (pseudo) forces** explains the motion in a non-inertial frame of reference.



Uniform circular motion with a string
Centrifugal force

$$F = m\omega^2 r$$

Centripetal force

$$R = -F$$

Inertial and Non-inertial Frames of Reference

Newton's second law in an inertial frame of reference:

$$\mathbf{F} = m\mathbf{a}$$

Modified Newton's second law in a non-inertial frame of reference:

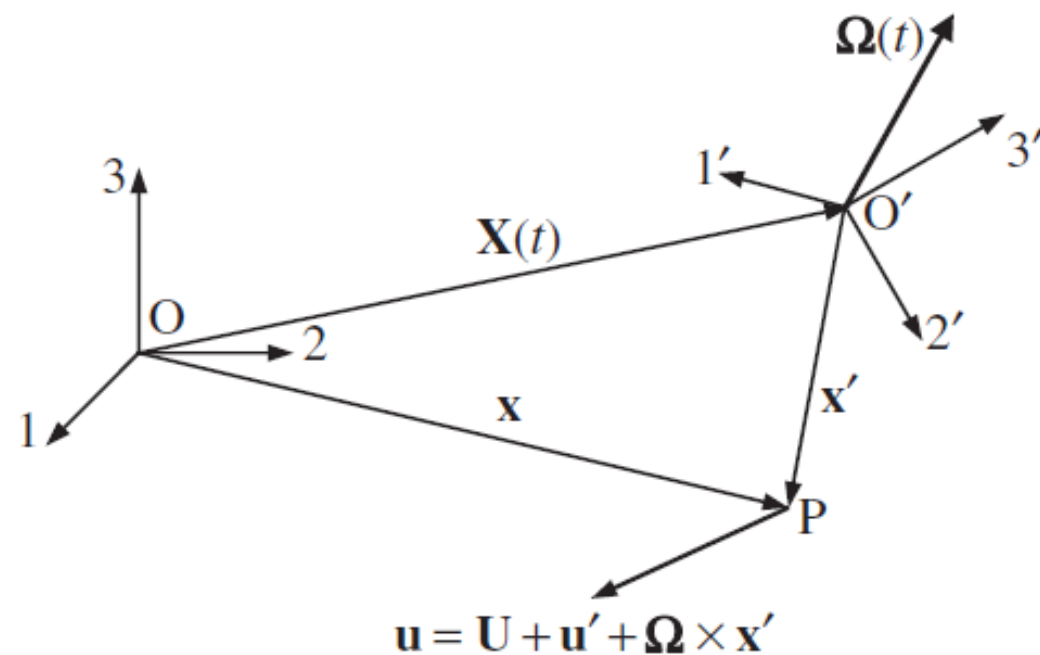
$$\mathbf{F} + \mathbf{F}_{fictitious} = m\mathbf{a}$$

$$\mathbf{a} = \frac{d\mathbf{u}}{dt} = \frac{d}{dt}(\mathbf{U} + \mathbf{u}' + \boldsymbol{\Omega} \times \mathbf{x}') = \frac{d\mathbf{U}}{dt} + \mathbf{a}' + 2\boldsymbol{\Omega} \times \mathbf{u}' + \frac{d\boldsymbol{\Omega}}{dt} \times \mathbf{x}' + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{x}')$$

\mathbf{U} and $\boldsymbol{\Omega}$ are the translation and angular velocities respect to a stationary frame of reference, respectively.

On the RHS, acceleration of the non-inertial frame with respect to inertial frame, fluid particle acceleration viewed in non-inertial frame, Coriolis acceleration, acceleration caused by angular acceleration of the non-inertial frame, and the centripetal acceleration.

Details can be found in the class notes of chapters 1 & 2 (6.1 Part 1)



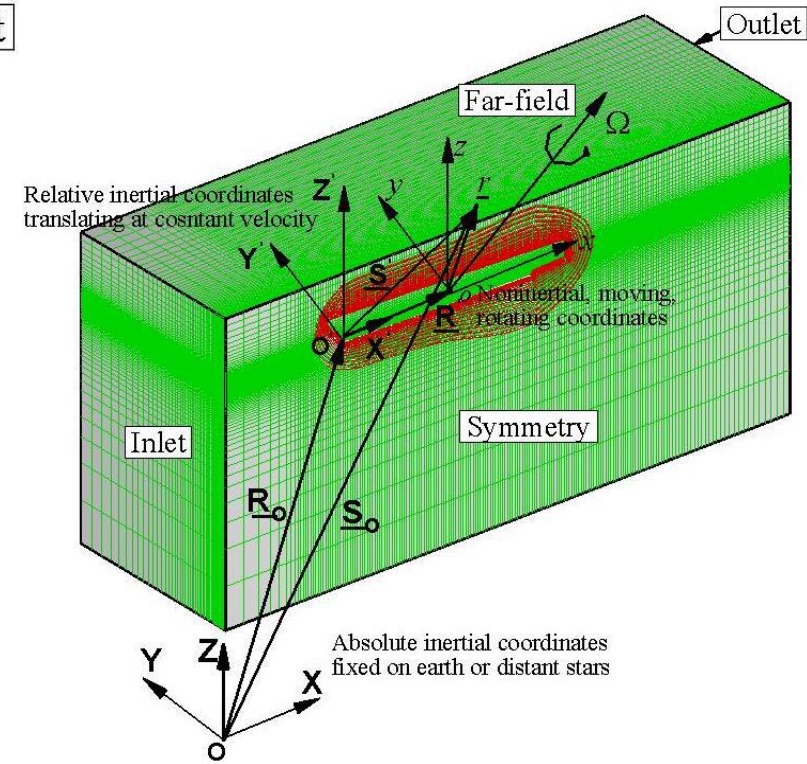
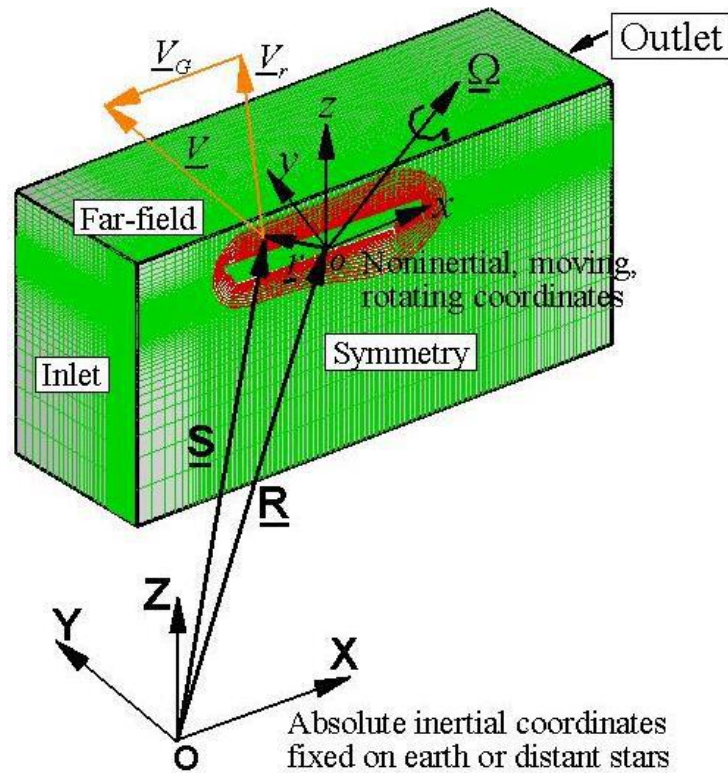
(Kundu et al., 2015)

Navier-Stokes Equation in Different Frames of Reference

Governing differential equations of fluid motion in

- ❑ Absolute inertial earth-fixed coordinates (X, Y, Z) ,
- ❑ Relative inertial coordinates (X', Y', Z') that translate at a constant velocity with respect to (X, Y, Z) ,
- ❑ Non-inertial ship-fixed coordinates (x, y, z)

for an arbitrary moving but non-deforming control volume (CV).



(Xing et al., 2008)

Navier-Stokes Equation in Different Frames of Reference

Correlations of (X, Y, Z) , (X', Y', Z') , and (x, y, z)

- \underline{r} is the instantaneous position vector of any grid point or fluid particle in (x, y, z) .
- The position vector of $o(x, y, z)$ is \underline{R} (X, Y, Z) . $\underline{S} = \underline{R} + \underline{r}$ is the instantaneous position vectors of any grid point or fluid particle in (X, Y, Z) .
- The position vector of $o(x, y, z)$ is \underline{R}' in (X', Y', Z') . $\underline{S}' = \underline{R}' + \underline{r}$ and $\underline{S}_o = \underline{R}_o + \underline{S}'$ are the instantaneous position vectors of any grid point or fluid particle in (X', Y', Z') and (X, Y, Z) , respectively.
- The velocity correlation in (X, Y, Z) and (x, y, z) :

$$\underline{V} = \underline{V}_r + \underline{V}_{CS}$$

- The velocity correlation between (X, Y, Z) and (X', Y', Z') :

$$\underline{V} = \frac{d(\underline{R}_o + \underline{S}')}{dt} = \dot{\underline{R}}_o + \underline{V}'_r$$

Navier-Stokes Equation in Different Frames of Reference

- Absolute inertial earth-fixed coordinates (X,Y,Z),

Standard incompressible Navier-Stokes equations in inertial reference frame for a fixed control volume

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g} \quad (1)$$

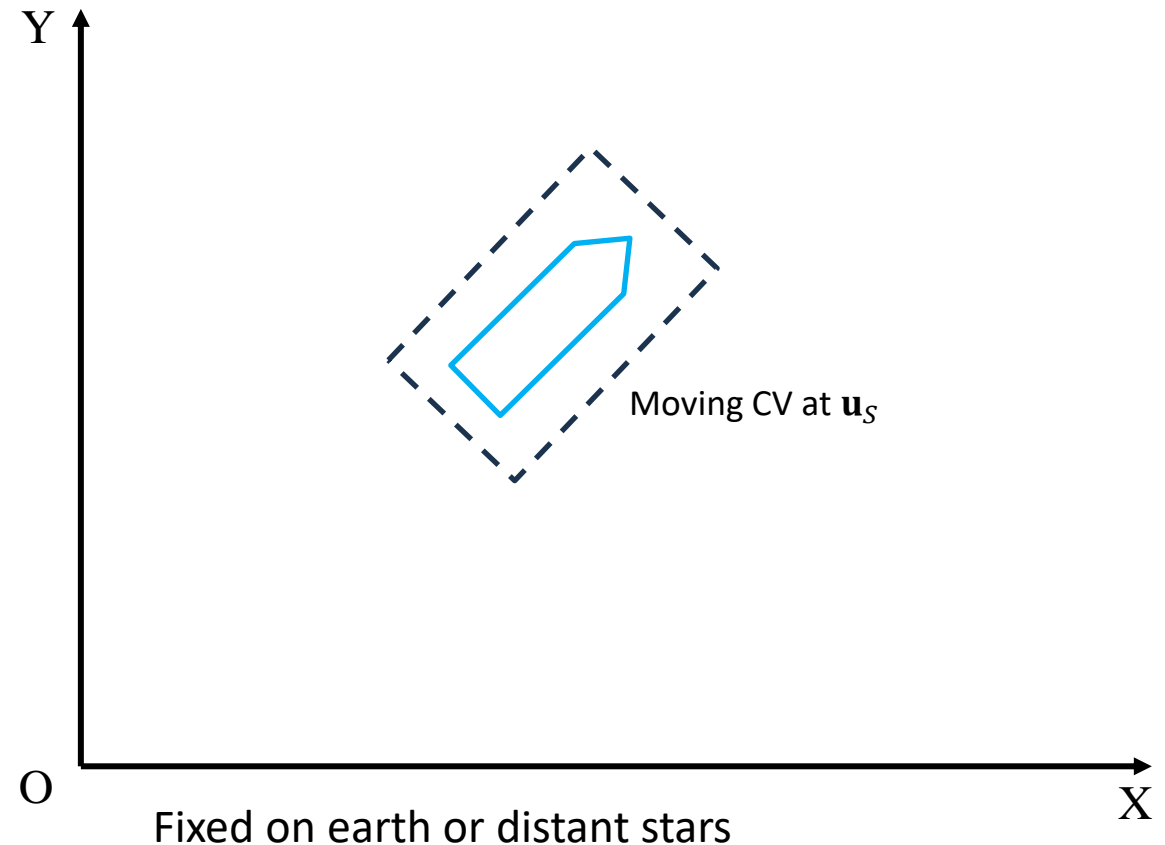
$$\nabla \cdot \mathbf{u} = 0 \quad (2)$$

Incompressible Navier-Stokes equations in inertial reference frame for a moving control volume at \mathbf{u}_S

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}_R \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g} \quad (3)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (4)$$

where $\mathbf{u}_R = \mathbf{u} - \mathbf{u}_S$, and \mathbf{u} is the fluid velocity relative to the same coordinate system in which the control volume motion \mathbf{u}_S is observed.



Navier-Stokes Equation in Different Frames of Reference

- Relative inertial coordinates (X', Y', Z') that translate at a constant velocity with respect to (X, Y, Z) ,

Incompressible Navier-Stokes equations in a relative inertial reference frame for a moving control volume at \mathbf{u}_S

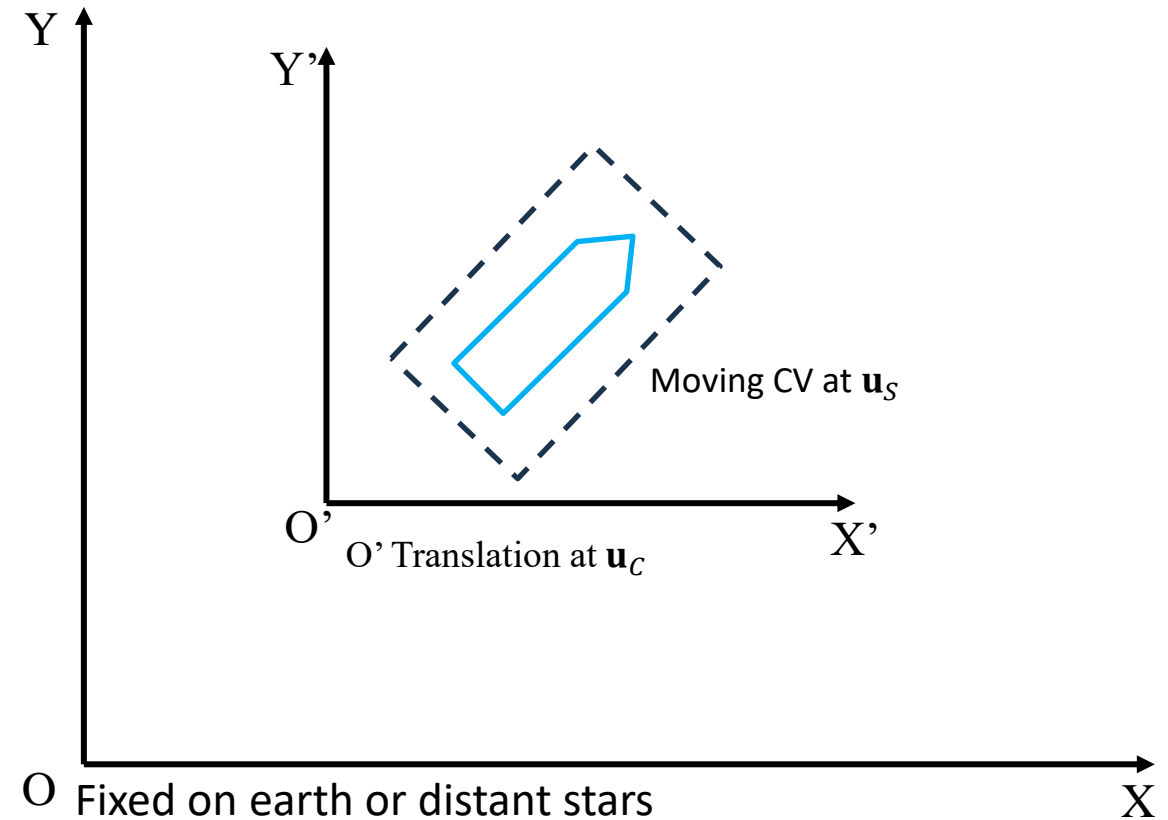
$$\frac{\partial \mathbf{u}'}{\partial t} + (\mathbf{u}'_R \cdot \nabla) \mathbf{u}' = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}' + \mathbf{g} \quad (5)$$

$$\nabla \cdot \mathbf{u}' = 0 \quad (6)$$

where $\mathbf{u}'_R = \mathbf{u}' - \mathbf{u}'_S$, and $\mathbf{u}' = \mathbf{u} - \mathbf{u}_C$ and $\mathbf{u}'_S = \mathbf{u}_S - \mathbf{u}_C$ are the fluid and the control volume velocities relative to coordinate (X', Y', Z') , respectively.

Note that Eqs. (5) and (6) can be obtained by submitting $\mathbf{u} = \mathbf{u}' + \mathbf{u}_C$ and $\mathbf{u}_S = \mathbf{u}'_S + \mathbf{u}_C$ into Eqs. (3) and (4).

The time derivatives in the two inertial coordinates are the same. The gradient, divergence, and Laplacian operators are frame invariant.



Navier-Stokes Equation in Different Frames of Reference

□ Non-inertial ship-fixed coordinates (x,y,z)

Incompressible Navier-Stokes equations in a non-inertial reference frame for a moving control volume at \mathbf{u}_S

$$\frac{\partial \mathbf{u}_r}{\partial t} + (\mathbf{u}_r \cdot \nabla) \mathbf{u}_r = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}_r + \mathbf{g} - \mathbf{a}_{rel} \quad (7)$$

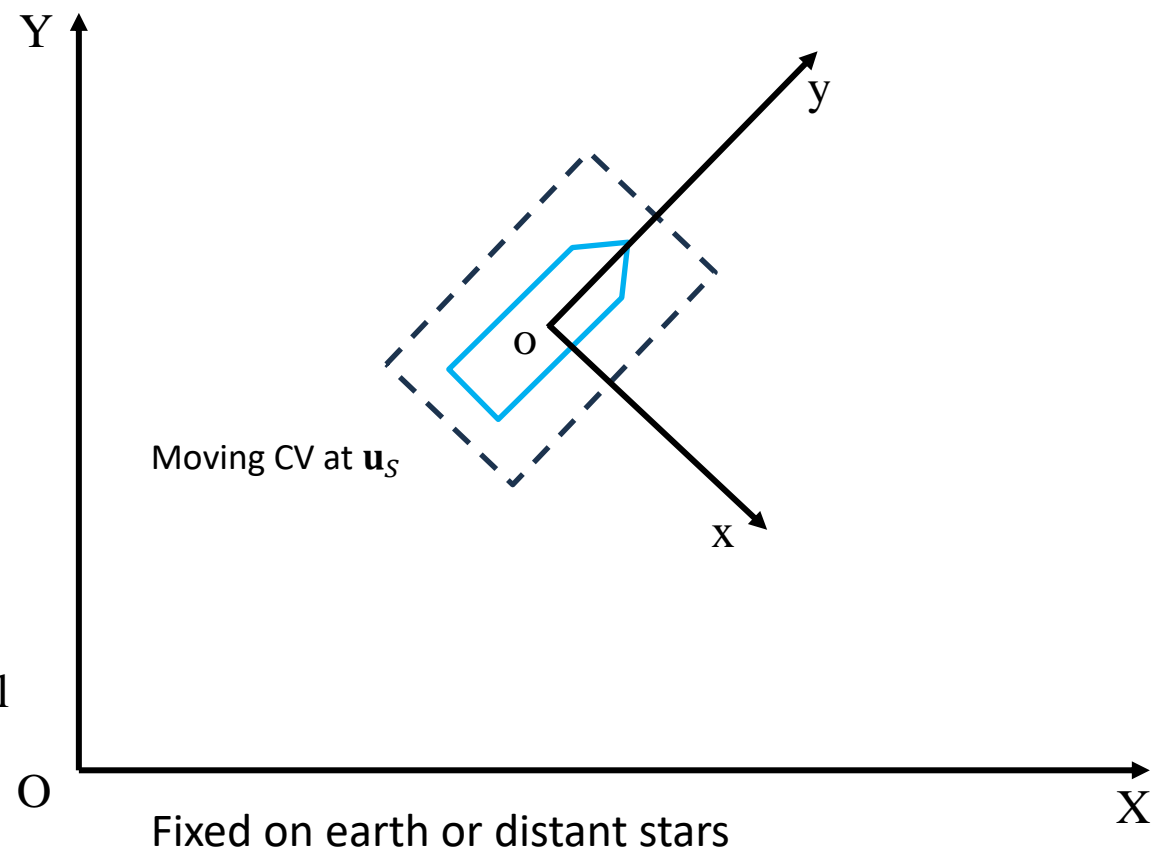
$$\nabla \cdot \mathbf{u}_r = 0 \quad (8)$$

where \mathbf{u}_r is the fluid velocity in coordinate (x,y,z), and

$$\mathbf{a}_{rel} = \frac{d\mathbf{U}}{dt} + 2\boldsymbol{\Omega} \times \mathbf{u}_r + \frac{d\boldsymbol{\Omega}}{dt} \times \mathbf{x} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{x})$$

The acceleration, \mathbf{a}_{rel} , arises from the motion of the non-inertial frame. \mathbf{U} is the translation velocity and $\boldsymbol{\Omega}$ is the rotation angular velocity with respect to the stationary frame of reference.

Note since the coordinate is fixed on the CV and moves with the same velocity, the control volume velocity relative to coordinate (x,y,z) is zero.



Navier-Stokes Equation in Different Frames of Reference

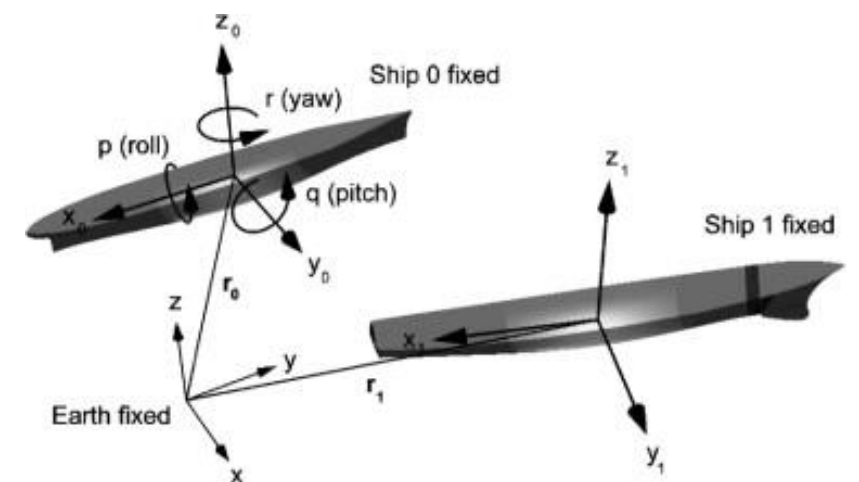
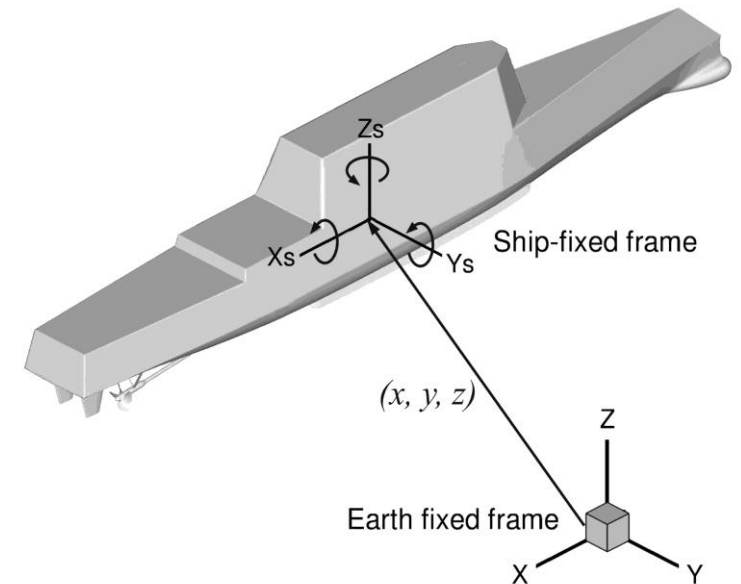
Remarks

- ❑ The continuity equation is in the same form in all frames of reference.
- ❑ The NS equations in the absolute and relative inertial frames are in the same form, except that \mathbf{u} and \mathbf{u}_S are replaced by $\mathbf{u} = \mathbf{u}' + \mathbf{u}_C$ and $\mathbf{u}_S = \mathbf{u}'_S + \mathbf{u}_C$. If the constant $\mathbf{u}_C=0$, Eqs. (5) and (6) reduce to Eqs. (3) and (4).
- ❑ The NS equations in (X,Y,Z) and (x,y,z) clearly take differently forms using absolute inertial earth-fixed or non-inertial ship-fixed coordinate system.
- ❑ Compared to Eq. (7) in the non-inertial frame, application of Eq. (3) simplifies the specification of boundary conditions, saves computational cost by reducing the solution domain size, and can be easily applied to simulate multi-objects such as ship-ship interactions.
- ❑ In CFDShip-Iowa V4.5/5.5, the NS equations are solved in the absolute inertial earth-fixed coordinate system.
- ❑ In general, implementation of Eq. (3) to simulate captive, semi-captive, or full 6DOF ship motions is straightforward. However, solutions to equations of motion for rigid body dynamics are needed. The grid usually is fixed for the non-inertial frame.

6DoF Solid Body Motions

Coordinate systems

- ❑ Two coordinate system types are used for the motions.
- ❑ Neglecting accelerations on points in the earth, the earth fixed coordinate system is an inertial reference frame, and the fluid flow equations are computed in that system.
- ❑ The inertial system might be moving at a constant velocity \mathbf{U}_{inf} with respect to the earth, so that a conveniently defined \mathbf{U}_{inf} minimizes the translation of the ships with respect to the inertial frame.
- ❑ A non-inertial coordinate system is attached to each of the ships under consideration.

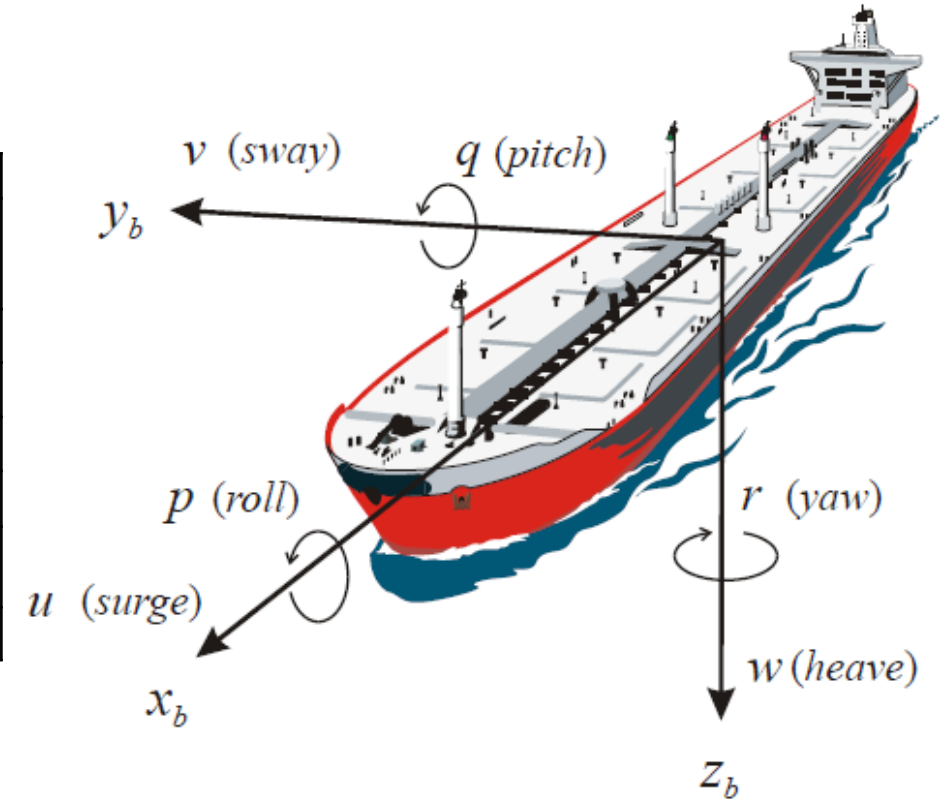


6DoF Solid Body Motions

6DoF motions of ships

DOF		forces and moments	linear and angular velocities	positions and Euler angles
1	motions in the x-direction (surge)	X	u	x
2	motions in the y-direction (sway)	Y	v	y
3	motions in the z-direction (heave)	Z	w	z
4	rotation about the x-axis (roll, heel)	K	p	ϕ
5	rotation about the y-axis (pitch, trim)	M	q	θ
6	rotation about the z-axis (yaw)	N	r	ψ

The notation is adopted from **SNAME (1950)**.



(Lecture Notes of T. I. Fossen)

6DoF Solid Body Motions

Kinematics

- The motion described by the translations and rotations with respect to the earth-fixed inertial frame, using the linear translations and the Euler angles for roll, pitch and yaw

$$\boldsymbol{\eta}_i = (\boldsymbol{\eta}_{i1} \boldsymbol{\eta}_{i2}) = (x_{1,i} x_{2,i} x_{3,i} x_{4,i} x_{5,i} x_{6,i}) = (x_{1,i} x_{2,i} z_{3,i} \phi_i \theta_i \psi_i)$$

**Positions
(earth system)**

- The linear (surge, sway, and heave) and angular velocity (roll, pitch, and yaw) with respect to the ship-fixed frame,

$$\mathbf{v}_i = (\mathbf{v}_{i1}, \mathbf{v}_{i2}) = (u_i, v_i, w_i, p_i, q_i, r_i)$$

**Velocities
(ship system)**

- The angular velocities in the ship system relate to the time rate of change of the Euler angles by

$$\mathbf{v}_{i2} = \begin{bmatrix} 1 & 0 & -\sin\theta_i \\ 0 & \cos\phi_i & \cos\theta_i \sin\phi_i \\ 0 & -\sin\phi_i & \cos\theta_i \cos\phi_i \end{bmatrix} \dot{\boldsymbol{\eta}}_{i2} = \mathbf{J}_{i2}^{-1} \dot{\boldsymbol{\eta}}_{i2}, \quad \dot{\boldsymbol{\eta}}_{i2} = \begin{bmatrix} 1 & \sin\phi_i \tan\theta_i & \cos\phi_i \tan\theta_i \\ 0 & \cos\phi_i & -\sin\phi_i \\ 0 & \sin\phi_i / \cos\theta_i & \cos\phi_i / \cos\theta_i \end{bmatrix} \mathbf{v}_{i2} = \mathbf{J}_{i2} \mathbf{v}_{i2}$$

6DoF Solid Body Motions

Kinematics

- A vector in the earth system can be expressed in the ship coordinate system by

$$\mathbf{a}_i = \begin{bmatrix} \cos\psi_i \cos\theta_i & \sin\psi_i \cos\theta_i & -\sin\theta_i \\ -\sin\psi_i \cos\phi_i + \sin\phi_i \sin\theta_i \cos\psi_i & \cos\psi_i \cos\phi_i + \sin\phi_i \sin\theta_i \sin\psi_i & \sin\phi_i \cos\theta_i \\ \sin\theta_i \sin\psi_i + \cos\phi_i \sin\theta_i \cos\psi_i & -\sin\phi_i \cos\psi_i + \cos\phi_i \sin\theta_i \sin\psi_i & \cos\theta_i \cos\phi_i \end{bmatrix} \mathbf{a}_e = \mathbf{J}_{i1}^{-1} \mathbf{a}_e$$

- For velocities

$$\dot{\boldsymbol{\eta}}_{i1} = \mathbf{J}_{i1} \mathbf{v}_{i1}, \quad \mathbf{v}_{i1} = \mathbf{J}_{i1}^{-1} \dot{\boldsymbol{\eta}}_{i1} = \mathbf{J}_{i1}^T \dot{\boldsymbol{\eta}}_{i1}$$

- The forces and moments with respect to the ship-fixed frame, the surge, sway, and heave forces and the roll, pitch, and yaw moments:

$$\boldsymbol{\tau}_i = (\boldsymbol{\tau}_{i1}, \boldsymbol{\tau}_{i2}) = (X_i, Y_i, Z_i, K_i, M_i, N_i)$$

6DoF Solid Body Motions

Forces and moments

- ❑ The forces and moments are computed in the earth-fixed inertial system, where the fluid flow equations of motion are solved.
- ❑ The hydrostatic and piezometric pressure, and friction forces in the earth system for ship- i are computed from

$$\mathbf{F}_{pe,i} = - \int_{ship-i} \left(p - \frac{z}{Fr^2} \right) d\mathbf{a}_e; \quad \mathbf{F}_{fe,i} = \frac{1}{2Re} \int_{ship-i} (\nabla \mathbf{u} + \nabla \mathbf{u}^T) \cdot d\mathbf{a}_e; \quad \mathbf{a}_e \text{ is the outward pointing area vector.}$$

$$\text{Total force } \mathbf{F}_e = \int_{ship} \left[\left(\frac{\nabla \mathbf{u} + \nabla \mathbf{u}^T}{2Re} \right) - \left(p - \frac{z}{Fr^2} \right) \mathbf{I} \right] \cdot d\mathbf{a}_e + m\mathbf{g}$$

- ❑ The total moments obtained by integrating the elemental forces with the distance to the center of gravity \mathbf{r}

$$\mathbf{L}_e = \int_{ship} \mathbf{r} \times \left\{ \left[\left(\frac{\nabla \mathbf{u} + \nabla \mathbf{u}^T}{2Re} \right) - \left(p - \frac{z}{Fr^2} \right) \mathbf{I} \right] \cdot d\mathbf{a}_e \right\} + \mathbf{x}_G \times m\mathbf{g}$$

- ❑ The forces and moments are **projected** to the ship coordinate system

$$\mathbf{F}_i = \mathbf{J}_{i1}^{-1} \mathbf{F}_{ei} = (X_i, Y_i, Z_i), \quad \mathbf{L}_i = \mathbf{J}_{i1}^{-1} \mathbf{L}_{ei} = (K_i, M_i, N_i)$$

6DoF Solid Body Motions

Rigid-body equations

- The rigid-body equations in the ship system are solved for ship motions. The coordinate of the ship is chosen to align with the principal axes of inertia, and the moment of inertia tensor is diagonal.

$$\mathbf{I}_i = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}_i = \begin{bmatrix} mr_{g,x}^2 & 0 & 0 \\ 0 & mr_{g,y}^2 & 0 \\ 0 & 0 & mr_{g,z}^2 \end{bmatrix}_i, \quad r_{g,i} \text{ is the radius of gyration.}$$

- The origin of the coordinate system of each ship is located at CG, the rigid body equations of motion

$$\begin{aligned} m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] + \zeta_u u &= X \\ m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})] + \zeta_v v &= Y \\ m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] + \zeta_w w &= Z \\ [I_x \dot{p} + (I_z - I_y)qr + m\{y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)\}] + \zeta_p p &= K \\ [I_y \dot{q} + (I_x - I_z)rp + m\{z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)\}] + \zeta_q q &= M \\ [I_z \dot{r} + (I_y - I_x)pq + m\{x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)\}] + \zeta_r r &= N \end{aligned}$$

$\mathbf{x}_G = \mathbf{x}_{rot} - \mathbf{x}_{cg}$ is the distance between the CG and rotation point, and the principal moments of inertia:

$$\begin{aligned} I_x &= I_{xcg} + m(y_G^2 + z_G^2) \\ I_y &= I_{ycg} + m(x_G^2 + z_G^2) \\ I_z &= I_{zcg} + m(x_G^2 + y_G^2) \end{aligned}$$

6DoF Solid Body Motions

Solutions to 6DoF rigid-body equations of motion

- ❑ The dynamic equations of motion are solved numerically using a predictor/corrector implicit approach. Any number of degrees of freedom can be imposed and the rest predicted.
- ❑ Predictor step (for any DoF φ , first-order accurate),

$$\begin{aligned}\dot{\phi}^n &= \dot{\phi}^{n-1} + \Delta t \ddot{\phi}^{n-1} \\ \phi^n &= \phi^{n-1} + \Delta t \dot{\phi}^{n-1}\end{aligned}$$

the forces and moments from the previous time step are used to compute the accelerations $\ddot{\varphi} = \ddot{u}, \ddot{v}, \ddot{w}, \ddot{p}, \ddot{q}, \ddot{r}$, and the velocities $(u_i, v_i, w_i, p_i, q_i, r_i)$ are iterated to convergence. Then the grids are translated and rotated accordingly, and the fluid flow field is solved.

- ❑ Corrector step (for any DoF φ , third order accurate)

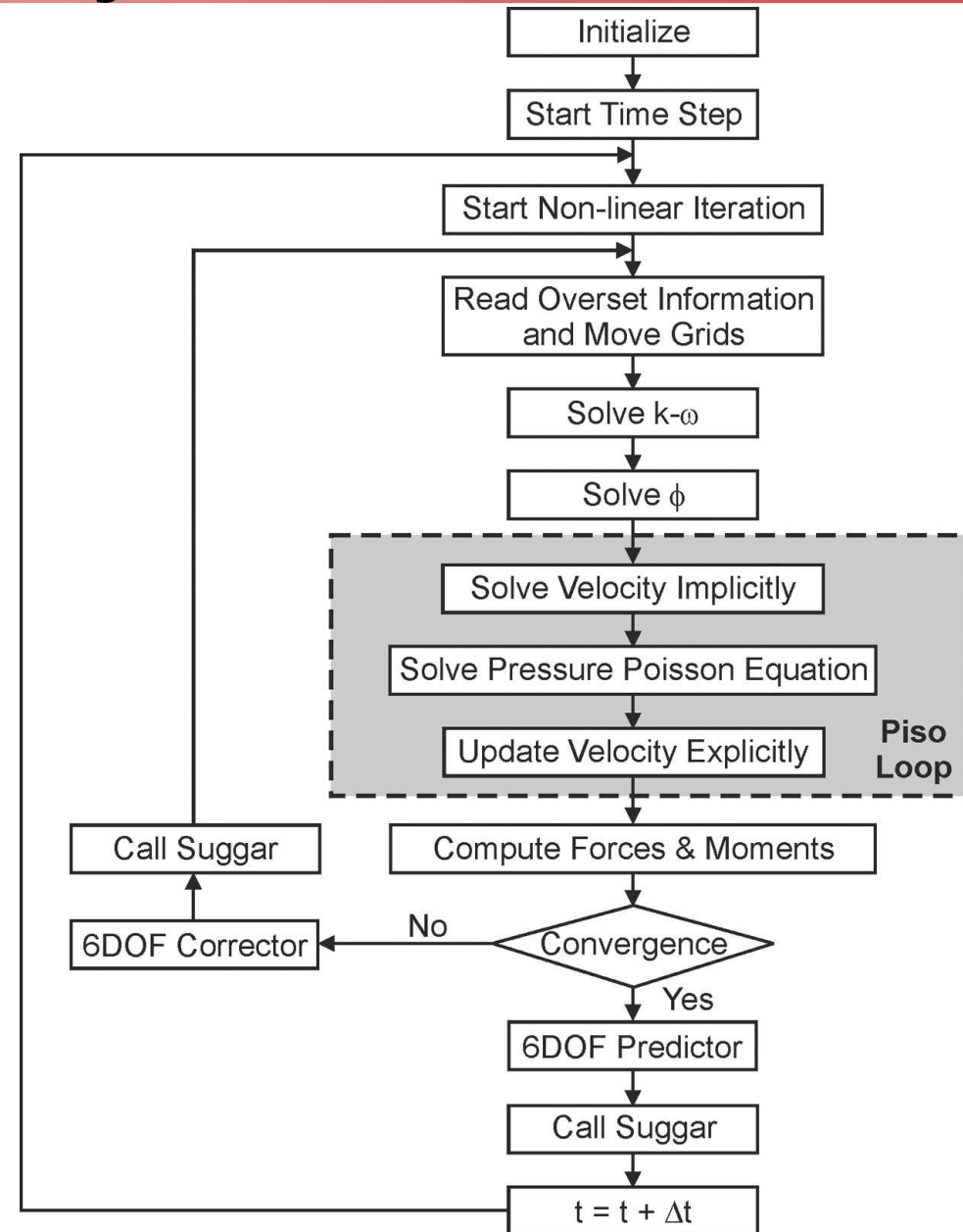
$$\begin{aligned}\dot{\phi}^n &= \dot{\phi}^{n-1} + \Delta t(5\ddot{\phi}^n + 8\ddot{\phi}^{n-1} - \ddot{\phi}^{n-2}) \\ \phi^n &= \phi^{n-1} + \Delta t(5\dot{\phi}^n + 8\dot{\phi}^{n-1} - \dot{\phi}^{n-2})\end{aligned}$$

Time integration is implicit, and global iterations within each time step are necessary.

6DoF Solid Body Motions

Dynamic overset approach

- ❑ Relative motion between different grids is involved, it is necessary to re-compute the overset domain connection coefficients at the run time.
- ❑ The Suggar code is used to obtain the overset domain connectivity between the set of overlapping grids.



Flow Solvers with Solid Body Motions in CFDShip-Iowa

- ❑ CFDShip-Iowa V6.2 is an orthogonal curvilinear grid solver extended from the Cartesian grid solver Version 6.1.
 - 6DoF equations of motion are not implemented.
 - For flows with solid body motions, a non-inertial reference frame can be used, where the fluid moves, and the body/grid is fixed.
- ❑ CFDShip-Iowa V4.5/5.5,
 - NS equations are solved in the absolute inertial earth-fixed coordinate system.
 - 6DoF equations of motion are solved in the ship-fixed coordinates.
 - Dynamic overset grids technique is used to calculate the overset domain connectivity coefficients at run time.

Slamming plate/wedge test

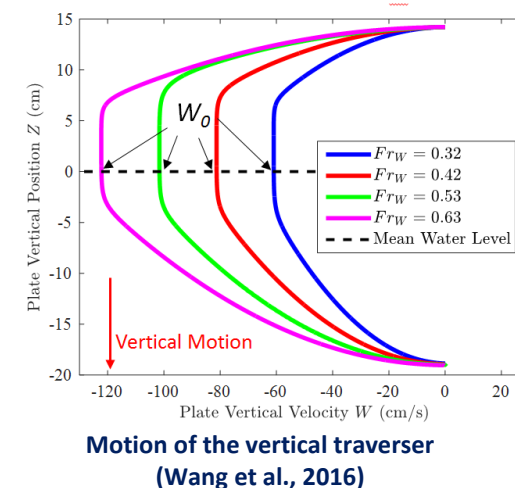
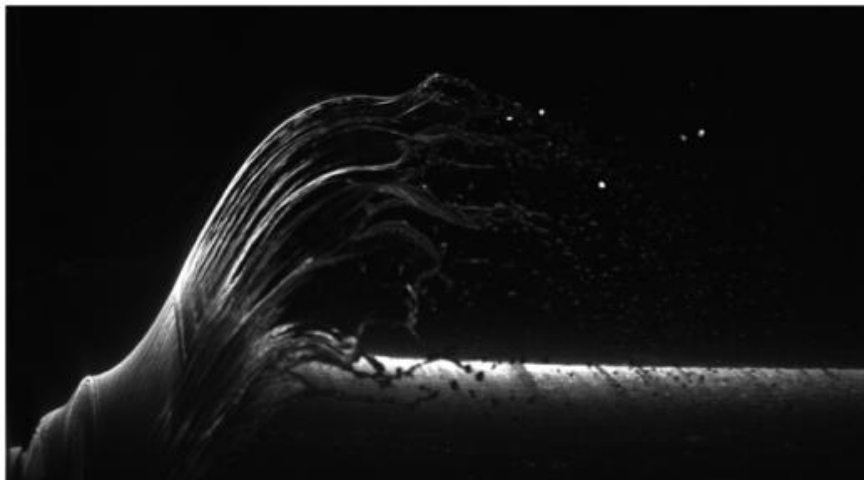
- For CFDShip-Iowa V6.2, additional source term is added to the NS equation for the non-inertial frame

$$\frac{\partial \mathbf{u}_r}{\partial t} + (\mathbf{u}_r \cdot \nabla) \mathbf{u}_r = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}_r + \mathbf{g} - \mathbf{a}_{rel}$$

where $\boldsymbol{\Omega}=0$, $\mathbf{a}_{rel} = \frac{d\mathbf{U}}{dt}$, the acceleration is only due to the slamming motion in the vertical direction.

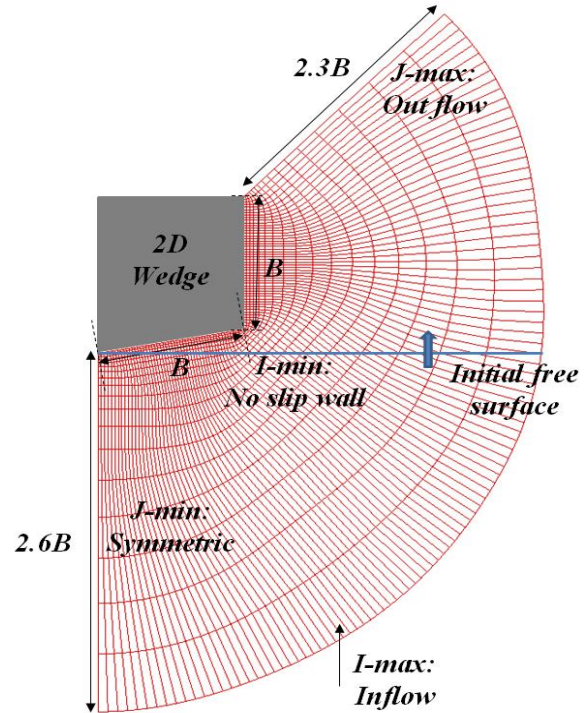
The grid is fixed.

- For CFDShip-Iowa V5.5, dynamic overset grid is used with 1DoF motion in the vertical direction.

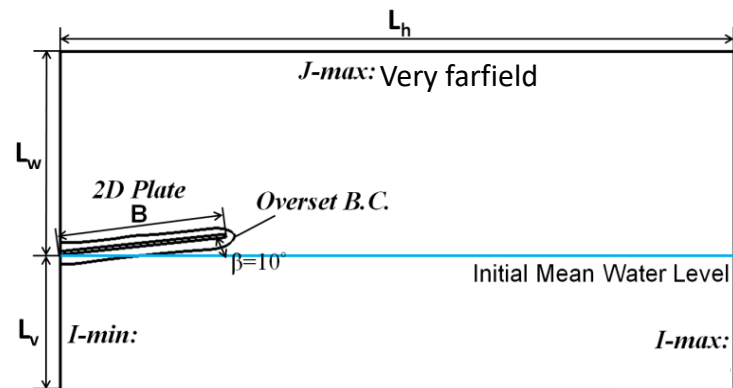


Flow Solvers with Solid Body Motions in CFDShip-Iowa

Slamming wedge test



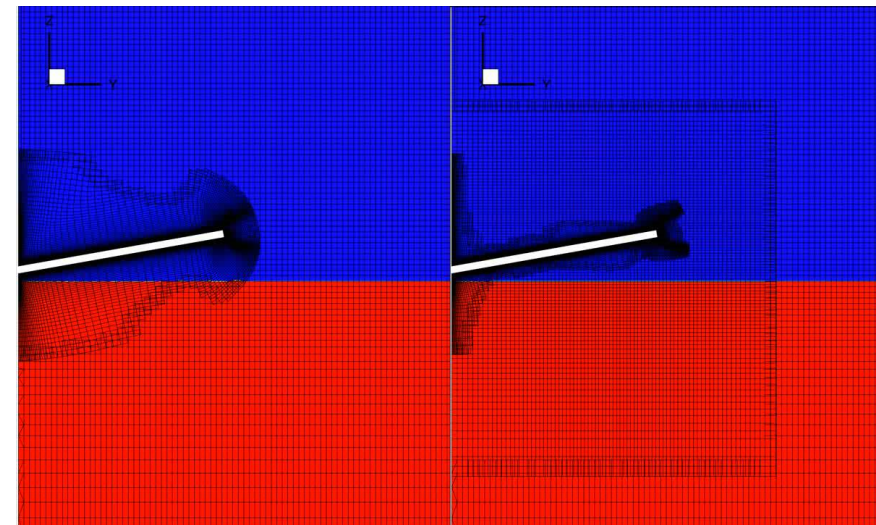
Computational domain and boundary conditions for V6.2



J -min: Computational domain and boundary conditions for V5.5



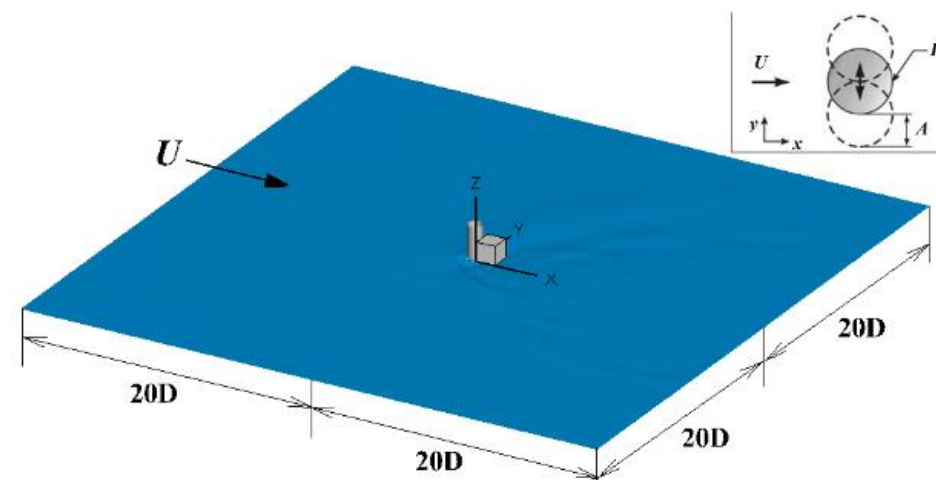
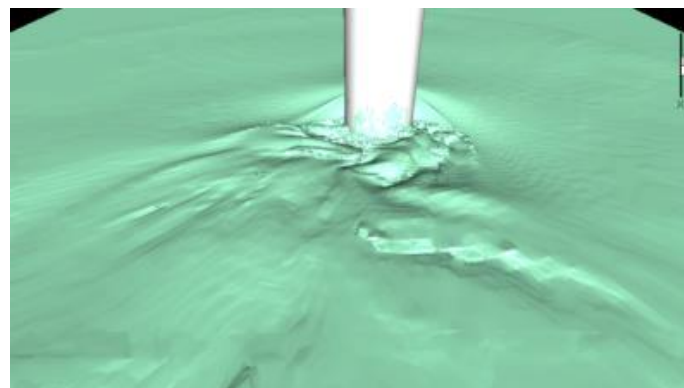
[Movie](#)



[Movie](#)

Sway motion of a truncated cylinder

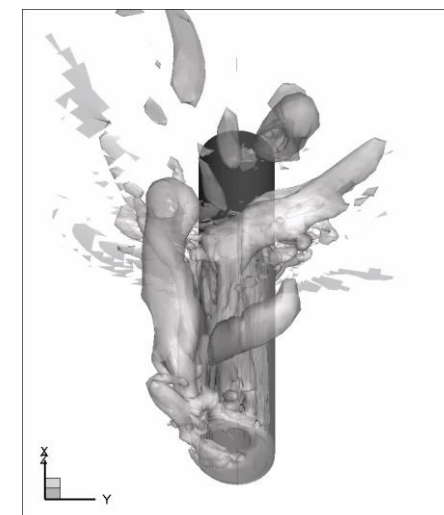
Simulation using V6.2, $\Omega = 0$ and the acceleration term, $\mathbf{a}_{rel} = \frac{d\mathbf{U}}{dt}$, for the transverse motion is added to the momentum equation



Computational domain and boundary conditions for V6.2



[Movie](#)



Vortex Structure using V6.2

[Movie](#)

Flow Solvers with Solid Body Motions in CFDShip-Iowa

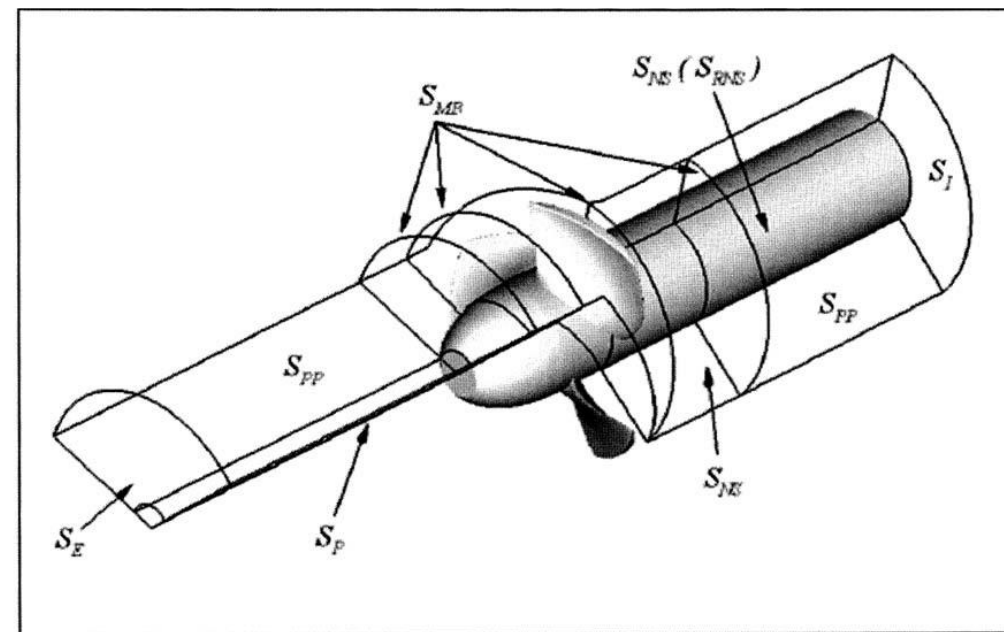
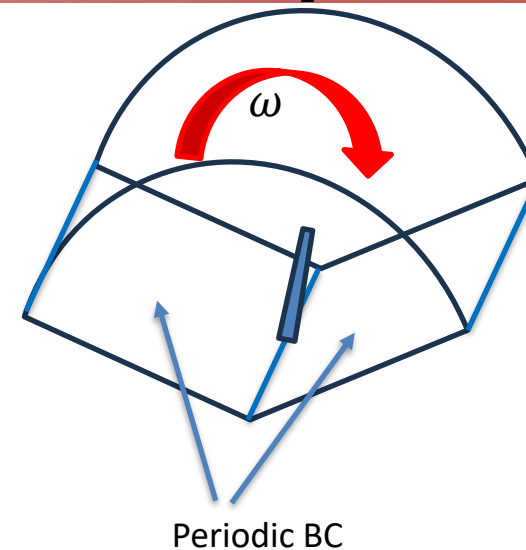
Marine propeller in rotating frame

- For steady uniform flow in a frame that rotates with the blade

$$(\mathbf{u}_r \cdot \nabla) \mathbf{u}_r = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}_r + \mathbf{g} - 2\boldsymbol{\Omega} \times \mathbf{u}_r - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{x})$$

$2\boldsymbol{\Omega} \times \mathbf{u}_r$ Coriolis acceleration; $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{x})$
Centrifugal acceleration

- Two additional source terms are included.
- Relative velocity, \mathbf{u}_r , in the non-inertial frame is used
- The absolute velocity $\mathbf{u} = \mathbf{u}_r + \boldsymbol{\Omega} \times \mathbf{x}$



CFDShip-Iowa V3.0 (Kim et al., 2006)

Flow Solvers with Solid Body Motions in CFDShip-Iowa

Free running KCS using CFDShip-Iowa V5.5

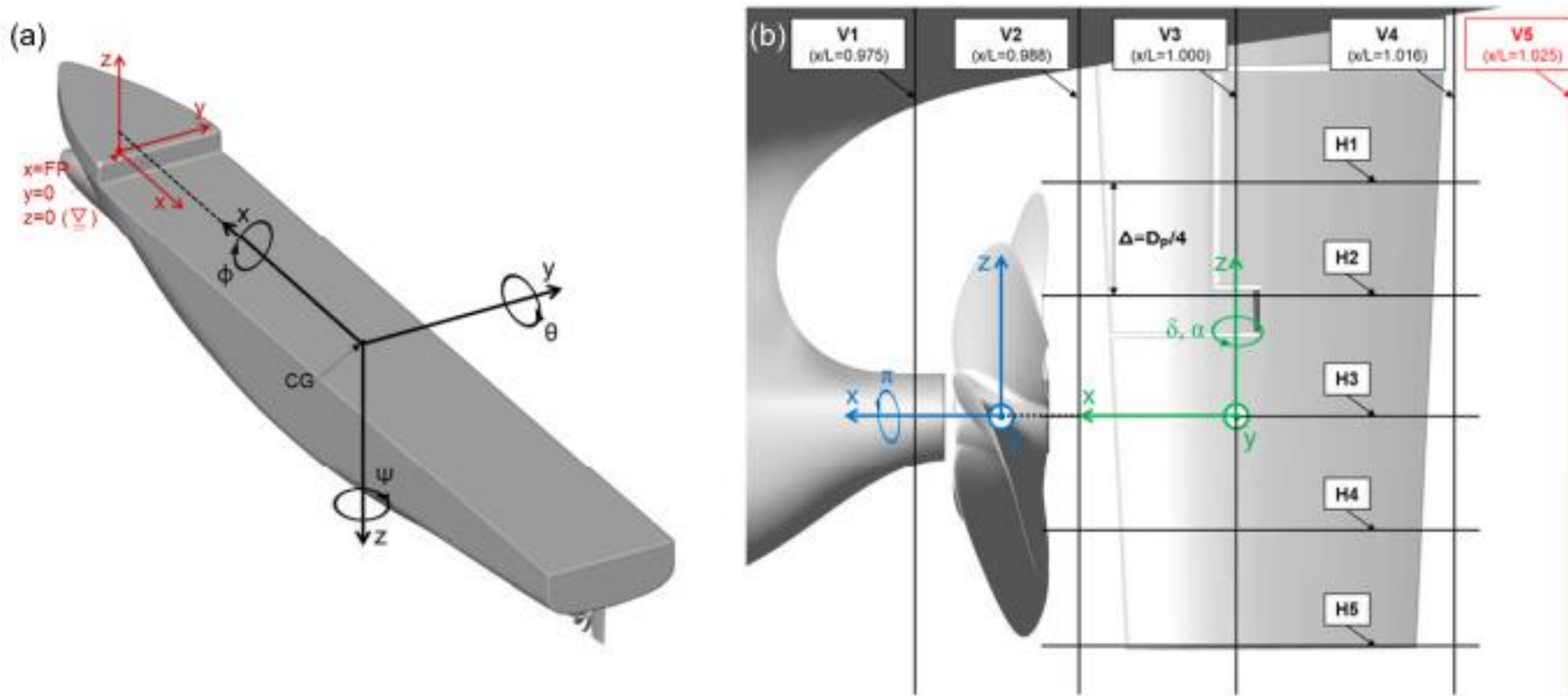


Figure 2-2. Coordinate system: (a) ship (black) and carriage (red); (b) propeller (blue) and rudder (green)

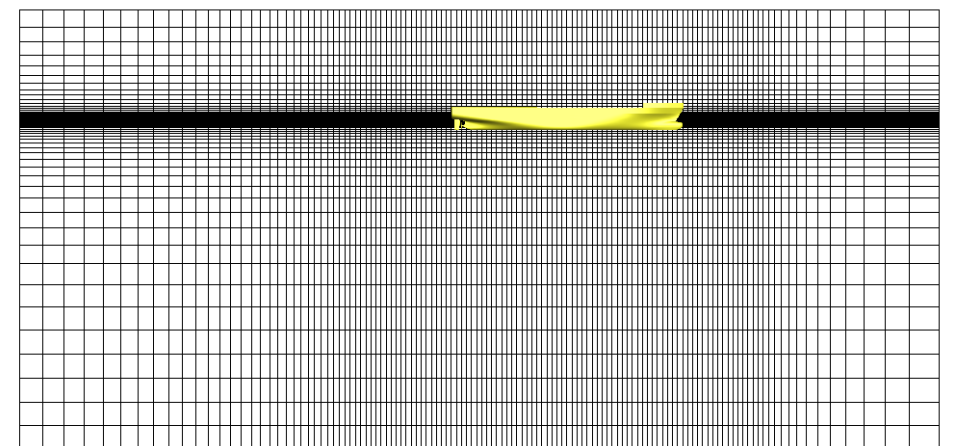
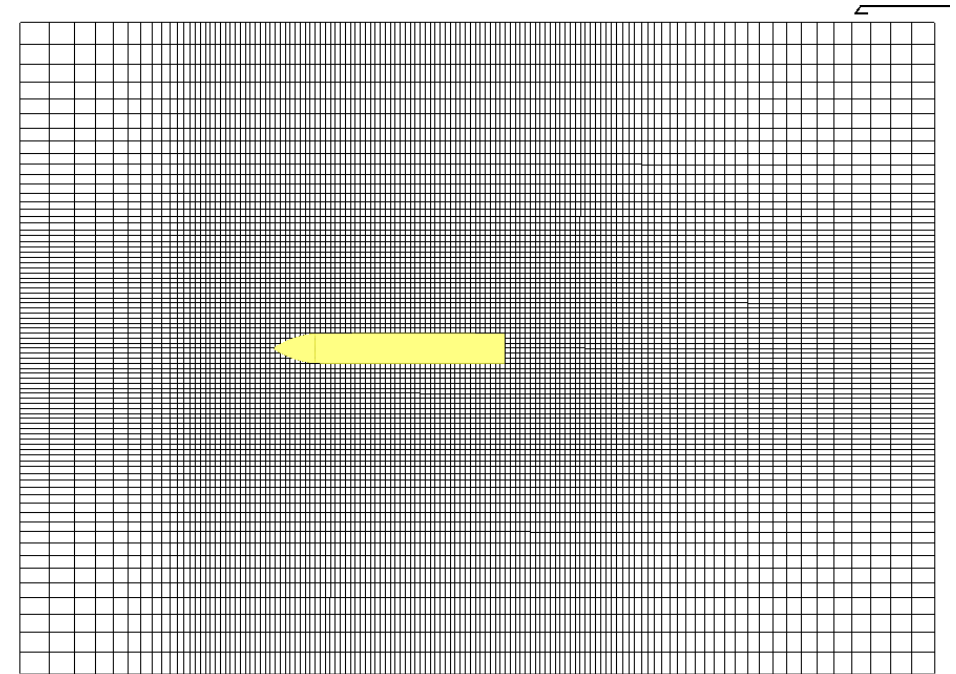
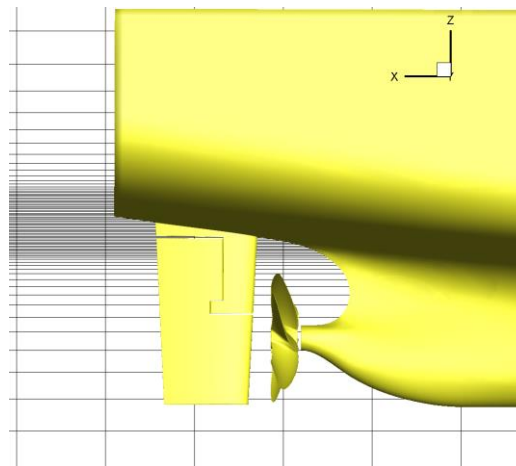
Flow Solvers with Solid Body Motions in CFDShip-Iowa

Free running KCS using CFDShip-Iowa V5.5

6DoF motion setup

	surge	sway	heave	roll	Pitch	Yaw
Hull	y	y	y	y	y	y
Propeller	y	y	y	y	y	y
Rudder	y	y	y	y	y	y
Refinement	y	y	n	n	n	y
Background	y	y	n	n	n	y

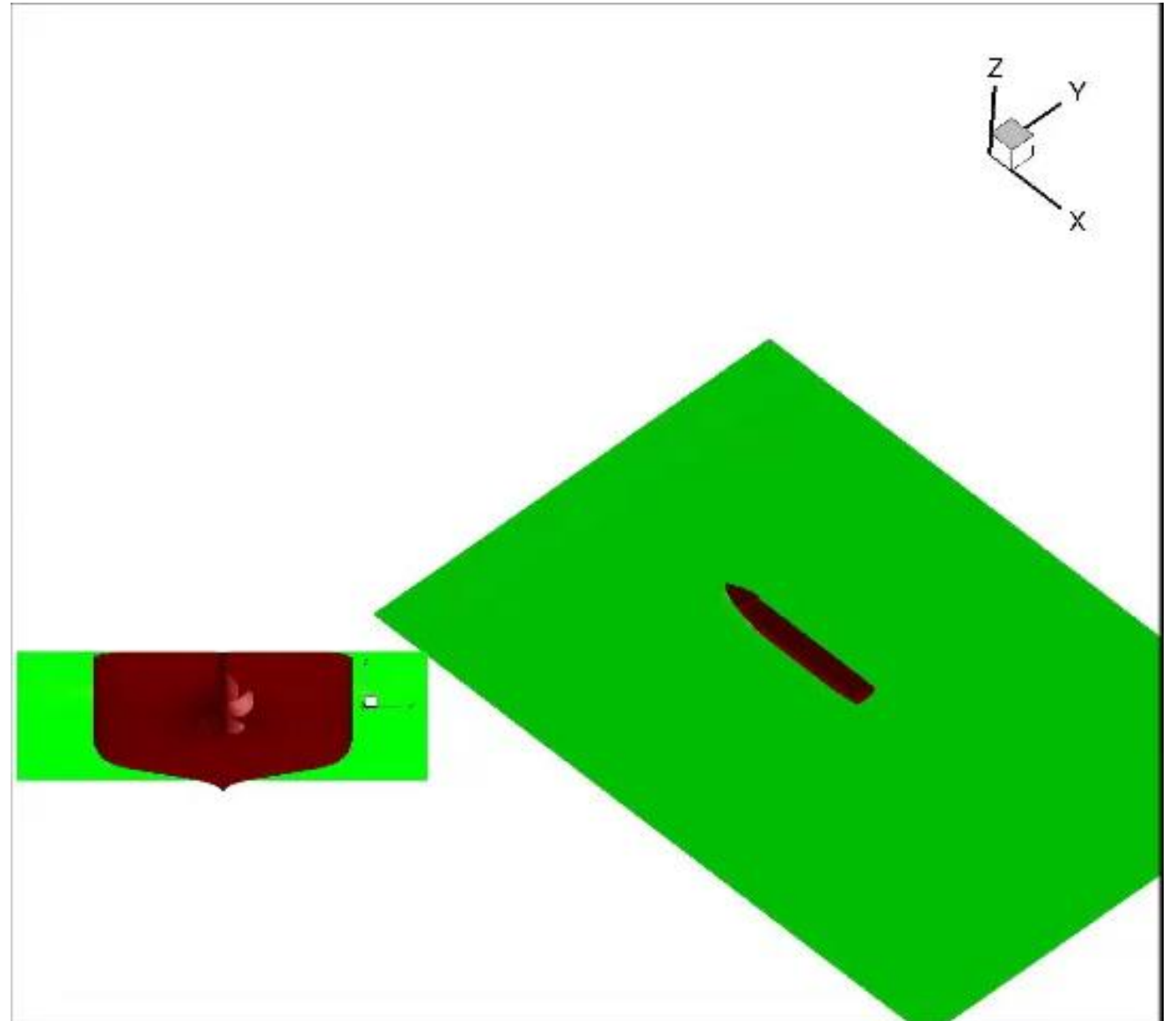
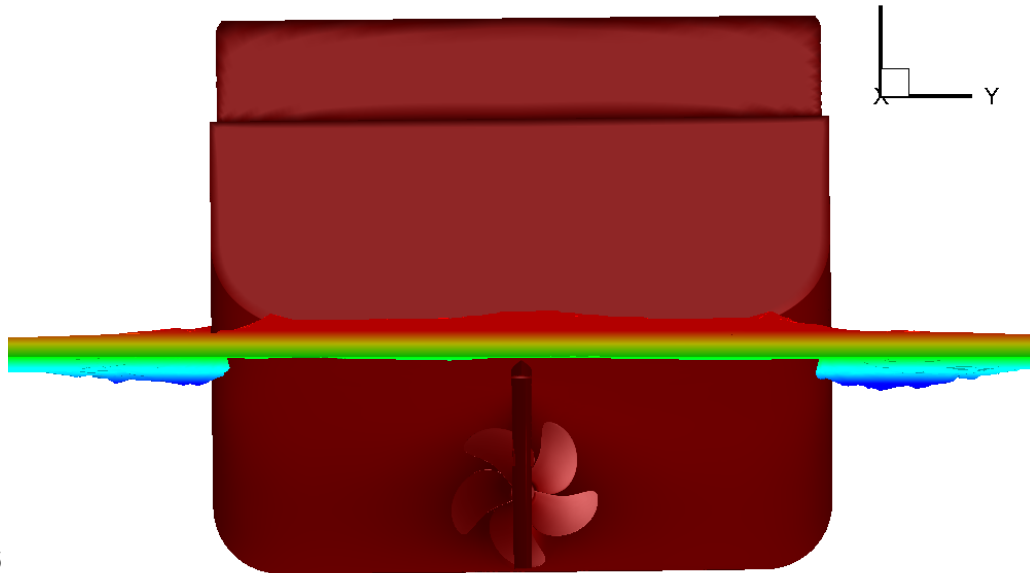
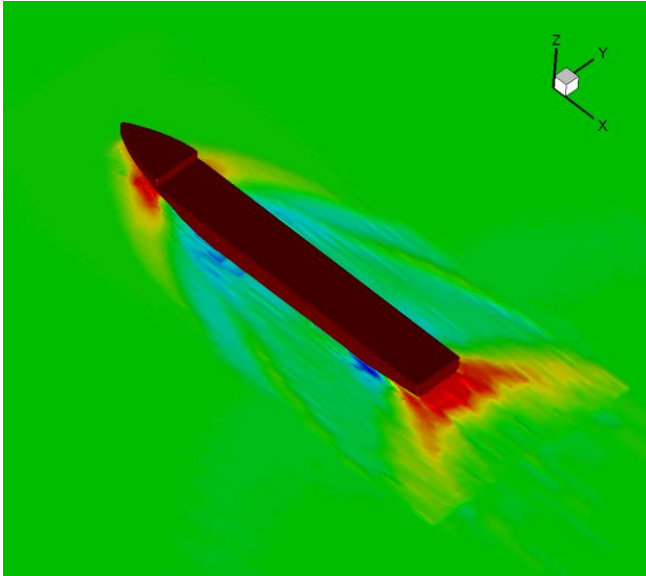
- Refinement and background blocks only move in the x-y plane with 3DoF motions, surge, sway and yaw.
- No relative motion the between the bodies(hull, rudder, and propeller) and the background and refinement grids in the X-Y plane.



Flow Solvers with Solid Body Motions in CFDShip-Iowa

Free running KCS using CFDShip-Iowa V5.5

[Movie](#)



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