Initial and Boundary Conditions for Viscous-Flow Problems

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ME:6260 Viscous flow (Spring 2024)

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Outline

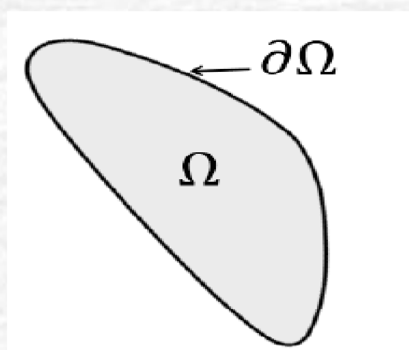
- Why are ICs and BCs needed?
- Initial conditions
- Boundary conditions: definition and types
- Examples of Boundary conditions
  - Solid wall: no slip, FSI, moving contact line
  - Single phase flows: free surface BCs
  - Two-phase interface internal jump conditions
  - Inlet/exit/outer; Fairfield/Open
- Simulations using CFDShip-Iowa

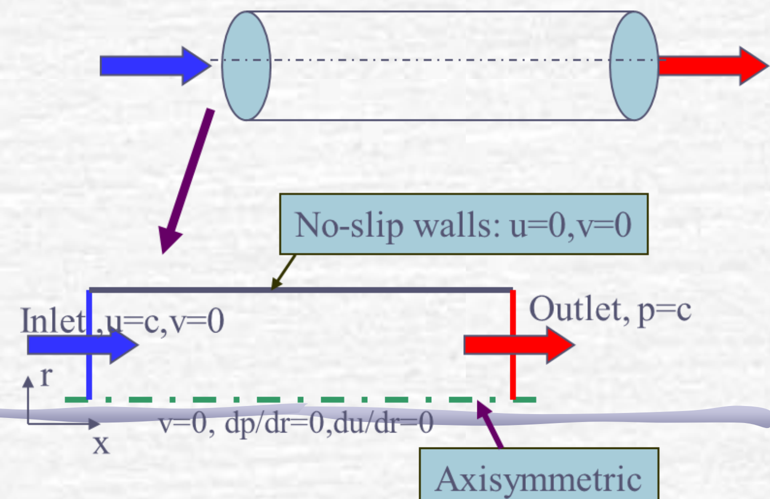
Why are ICs and BCs needed?

- Solutions to ODEs are usually not unique (integration constants exist), which is also a problem for PDE's.
- PDE's are usually specified through a set of ICs and BCs.
- A BC expresses the behavior of a function on the boundary of the domain. An IC specifies the value of the function in time direction, at time t = 0.0.
- The GDEs to be discussed next constitute an IBVP for a system of 2nd order nonlinear PDE, which require IC and BC for their solutions, depending on physical problem and appropriate approximations.

$$\frac{d^2T}{dx^2} = 0, T(x) = c_1 x + c_2$$

$$1).\ x = 0, T = T_0$$
$$2).\ x = l, T = T_l$$





Initial Conditions

- Initial conditions (ICs, steady/unsteady flows)
  - ICs should not affect final results and only affect convergence path, i.e. number of iterations (steady) or time steps (unsteady) need to reach converged solutions.
  - More reasonable guess can speed up the convergence
  - For complicated unsteady flow problems, CFD codes are usually run in the steady mode for a few iterations for getting a better initial conditions

Boundary Conditions

Types of BCs: can be defined/categorized mathematically, physically, and numerically.

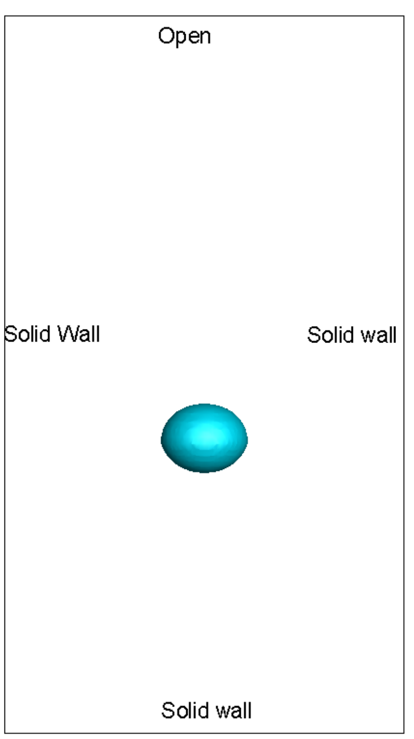
- <u>Mathematical definitions</u>

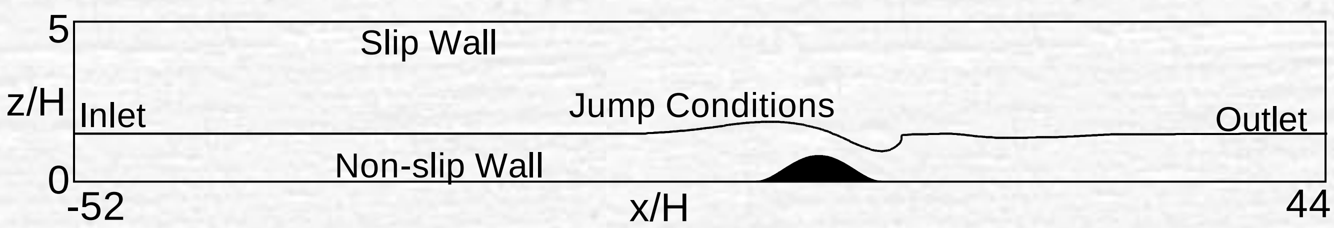
Name	Form
Dirichlet	$\phi = f$
Neumann	$\frac{\partial \phi}{\partial n} = f$
Robin	$C_0 \phi + C_1 \frac{\partial \phi}{\partial n} = f$
Mixed	$\phi = f \quad , \quad C_0 \phi + C_1 \frac{\partial \phi}{\partial n} = f$
Cauchy	$\phi = f$ and $C_0 \frac{\partial \phi}{\partial n} = g$

- <u>For flow variables</u>
  - Kinematic BCs: motion without regard for the cause
  - Dynamic BCs: the causes of motion

Boundary Conditions

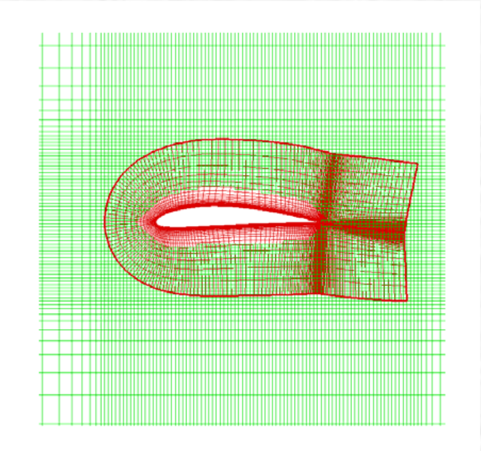
- Physical domain boundaries:
  - Solid Surface
    - Fixed, moving wall, Deforming wall, FSI
    - Permeable Interface, Porous Surface
  - Free Surface, Wave Boundary
  - Two-Phase Interface Internal Jump conditions
  - Inlet/exit/outer
  - Fairfield/Open

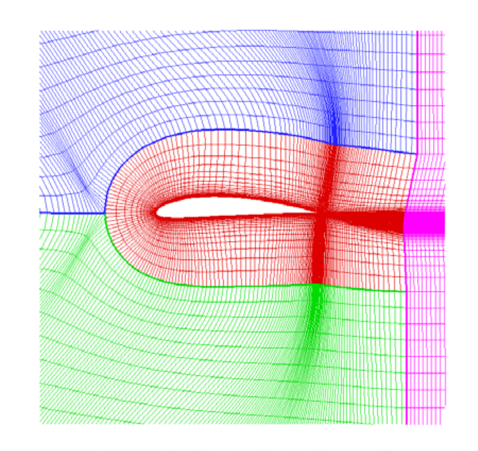




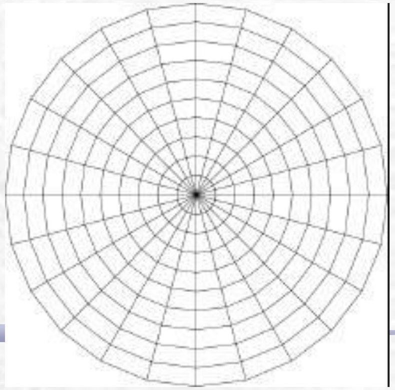
Boundary Conditions

- For grid and numerical treatment:
  - Symmetric BC
  - Periodic BC
  - Numerical beach, absorbing BC
  - Multiblock/Overset overlapping grid BC
  - Convection BC
  - Pole BC (singularity)
  - Global mass conservation enforcing BC

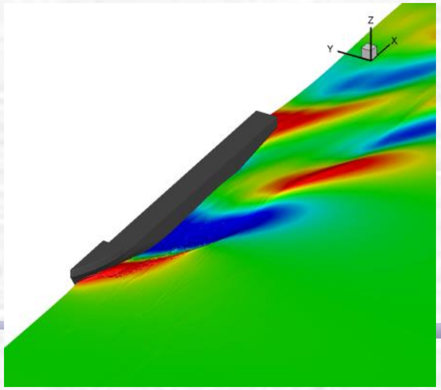




Overset and patched multiblock grids for airfoil.



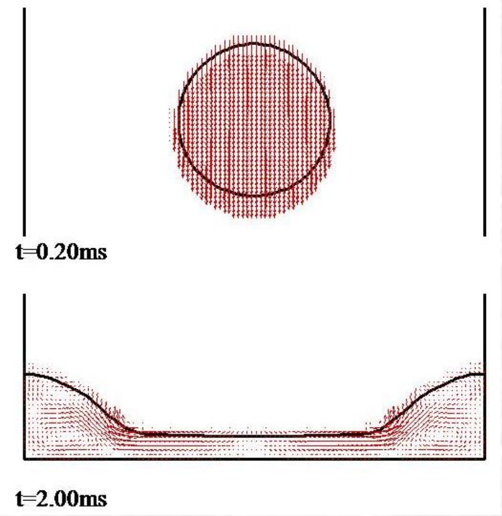
Pole BC



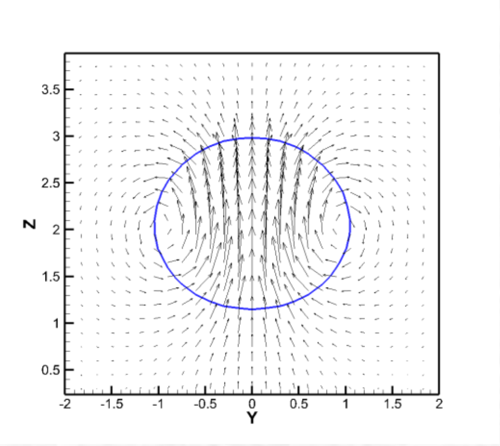
Symmetric BC

Examples of Boundary conditions

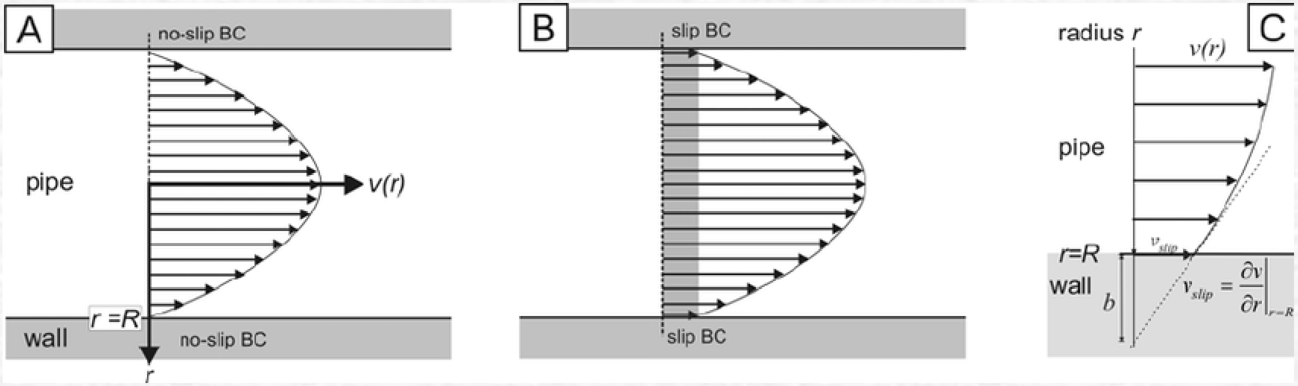
1. Solid Surface
   - Fixed, moving wall
   - Permeable interface, porous surface
   - Deforming wall, FSI
2. Single phase flows: Free surface BCs
3. Multiphase flows: Two-phase interface jump conditions
4. Inlet/exit/outer



Free surface flow: droplet



Two-phase interfacial flow: bubble



Pipe flow with no-slip (A) and slip (B) boundary conditions. (Berg et al., 2021).

Examples of Boundary conditions

1. Solid Surface

<u>No-slip BCs</u>: No-slip BC widely used for most macroscopic flows without loss of accuracy

- ℓ = mean free path of a moving molecular particle << fluid motion; therefore, macroscopic view is "no slip" condition, i.e. no relative motion or temperature difference between liquid and solid.

$$\underline{V}_{liquid} = \underline{V}_{solid} \qquad T_{liquid} = T_{solid}$$

- Exception for gas and contact line problem

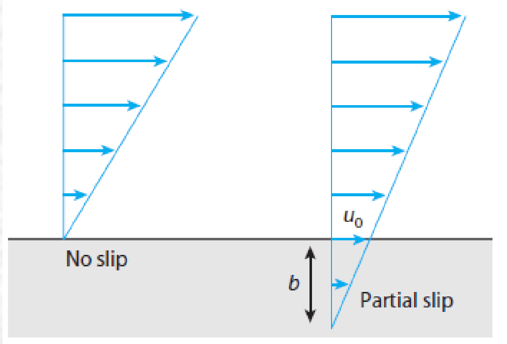
<u>Smooth wall</u>:
Specular reflection
Conservation of tangential momentum
u_w=0=fluid velocity at wall

<u>Rough wall</u>:
Diffuse reflection. Lack of reflected tangential momentum balanced by u_w

Examples of Boundary conditions

Slip-wall BCs:

$$u_w = l \left.\frac{du}{dy}\right|_w$$



$$\tau_w = \mu \left.\frac{du}{dy}\right|_w \qquad l = \frac{\mu}{{}^2\!/_3\, \rho a} \qquad \textit{low density limit}$$

$$u_w = \frac{3}{2}\frac{\mu}{\rho a}\frac{\tau_w}{\mu} \qquad Ma = U/a \qquad C_f = \tau_w \Big/ \tfrac{1}{2}\rho U^2$$

$$u_w / U = .75 Ma\, C_f$$

High Re: $C_f \sim 0.005$, Say $Ma \sim 20$ $\longrightarrow$ $\frac{u_w}{U} < 0.01$

Low Re: $C_f \sim .6 Re_x^{-1/2}$ $\qquad Re_x = Ux/\nu$

$$\frac{u_w}{U} = \frac{.4Ma}{Re_x^{\frac{1}{2}}}$$

Significant slip possible at low Re, high Ma:
"Hypersonic LE Problem"

Similar for T:

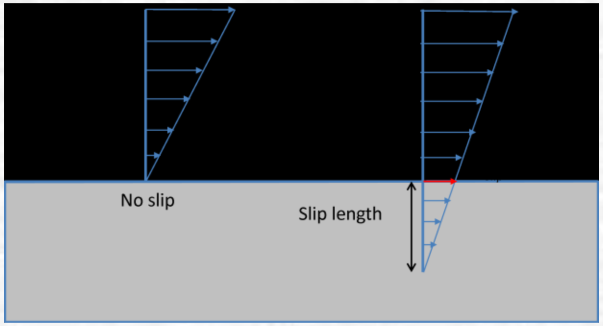
High Re: $T_{gas} = T_w$

Low Re $\qquad \frac{T_{gas} - T_w}{(T_r - T_w)} = .87 Ma C_f$ $\qquad$ *air*

$(T_r - T_w)$ = driving ΔT; Ref. T (T_r)

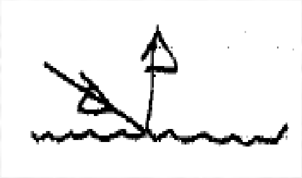
No-slip boundary condition and contact line problem*

- The **no-slip boundary condition** (BC) is usually used at the solid surfaces for the numerical simulation of viscous flows.
- It is assumed that fluid **velocity is zero relative** to the solid surface.
- This assumption is accurate and acceptable for most macroscopic fluid flows but may be invalid and pose problems for **microscopic-scale flows**.
  - For small scale flows where the mean free path of the fluid is close to the characteristic length (Rothstein, 2010), e.g., flows of rarified gases, gas molecules on the solid surface can move freely.
- The no-slip BC also fails for viscous flows with a **moving contact line**.
  - The moving contact line is defined as the interface between two immiscible fluids that intersects with the solid surface.
  - For example, the air-water interface on the ship surface will not move if the no-slip BC is used.



Smooth wall:
Specular reflection
Conservation of tangential momentum
u_w=0=fluid velocity at wall

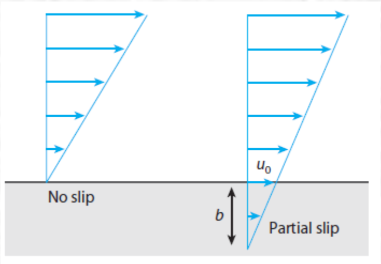
Rough wall:
Diffuse reflection. Lack of reflected tangential momentum balanced by u_w

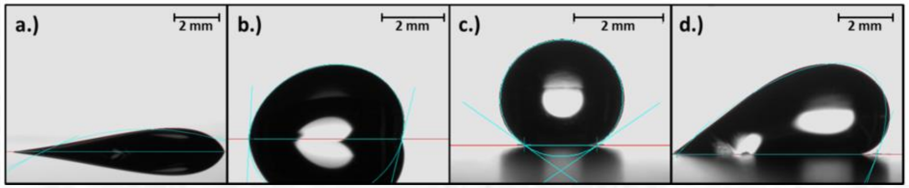


*Wang, Z. and Stern, F., "Moving contact line and no-slip boundary conditions for high-speed planing hulls," X International Conference on Computational Methods in Marine Engineering," *Special Issue of Ships and Offshore Structures, under review, 2024.*

No-slip boundary condition and contact line problem

- For small scale flows, **slip BC** with a finite slip length is usually used: $u_0 = b\left|\frac{\partial u}{\partial y}\right|$. The contact line movement is also dependent on the contact angles when **surface tension force** is dominant, but the mechanism is not fully understood.
- Most previous studies have been focused on **small scale flows**, such as flows within microfluidic or nanofluidic devices, and small bubbles/droplets (Mohammad Karim, 2022; Rothstein, 2010).
- Few studies have been reported for **large scale flows**, such as ship flows.
- For large scale flows with **high Reynolds numbers**, very small grid spacing is usually used near the wall in order to resolve the boundary layer, the numerical treatments used for the small-scale flows are not suitable.

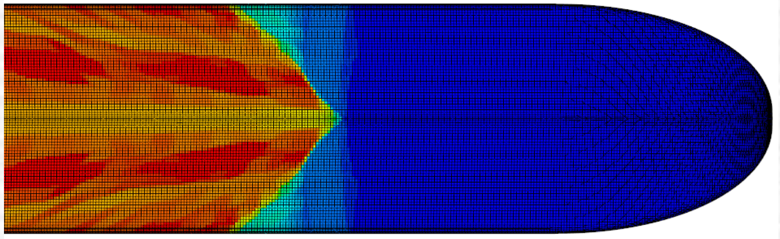




Contact angles for a droplet

No-slip boundary condition and contact line problem

- One of the issues caused by the no-slip BC is the **numerical ventilation** (Cucinotta et al., 2021), especially serious for the **algebraic volume-of-fluid** (VOF) method.
- **VOF slip velocity** is used by Wheeler et al. (2021) to minimize the numerical ventilation effect.
- In the present study, a numerical strategy to handle the no-slip BC and moving contact line problem for high-speed planing hulls is proposed.
- A **blanking distance** from the solid surface is used when solving the interface modeling equations, which is chosen based on the y+ values and the velocity profiles in the boundary layer.
- Numerical tests show if the blanking distance is **y+ < 30**, the air-water interface will be unstable and numerical ventilation will occur. For the blanking distance **y+ > 30** (outside the buffer layer), a smooth air-water contact line can be obtained without air entrainment.

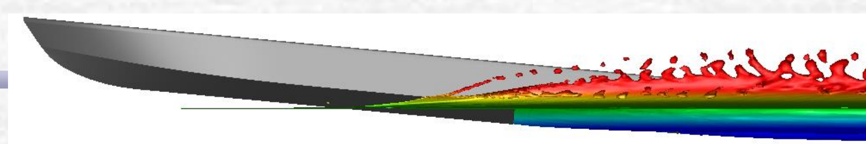


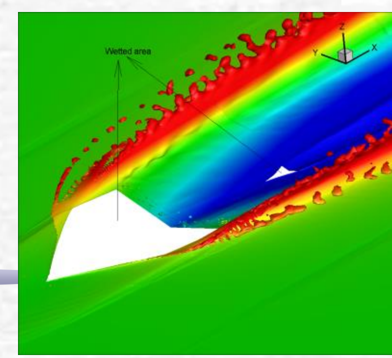
Numerical ventilation using STAR-CCM+ (Wheeler et al., 2021)

No-slip boundary condition and contact line problem

No-slip Wall Boundary Conditions for Ship Flows

- High Reynolds number multi-phase turbulent flows are involved in ship flows and no-slip wall BC with proper turbulence models (e.g., RANS) and very small grid spacing ($y_+ \sim 1$) near the surface of the hull are needed in order to resolve the boundary layer.
- Numerical ventilation usually does not cause serious problems for the CFD prediction of **displacement hulls**.
- For **small planing craft**, however, the **wetted area** of planing hulls can change abruptly and significantly due to slamming, and the size of which is also comparable to the jets and sprays generated.
- Special numerical treatment is needed to handle the no-slip BC and moving contact line problem for high-speed planing hulls, which is critical for the correct prediction of jets, sprays, wave breaking, and ventilation near and around hulls, and their effects on forces and ship motions.

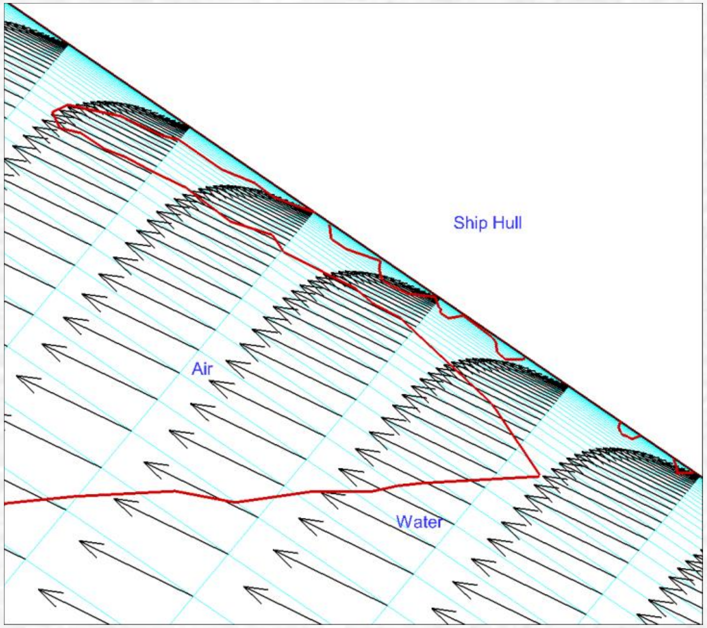




No-slip boundary condition and contact line problem

Moving Contact Line and wave blanking

- A **geometric VOF** method (Wang et al., 2012) is used for the interface modeling with a distinct and sharp air-water interface defined
- This differs from the smeared air-water interface usually obtained using the algebraic VOF method
- Fluid particles near the wall move at a slower speed than those away from it.
- Air entrainment will occur when an air-water interface is present, ultimately leading to numerical ventilation.

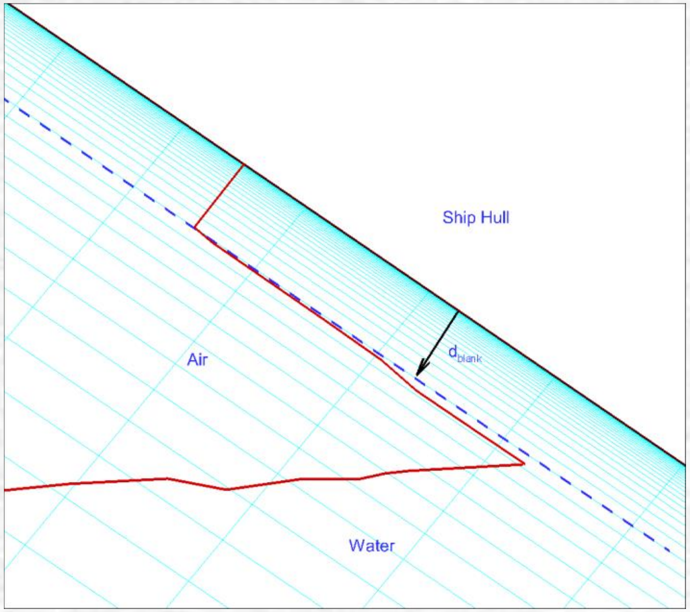


Velocity profile and air-water interface on a ship hull.

No-slip boundary condition and contact line problem

Moving Contact Line and wave blanking

- Herein, a wave blanking distance, d_{blank}, is used when solving the VOF interface equations.
  - ✓ The d_{blank} is defined as the distance in the normal direction of the wall.
  - ✓ The VOF interface equations will not be solved if the computational cells are located within the blanking distance
  - ✓ The values of the VOF and distance functions in these cells will be extrapolated from the cells outside of the blanking region.
  - ✓ The d_{blank} will be chosen based on the $y+$ values and the velocity profiles in the boundary layer.



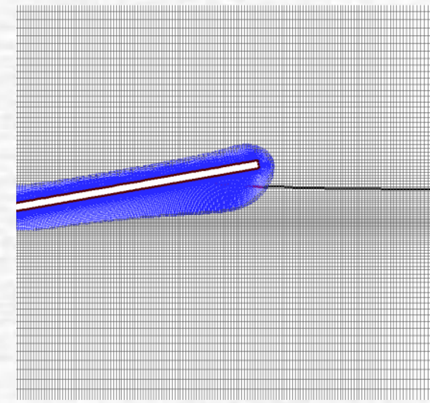
Wave blanking for interface contact line on the wall of a ship hull.

No-slip boundary condition and contact line problem

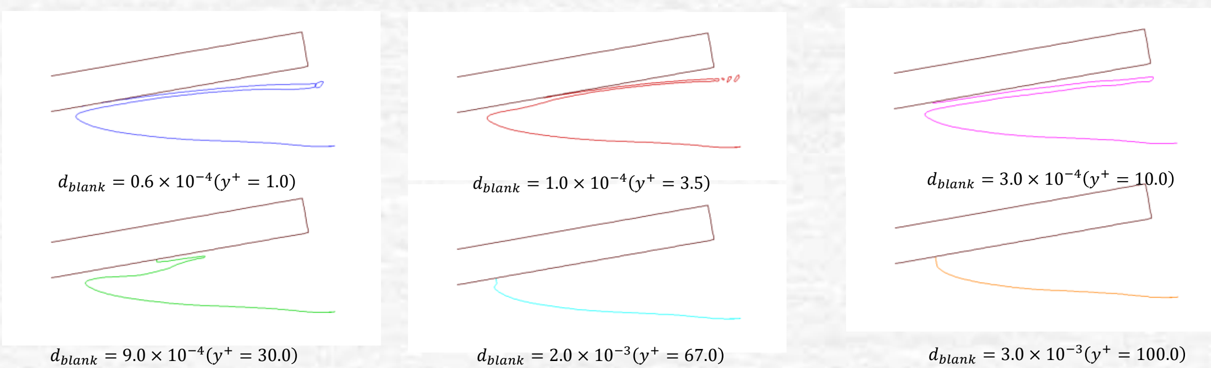
Numerical Tests: slamming plate

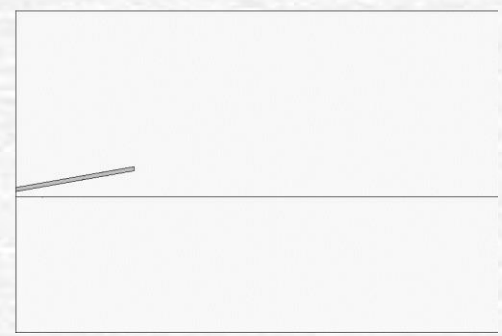
A deadrise angle of 10◦ and a pitch angle of 0◦ are considered; Froude number of *Fr* = 0.42.

- Very thin water jets are created for small blanking distances with y+ < 30. Air can be entrapped under the plate when the jet breaks up.
- The nonphysical air entrainment will affect the accuracy of the force calculations and computational stability.
- For the large blanking distances with y+ > 30, water jet is not formed, and the contact line is smooth, especially for y+ > 100.



Grids of a flat plate slamming onto a water surface

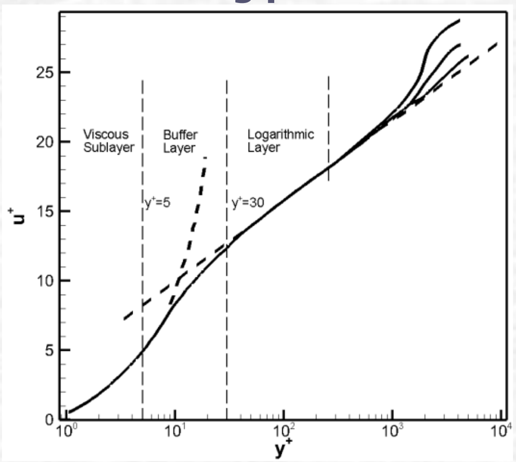




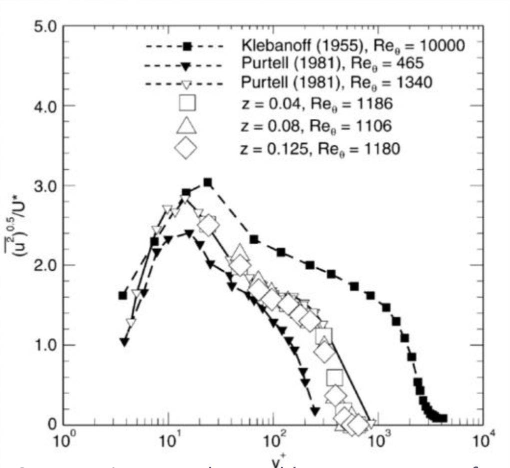
Movie

No-slip boundary condition and contact line problem

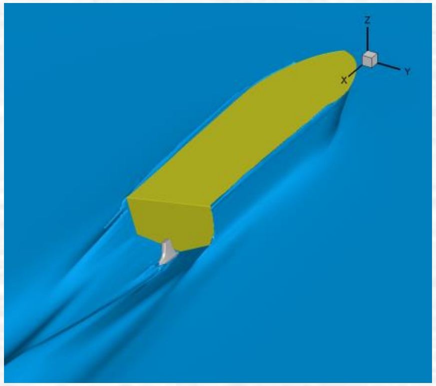
Numerical Tests: slamming plate



Boundary layer velocity profile



Stream-wise normal Reynolds stress near a surface-piercing flat plate (Longo et al., 1998).



GPPH, wave blank , $y+$ = 2320,

- Note that the wave blanking distances chosen based on the $y+$ values correspond to different regions of the wall boundary layer velocity distribution.
- If the wave blanking distance is inside the buffer layer and viscous sub-layer regions, non-smooth contact line with a very thin water jet and non-physical ventilation will occur.
- The flows within these layers are not stable and very large Reynolds stress can be observed as shown for a surfacing-piercing flat plate.
- Therefore, the blanking distance should be chosen outside of the buffer layer with $y+ > 30$ for a smooth air-water interface and to avoid nonphysical ventilation

No-slip boundary condition and contact line problem

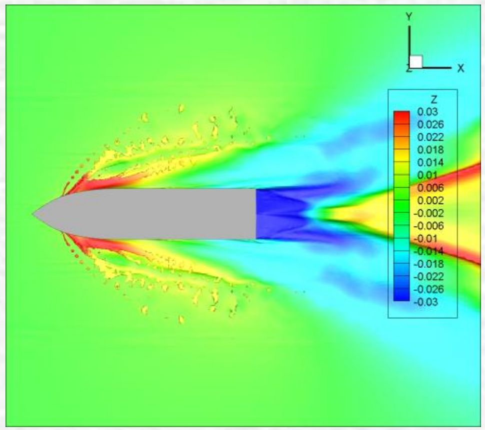
Application Examples: step planing hull

- A single step planing craft is chosen.
- The Found number is $Fr_b = 1.463$.
- 2DoF motions of heave and pitch are considered using the dynamic overset grid.
- The grid size is 11.34M with 6 blocks of overset grids.
- All the wave blanking distances used herein outside of the buffer layer with $y+ > 30$.
- Details of the geometry, grids, and computational setup in the study by Park et al. (2022).

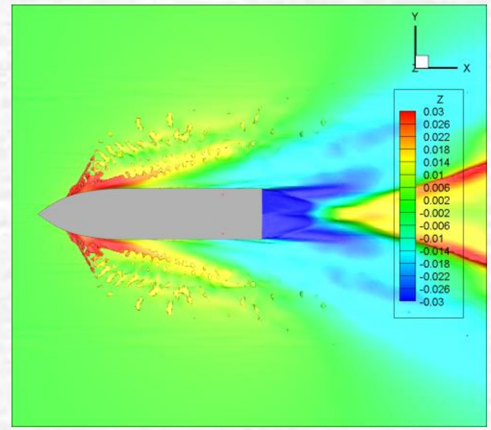
No-slip boundary condition and contact line problem

Application Examples: step planing hull

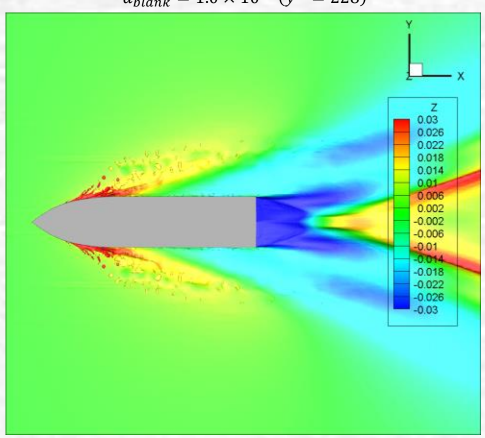
- Top view of the wave profiles of a single step planing hull



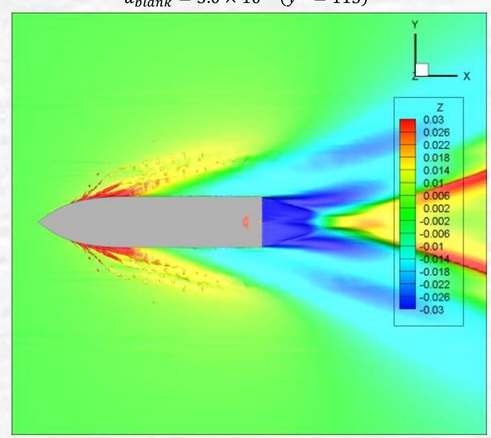
$d_{blank} = 1.0 \times 10^{-3} (y^{+} = 228)$



$d_{blank} = 5.0 \times 10^{-4} (y^{+} = 113)$



$d_{blank} = 3.0 \times 10^{-4} (y^{+} = 68)$

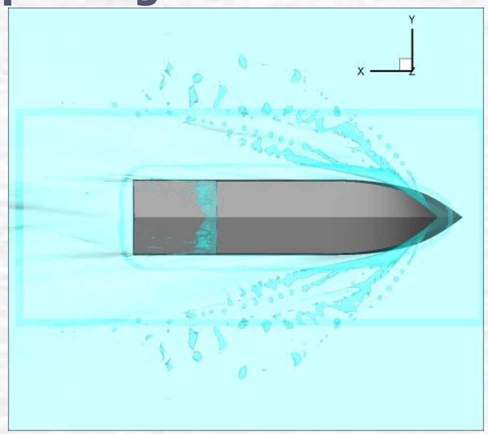


$d_{blank} = 2.0 \times 10^{-4} (y^{+} = 46)$

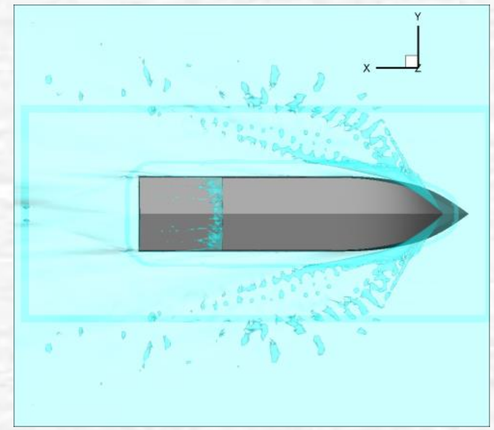
No-slip boundary condition and contact line problem

Application Examples: step planing hull

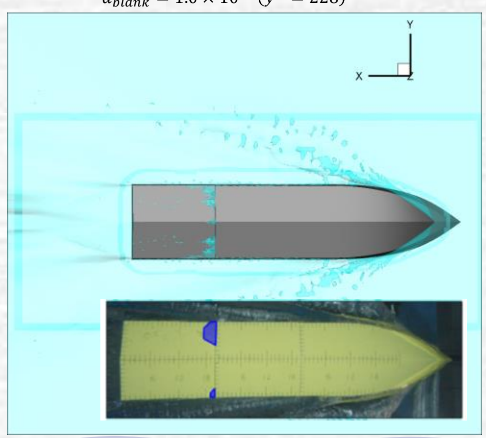
- The wave spread increases with the wave blanking distance.
- The air ventilation region under the step of the hull also increases with the wave blanking distance.
- Generally, the size of the ventilation region is comparable to the experimental observation



$d_{blank} = 1.0 \times 10^{-3} (y^+ = 228)$



$d_{blank} = 5.0 \times 10^{-4} (y^+ = 113)$



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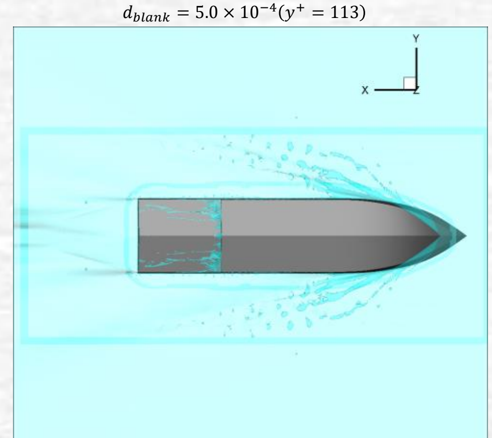
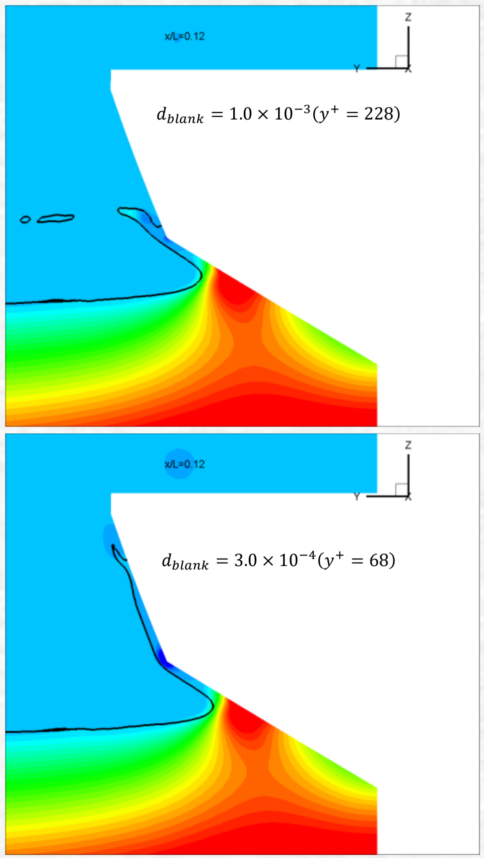


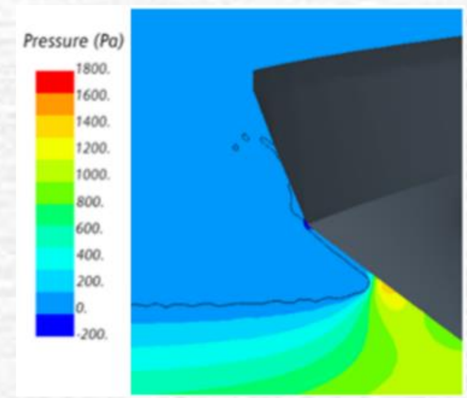
$d_{blank} = 2.0 \times 10^{-4} (y^+ = 46)$

No-slip boundary condition and contact line problem

Application Examples: step planing hull

- The water jet separates from the hull with the increase of the wave blanking distance.
- For a small wave blanking distance, a water film is formed and stuck on the hull.
- The computation using a large wave blanking distance is more stable as compared to that using a small one.





StarCCM+

The slices of the wave profile and pressure distribution cut in the stream-wise direction.

No-slip boundary condition and contact line problem

Application Examples: step planing hull

Comparison of the force and motions for singe step hull Fr_b=1.463.

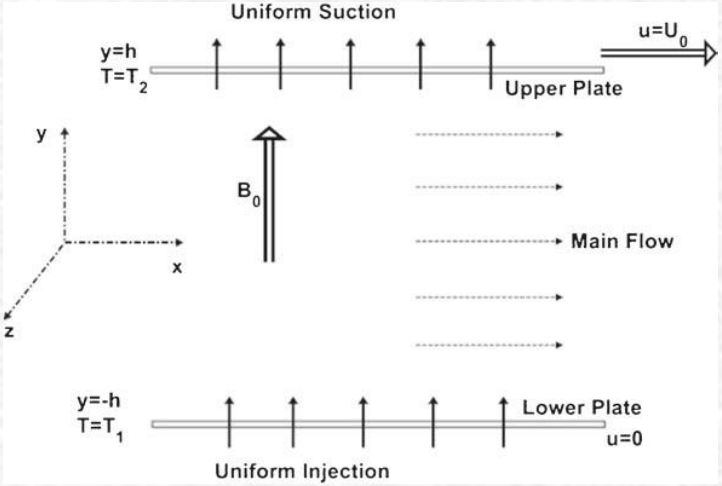
	Force (N)	Error (%)	Heave (mm)	Error (%)	Pitch (deg)	Error (%)
EFD	49.469		0.445		4.216	
y+=228	47.69	3.596	4.336	874.4	3.542	15.99
y+=113	48.51	1.938	4.305	867.4	3.589	14.87
y+=68	48.32	2.322	3.889	754.7	3.725	11.64
y+=46	48.58	1.797	4.065	793.4	3.723	11.69

- Generally, both forces and motions improved with decrease of the wave blank value
- For y+<100, results are comparable.
- $30 < y+ < 200$ in consideration of accuracy and stability, and a value of $y+ \sim 100$ can be used in practice.

Examples of Boundary conditions

Permeable interface, porous surface:

Suction or Injection



$u = 0, no\ slip$

$v = v_s$ or $v = v_i$, $flow\ through\ the\ wall$

$T_{fluid} = T_{wall},\ \ no\ temperature\ jump$

$q_w = hT_y|_w = \rho_i v_i c_p (T_w - T_i)$, $energy\ at\ the\ wall$

Examples of Boundary conditions

Fluid Structure Interaction (FSI) BCs:

Kinematic continuity between the fluid and the structure is ensured by the non-slip wall condition:

$$u_w = \frac{\partial x}{\partial t}, v_w = \frac{\partial y}{\partial t}, w_w = \frac{\partial z}{\partial t}$$

where $\boldsymbol{u}_w = \{u_w, v_w, w_w\}$ is the velocity of the fluid particle and $\boldsymbol{x} = \{x, y, z\}$ are the coordinates of the solid wall.

The *continuity of the momentum* is ensured by the continuity of the stress across the fluid-structure interface:

$$\tau_{ij} \cdot n_i|_w = \tau_{ij} \cdot n_i|_s$$

The *energy conservation*, considering the first law of thermodynamics (not currently implemented within CFDShip-Iowa)

$$\delta Q - \delta W = dE$$

The energy equation for adiabatic CV,

$$-\frac{\delta W}{dt} = \frac{dE}{dt} = \frac{\partial}{\partial t}\iiint_{V(t)} e\rho \, dV + \iint_{S(t)} e\rho(\boldsymbol{u} \cdot \hat{n}) dS$$

$e = k_e + p_{e\varepsilon} + p_{eg}$. k_e , $p_{e\varepsilon}$ and p_{eg} are the kinetic and elastic and gravitational potential energies.

Examples of Boundary conditions

Fluid Structure Interaction (FSI) BCs:

The work rate flows are exchanged between the water CV and the structure CV.

$$\dot{W} = \underbrace{\dot{W}_{shaft}}_{pump/turbine} + \underbrace{\dot{W}_{p}}_{pressure} + \underbrace{\dot{W}_{v}}_{viscous}$$

$\dot{W}_p$ and $\dot{W}_v$ are the pressure and viscous work rates done by the CV.

Within the fluid CV, $\dot{W}_{shaft} = 0$. Within the structure CV, $\dot{W}_{shaft}$ is the work rate done by the plate's mount on the system and is hereafter named $\dot{W}_M$.

The pressure and viscous work done on solid = pressure and viscous work done on fluid and vice versa, which are equivalent to separate dE/dt for solid which includes $e = k_e + p_{e\varepsilon} + p_{eg}$ and fluid which only includes $e = k_e + p_{eg}$. In most cases net outflux of energy from the CV is zero.

Thus for the fluid $$\frac{dE}{dt} = \frac{\partial}{\partial t}\iiint_{V_w} \rho_w \left(gz + \frac{\|\boldsymbol{u}_w\|^2}{2}\right) dV_w$$

And for the solid $$\frac{dE}{dt} = \frac{\partial}{\partial t}\iiint_{V(t)} e\rho \, dV$$

Examples of Boundary conditions

Single flow free surface BCs:

- Free surface problems since interface is unknown and part of the solution, but effect gas on liquid idealized.
- Assume the upper fluid (air) is an “atmosphere” that merely exerts pressure on the lower fluid (water), with shear and heat conduction negligible.
- Kinematic FSBC: free surface is stream surface
- Dynamic FSBC: stress continuous across free surface (similar for mass and heat flux)

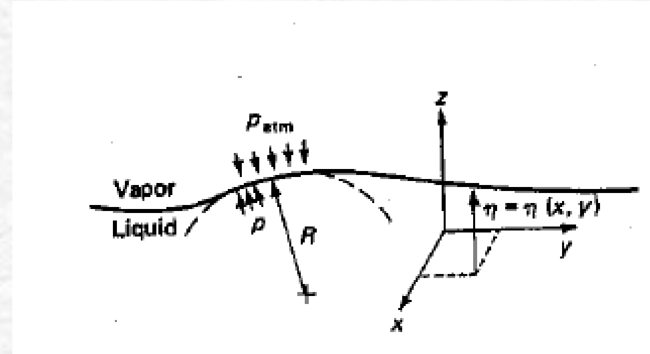
Approximations:

$p \approx p_a = 0$, *neglect air viscosity and surface tension*

$\xi_x \sim \xi_y \sim 0$, *small slope*

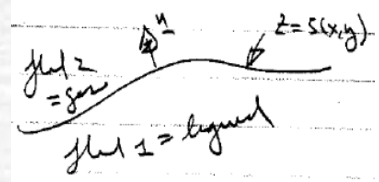
$w_x \sim w_y \sim w_z = 0, \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$

small normal velocity gradient



$F = \zeta(x, y) - z = surface\ function$

$\underline{n} = \nabla F / |\nabla F| = (\zeta_x, \zeta_y, -1) / \left[\zeta_x^2 + \zeta_y^2 + 1\right]^{\frac{1}{2}}$



$$\frac{DF}{Dt} = 0 = \frac{\partial F}{\partial t} + \underline{V} \cdot \nabla F$$

$$\frac{1}{|\nabla F|} \frac{\partial F}{\partial t} + \underline{V} \cdot \underline{n} = 0$$

$$\tau_{ij} n_j = \tau_{ij}^{*} n_j - p_{\gamma} \delta_{ij}$$

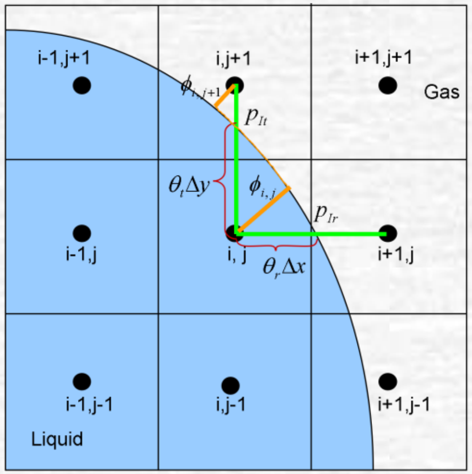
Fluid 1 stress — Fluid 2 stress — Surface tension pres.

Examples of Boundary conditions

<u>Single flow free surface BCs: (</u>*CFDShip-Iowa V6.0)*

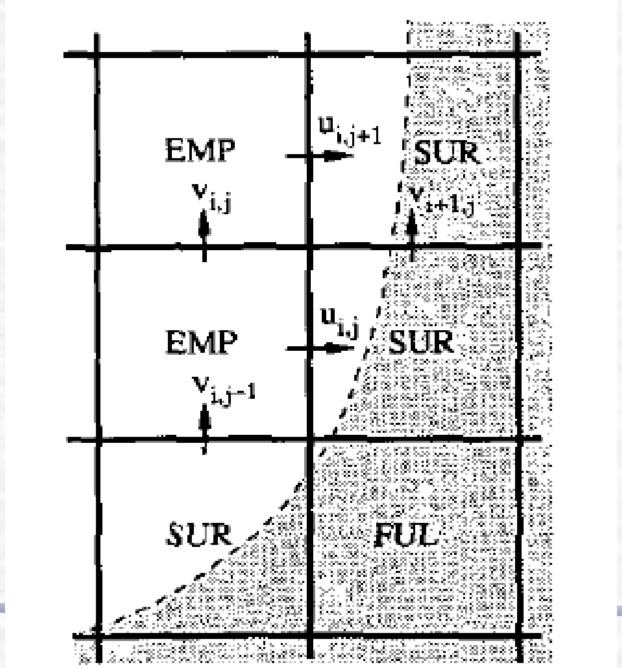
•<u>Pressure Poisson equation</u> on an irregular domain with Dirichlet boundary conditions [Gibou et al., JCP, 2002], [Balay et al., 1997]

$$p \approx p_a = 0$$



•<u>Velocity extension</u> to the void (gas) cells [Sussman et al., JCP, 2007], [Chen et al., JCP 1995]

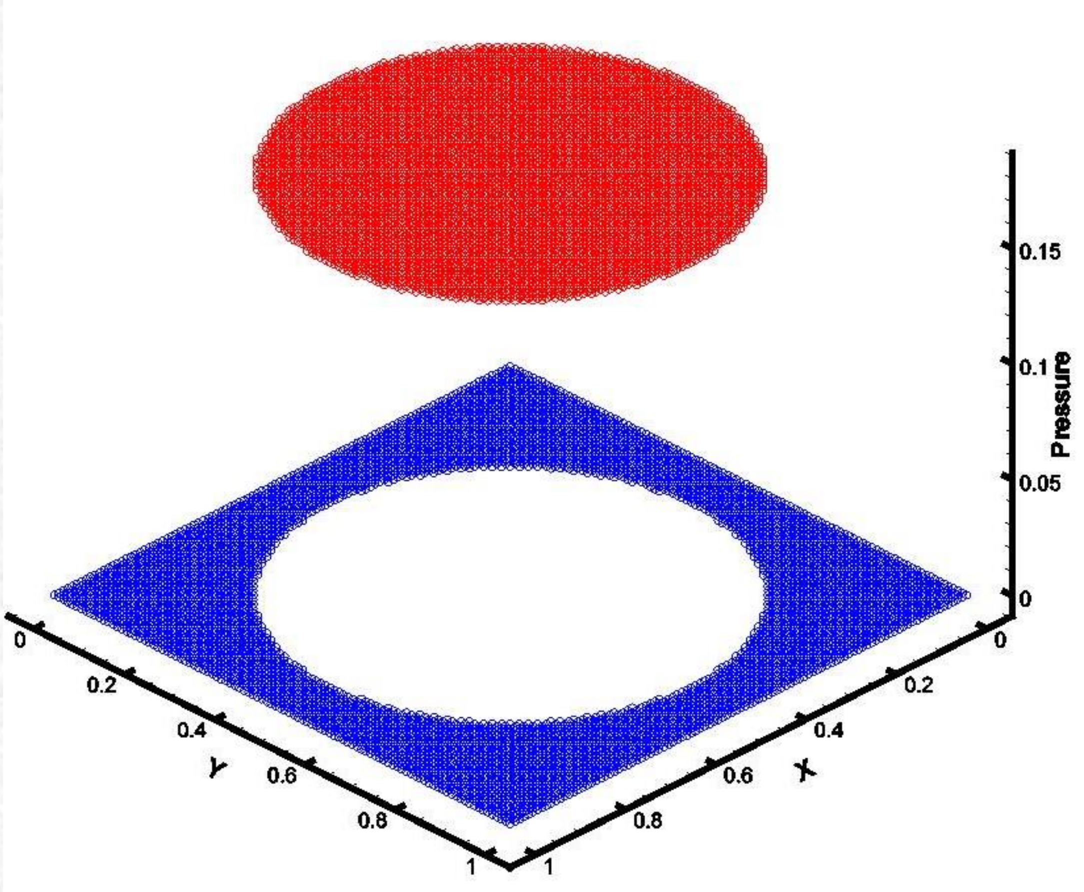
•<u>Physical property treatment</u>: density, viscosity, and surface tension



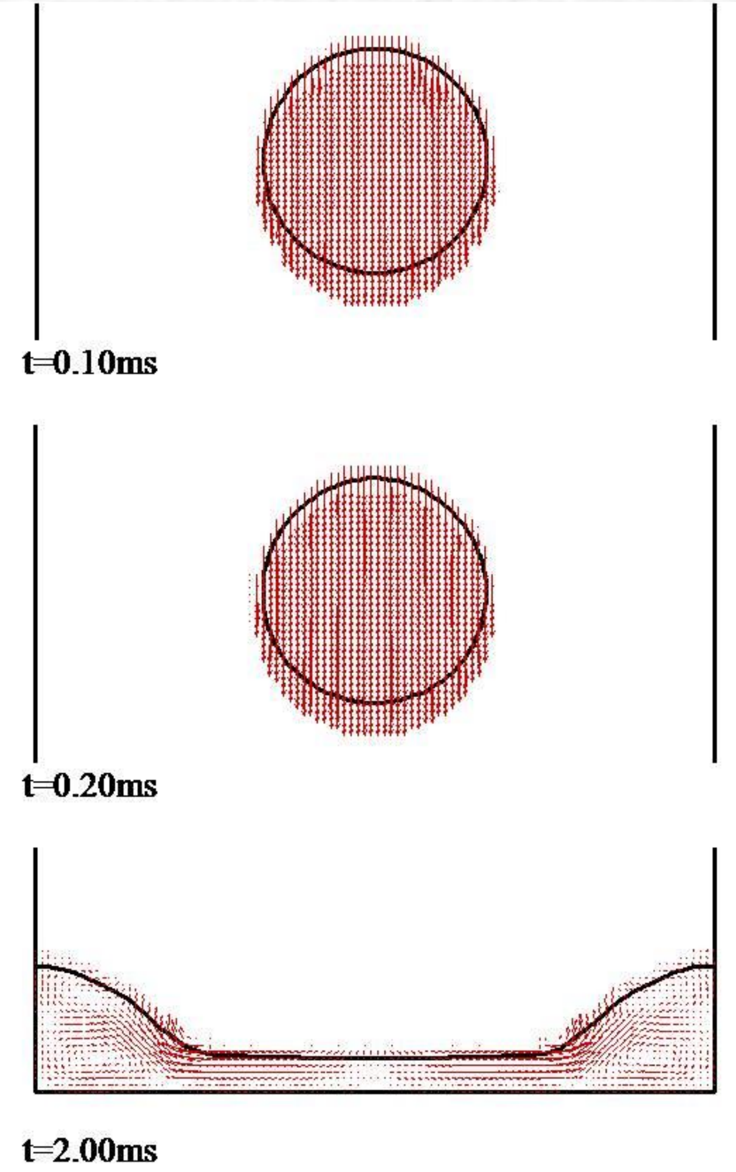
Chen et al., 1995

Examples of Boundary conditions

Single flow free surface BCs: *(CFDShip-Iowa V6.0).*

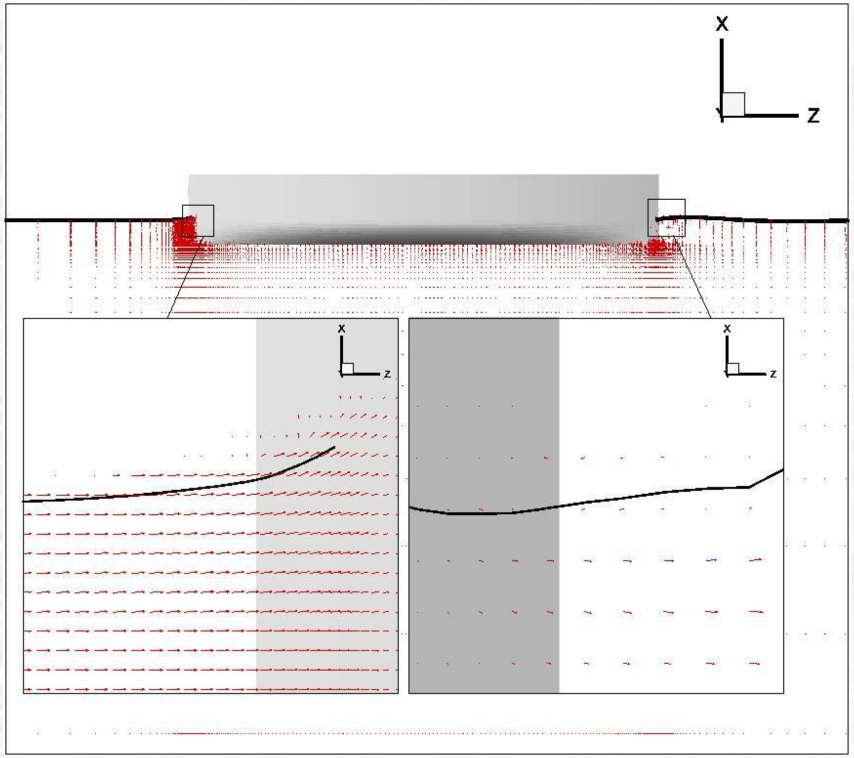


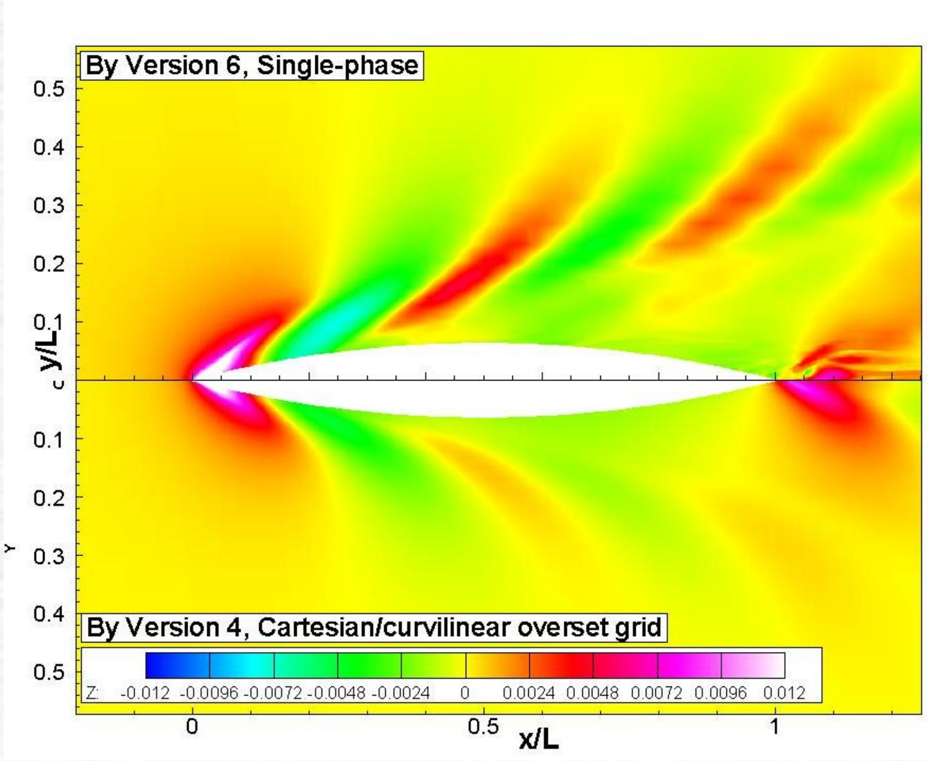
Static droplet pressure distribution



Examples of Boundary conditions

Single flow free surface BCs: *(CFDShip-Iowa V6.0).*

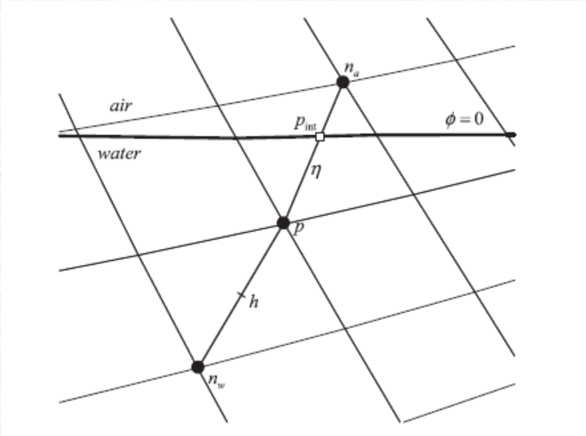




Examples of Boundary conditions

Single flow free surface BCs (CFDShip-Iowa V4.5)

- Free surface is tracked by level set method*.



- Velocity boundary conditions at the free surface

$$\nabla v \cdot \mathbf{n} = 0$$

- Pressure jump conditions

$$[p_{\text{abs}} - 2\mu(\nabla \mathbf{v} \cdot \mathbf{n}) \cdot \mathbf{n}] = \sigma\kappa + \nabla_i \sigma \cdot \mathbf{n}$$

(Carrica et al., 2007)

- Neglect air viscosity and surface tension, the pressure at the free surface

$$p \approx p_a = 0$$

- Dimensionless piezometric pressure, at the fluid/fluid interface the pressure

$$p = z/Fr^2$$

*Carrica, P. M., Wilson, R. V., & Stern, F. (2007). An unsteady single-phase level set method for viscous free surface flows. *International Journal for Numerical Methods in Fluids*, *53*(2), 229-256.

Examples of Boundary conditions

Single flow free surface BCs: *(CFDShip-Iowa V4.5).*

Velocity extension

- Total time derivative in grid points in which the level set function changes from air to water is replaced by

$$\frac{D\mathbf{u}}{Dt} = \frac{\mathbf{u}(\mathbf{r}_p, t) - \mathbf{u}(\mathbf{r}_p, t - \Delta t)}{\Delta t} + \frac{\phi(\mathbf{r}_p, t)}{\phi(\mathbf{r}_p, t) - \phi(\mathbf{r}_p, t - \Delta t)}\mathbf{u}(\mathbf{r}_p, t) \cdot \nabla\mathbf{u}(\mathbf{r}_p, t)$$

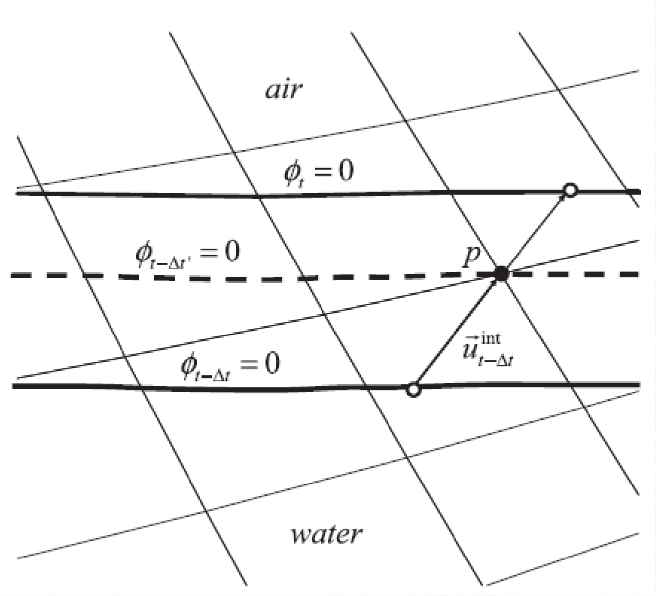
- Velocity extension in the normal direction in air

$$\nabla\mathbf{v} \cdot \mathbf{n} = 0$$

$$\mathbf{n} = \frac{\nabla\phi}{|\nabla\phi|}$$

- Convective extension only in air,

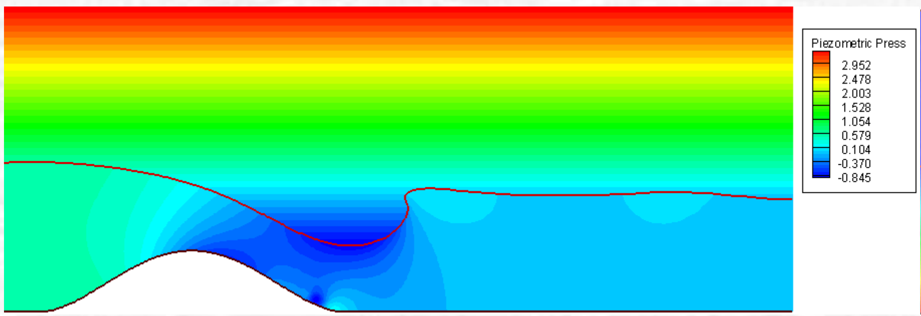
$$\mathbf{u}(\mathbf{r}_p, t - \Delta t) \cdot \nabla\mathbf{u}(\mathbf{r}_p, t - \Delta\mathrm{t}) = 0$$



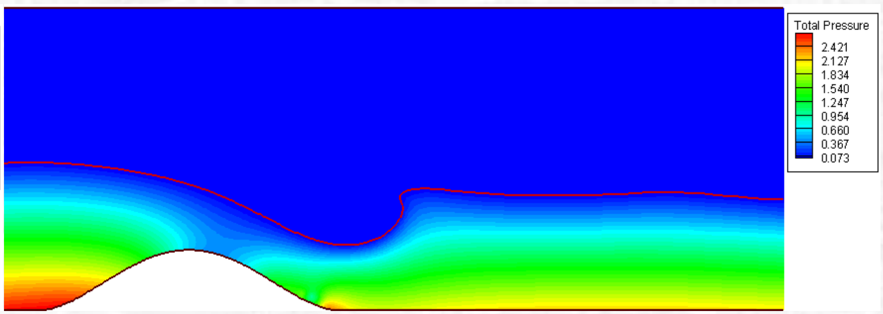
(Carrica et al., 2007)

Examples of Boundary conditions

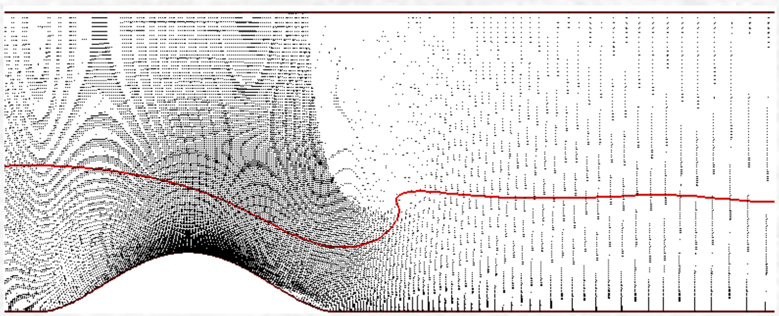
Single flow free surface BCs: *(CFDShip-Iowa V4.5).*



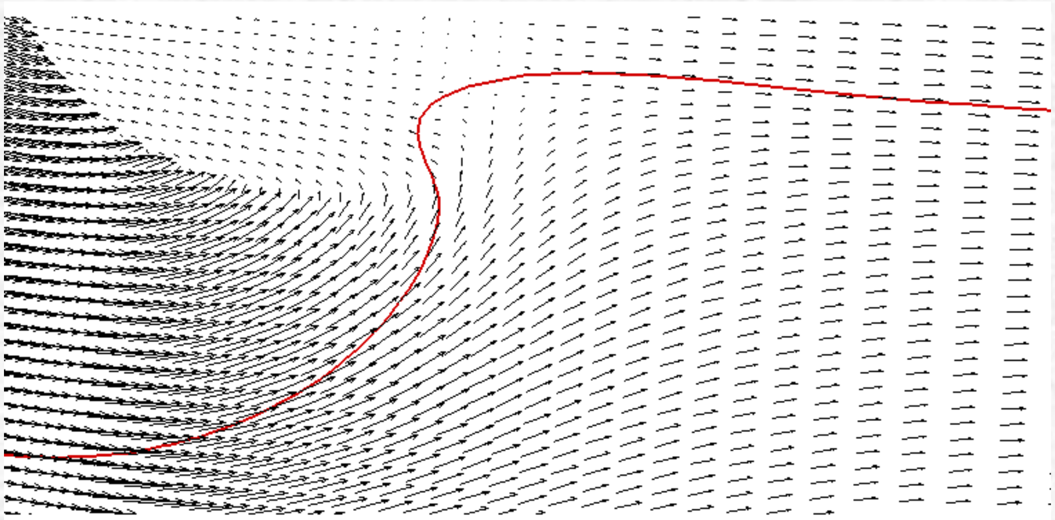
Piezometric pressure distribution of bump wave breaking



Total pressure distribution of bump wave breaking



Velocity vector field of bump wave breaking



Examples of Boundary conditions

Two-phase interface jump conditions:

The velocity fields in fluids 1 and 2 are continuous across the interface if there is no phase change and mass transfer across the interface,

$$\boxed{\mathbf{u_1} = \mathbf{u_2}} \quad (1)$$

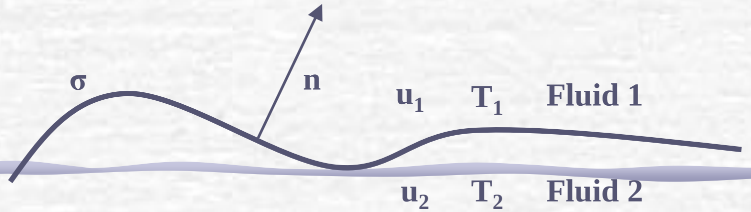
where $\mathbf{u}$ is the velocity vector. The interface velocity V_I is the normal velocity and is the same on both sides of the interface:

$$V_I = \mathbf{u_1} \cdot \mathbf{n} = \mathbf{u_2} \cdot \mathbf{n} \quad (\textit{kinematic condition}) \quad (2)$$

where $\mathbf{n}$ is the unit normal vector.

The continuity of the tangential velocities is analogous to the no-slip boundary condition on a wall,

$$\mathbf{u_1} - (\mathbf{u_1} \cdot \mathbf{n})\mathbf{n} = \mathbf{u_2} - (\mathbf{u_2} \cdot \mathbf{n})\mathbf{n} \quad (\textit{continuity of the tangential velocity}) \quad (3)$$



Examples of Boundary conditions

Stress conditions:

The stress tensor is defined in terms of the local fluid pressure and velocity field as

$$\mathbf{T} = -p\mathbf{I} + \boldsymbol{\tau} = -p\mathbf{I} + \mu[\nabla\mathbf{u} + (\nabla\mathbf{u})^T] \tag{4}$$

where $\mathbf{I}$ is the unit tensor, $\boldsymbol{\tau}$ is viscous stress tensor, p is pressure, and μ is the dynamic viscosity. The stress vector, the force (per unit area) exerted by the fluid on the interface, is defined as,

$$\mathbf{t}(\mathbf{n}) = \mathbf{n} \cdot \mathbf{T} \tag{5}$$

Note that the stress vector in the above equation generally includes both the *normal and tangential* stress components.

The *exact interface stress condition* is given in the stress balance equation below:

$$\mathbf{n} \cdot \mathbf{T_1} - \mathbf{n} \cdot \mathbf{T_2} = \sigma\mathbf{n}(\nabla \cdot \mathbf{n}) - \nabla\sigma \tag{6}$$

where $\nabla\sigma$ is tangential stress associated with gradients of the surface tension. The divergence of the unit normal is related to the mean curvature:

$$\boldsymbol{\nabla} \cdot \mathbf{n} = \kappa \tag{7}$$

The stress jump condition can be rewritten as

$$\mathbf{n} \cdot (\mathbf{T_1} - \mathbf{T_2}) = \sigma\kappa\mathbf{n} - \nabla\sigma \tag{8}$$

Examples of Boundary conditions

Note that both normal and tangential stresses must be balanced at the interface. The condition can be written separately as the "*normal stress balance*" and "*tangential stress balance*".

Normal stress balance

Projection of Eq. (8) along the unit normal **n** obtains,

$$\boxed{\mathbf{n} \cdot (\mathbf{T_1} - \mathbf{T_2}) \cdot \mathbf{n} = \sigma\kappa\mathbf{n} \cdot \mathbf{n} = \sigma\kappa} \qquad (9)$$

Tangential stress balance

Taking dot product of Eq. (8) with any unit tangential vector **t** yields the tangential stress balance,

$$\boxed{\mathbf{n} \cdot (\mathbf{T_1} - \mathbf{T_2}) \cdot \mathbf{t} = \nabla\sigma \cdot \mathbf{t}} \qquad (10a)$$

The surface tension σ depends on temperature and composition of the interface, which can be treated as a constant. The gradient of surface tension will vanish, and the tangential stress is continuous across the interface.

$$\boxed{\mathbf{n} \cdot (\mathbf{T_1} - \mathbf{T_2}) \cdot \mathbf{t} = 0} \qquad (10b)$$

Examples of Boundary conditions

Numerical approximation of the jump conditions

The viscous stress tensor $\boldsymbol{\tau}$ can be written as,

$$\boldsymbol{\tau} = \mu[\nabla\mathbf{u} + (\nabla\mathbf{u})^T] = \mu\begin{pmatrix}\boldsymbol{\nabla} u\\ \nabla v\\ \nabla w\end{pmatrix} + \mu\begin{pmatrix}\nabla u\\ \nabla v\\ \nabla w\end{pmatrix}^T \quad (11)$$

Using the jump notation $[x] = x_1 - x_2$, and $\mathbf{t}_I$ and $\mathbf{t}_{II}$ the orthogonal unit tangential vectors, the stress jump conditions Eqs. (9) and (10b) can be rewritten as three separate jump conditions,

$$[p - 2\mu(\nabla u \cdot \mathbf{n}, \nabla v \cdot \mathbf{n}, \nabla w \cdot \mathbf{n}) \cdot \mathbf{n}] = \sigma\kappa \quad (12)$$

$$[\mu(\nabla u \cdot \mathbf{n}, \nabla v \cdot \mathbf{n}, \nabla w \cdot \mathbf{n}) \cdot \mathbf{t}_I + \mu(\nabla u \cdot \mathbf{t}_I, \nabla v \cdot \mathbf{t}_I, \nabla w \cdot \mathbf{t}_I) \cdot \mathbf{n}] = 0 \quad (13)$$

$$[\mu(\nabla u \cdot \mathbf{n}, \nabla v \cdot \mathbf{n}, \nabla w \cdot \mathbf{n}) \cdot \mathbf{t}_{II} + \mu(\nabla u \cdot \mathbf{t}_{II}, \nabla v \cdot \mathbf{t}_{II}, \nabla w \cdot \mathbf{t}_{II}) \cdot \mathbf{n}] = 0 \quad (14)$$

The velocity is continuous and the tangential velocity derivatives are also continuous,

$$[\nabla\mathbf{u} \cdot \mathbf{t}_I^T] = 0 \quad \text{or} \quad [\nabla u \cdot \mathbf{t}_I] = [\nabla v \cdot \mathbf{t}_I] = [\nabla w \cdot \mathbf{t}_I] = 0 \quad (15)$$

$$[\nabla\mathbf{u} \cdot \mathbf{t}_{II}^T] = 0 \quad \text{or} \quad [\nabla u \cdot \mathbf{t}_{II}] = [\nabla v \cdot \mathbf{t}_{II}] = [\nabla w \cdot \mathbf{t}_{II}] = 0 \quad (16)$$

Examples of Boundary conditions

Numerical approximation of the jump conditions

The normal stress condition can be written as,

$$[p] - 2[\mu](\nabla u \cdot \mathbf{n}, \nabla v \cdot \mathbf{n}, \nabla w \cdot \mathbf{n}) \cdot \mathbf{n} = \sigma\kappa \tag{17}$$

The tangential jump conditions

$$[\mu\nabla\mathbf{u}] = [\mu](\nabla\mathbf{u})\begin{pmatrix}\mathbf{0}\\ \mathbf{t}_I\\ \mathbf{t}_{II}\end{pmatrix}^T\begin{pmatrix}\mathbf{0}\\ \mathbf{t}_I\\ \mathbf{t}_{II}\end{pmatrix} + [\mu]\mathbf{n}^{\boldsymbol{T}}\mathbf{n}(\nabla\mathbf{u})\mathbf{n}^{\boldsymbol{T}}\mathbf{n} - [\mu]\begin{pmatrix}\mathbf{0}\\ \mathbf{t}_I\\ \mathbf{t}_{II}\end{pmatrix}^T\begin{pmatrix}\mathbf{0}\\ \mathbf{t}_I\\ \mathbf{t}_{II}\end{pmatrix}(\nabla\mathbf{u})^T\mathbf{n}^{\boldsymbol{T}}\mathbf{n} \tag{18}$$

Note that the right-hand side of the above equation only involves velocity derivatives that are continuous across the interface. If $[\mu] = 0$, then $[\nabla\mathbf{u}] = \mathbf{0}$.

If the viscosity is smoothed to be continuous across the interface, the normal jump condition

$$[p] = \sigma\kappa \tag{19}$$

The tangential viscous stress jump condition,

$$[(\nabla\mathbf{u} \cdot \mathbf{n}^T) \cdot \mathbf{t}_I] + [(\nabla\mathbf{u} \cdot \mathbf{t}_I^T) \cdot \mathbf{n}] = 0 \tag{20}$$

According to Eq. (18), with a constant viscosity, all the velocity derivatives will be continuous across the interface which implies that both jump terms on the left hand side of the above equation are zero.

Examples of Boundary conditions

Vorticity condition across the interface

The vorticity in the **normal direction** is written as,

$$\mathbf{n}\cdot\boldsymbol{\omega} = \mathbf{n}\cdot(\nabla\times\mathbf{u}) = (\mathbf{t}_I\times\mathbf{t}_{II})\cdot(\nabla\times\mathbf{u}) \tag{21}$$

Using the identity $(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{c}\times\mathbf{d}) = (\mathbf{a}\cdot\mathbf{c})(\mathbf{b}\cdot\mathbf{d}) - (\mathbf{a}\cdot\mathbf{d})(\mathbf{b}\cdot\mathbf{c})$, Eq. (21) can be rewritten as,

$$\mathbf{n}\cdot\boldsymbol{\omega} = (\mathbf{t}_I\cdot\nabla)(\mathbf{t}_{II}\cdot\mathbf{u}) - (\mathbf{t}_I\cdot\mathbf{u})(\mathbf{t}_{II}\cdot\nabla) \tag{22}$$

which is **continuous** across the interface since the right hand side of the above equation only involves tangential derivatives of the velocity.

The vorticity in the **tangential directions**,

$$\mathbf{t}_I\cdot\boldsymbol{\omega} = \mathbf{t}_I\cdot(\nabla\times\mathbf{u}) = (\mathbf{t}_{II}\times\mathbf{n})\cdot(\nabla\times\mathbf{u}) = (\mathbf{t}_{II}\cdot\nabla)(\mathbf{n}\cdot\mathbf{u}) - (\mathbf{t}_{II}\cdot\mathbf{u})(\mathbf{n}\cdot\nabla) \tag{23}$$

$$\mathbf{t}_{II}\cdot\boldsymbol{\omega} = \mathbf{t}_{II}\cdot(\nabla\times\mathbf{u}) = (\mathbf{n}\times\mathbf{t}_I)\cdot(\nabla\times\mathbf{u}) = (\mathbf{n}\cdot\nabla)(\mathbf{t}_I\cdot\mathbf{u}) - (\mathbf{n}\cdot\mathbf{u})(\mathbf{t}_I\cdot\nabla) \tag{24}$$

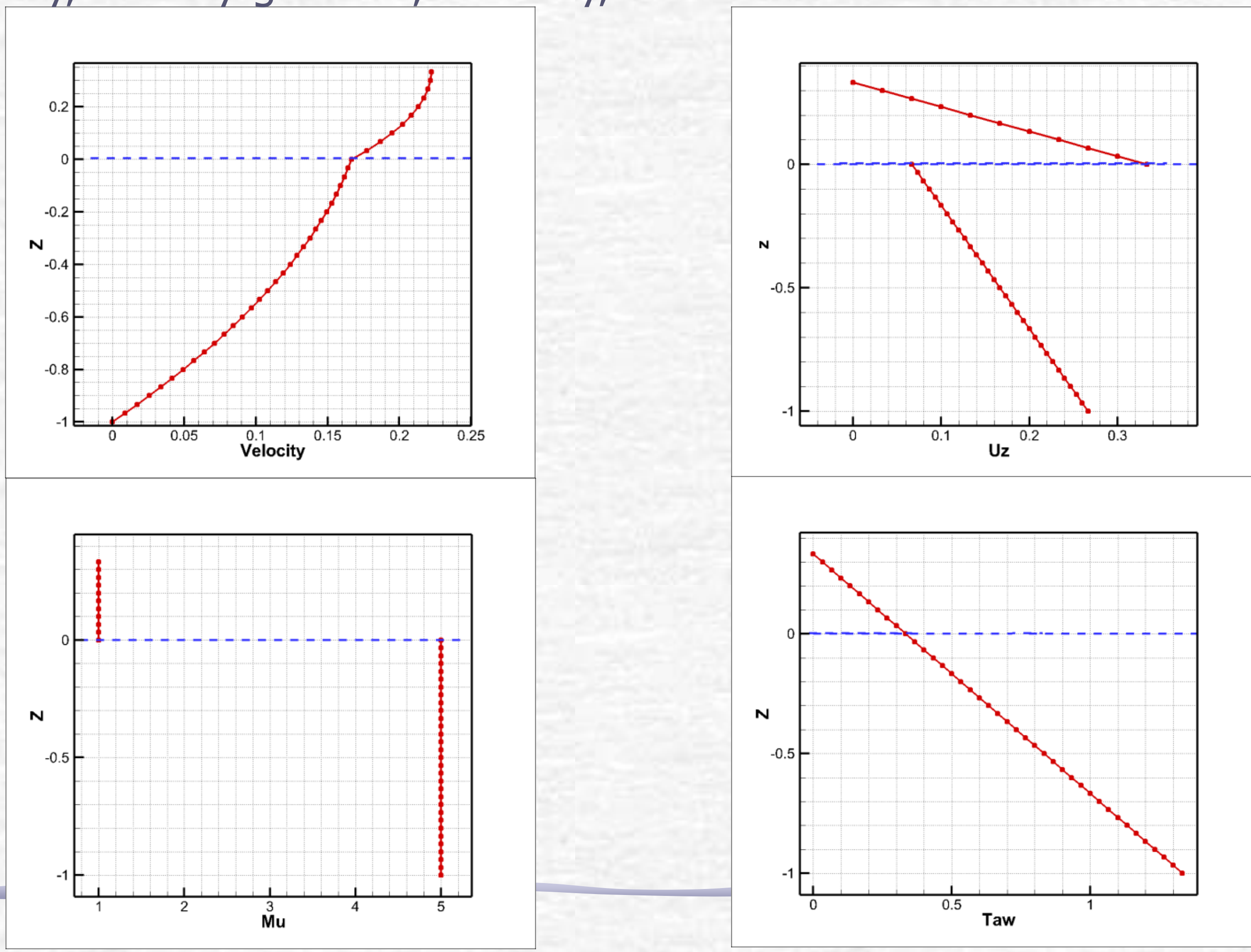
The tangential vorticities are generally **not continuous** across the interface since the normal derivatives, $(\mathbf{t}_{II}\cdot\mathbf{u})(\mathbf{n}\cdot\nabla)$ and $(\mathbf{n}\cdot\nabla)(\mathbf{t}_I\cdot\mathbf{u})$, are involved in the above equations, respectively.

However, as shown in Eq. (20), all the velocity derivatives will be continuous if the viscosity jump $[\mu] = 0$, then tangential vorticities will also be continuous.

Examples of Boundary conditions

For two immiscible fluids with different density and viscosity:

Velocity, velocity gradient, viscosity, and shear stress distribution



Velocity, *Uz*, viscosity, and Taw profile for a layered two fluid flow.

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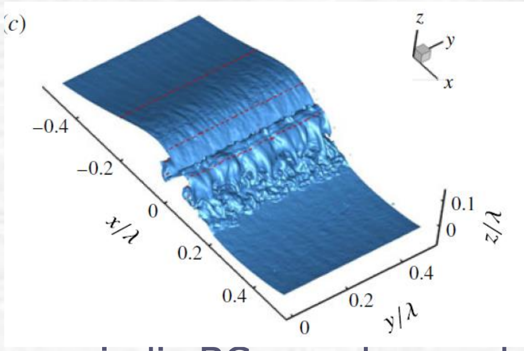
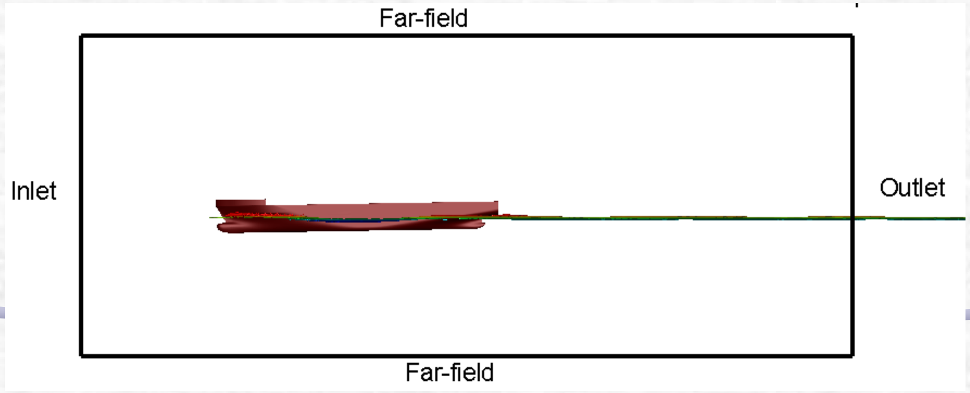
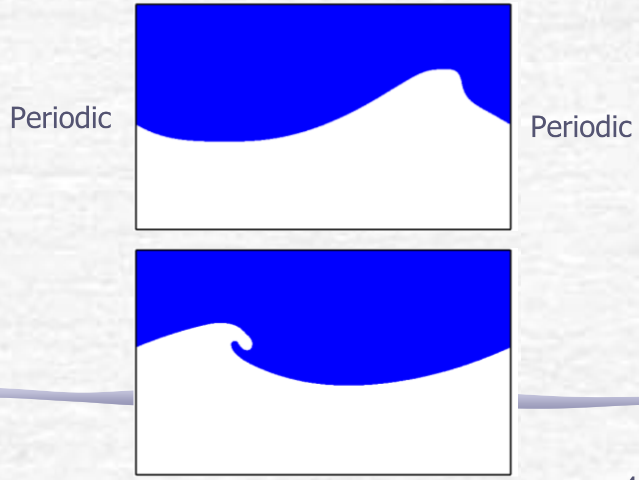
Examples of Boundary conditions

Inlet/outlet/exit/outer/far-field BCs:

- *Inlet:* V, p, T, specified, e.g., constant values are used, $V = V_{in}$, $p = 0$, $T = T_{in,0}$
- *Outer or far-field*: V, p, T, specified similarly as inlet
- *Exit:* depends on the problems, but
  - often use $\mathrm{U_{xx}} = 0$ and $\frac{\partial p}{\partial n} = 0$.
  - For external flow, zero stream wise diffusion
  - For fully developed internal flow and wave problem, periodic BCs can be used
  - For unsteady internal flow, global mass conservation enforcement may be needed: $U_{out} = U_{out}\frac{Q_{in}}{Q_{out}}$, where Q_{in} and Q_{out} is the total inlet and outlet and flux, respectively.







BCs in CFDShip-Iowa

	IBTYP	Description	U	V	W	P	k	ω	ν_t
Domain Truncation Boundaries	10	Inlet	UINF	VINF	WINF	$\partial P/\partial\xi_i = 0$	$k_{fst}=1\times10^{-7}$	$\omega_{fst}=9.0$	$\nu_{t,fst}$
	11	Exit	$\partial^2 U/\partial\xi_i^2 = 0$	$\partial^2 V/\partial\xi_i^2 = 0$	$\partial^2 W/\partial\xi_i^2 = 0$	$\partial P/\partial\xi_i = 0$	$\partial k/\partial\xi_i = 0$	$\partial\omega/\partial\xi_i = 0$	$\partial\nu_t/\partial\xi_i = 0$
	12	Far-field #1	UINF	$\partial V/\partial\xi_i = 0$	$\partial W/\partial\xi_i = 0$	0	$\partial k/\partial\xi_i = 0$	$\partial\omega/\partial\xi_i = 0$	$\partial\nu_t/\partial\xi_i = 0$
	13	Far-field #2	UINF	VINF	WINF	$\partial P/\partial\xi_i = 0$	$\partial k/\partial\xi_i = 0$	$\partial\omega/\partial\xi_i = 0$	$\partial\nu_t/\partial\xi_i = 0$
	14	Prescribed	*	*	*	*	*	*	*
Physical Boundaries	20	Absolute-frame no-slip	0	0	0	$\partial P/\partial\xi_i = 0$	0	$60/\mathrm{Re}\,\beta\Delta y^2$	0
	22	Relative-frame no-slip	$\dot{x}$	$\dot{y}$	$\dot{z}$	$\partial P/\partial\xi_i = 0$	0	$60/\mathrm{Re}\,\beta\Delta y^2$	0
	27	Impermeable slip (calculate forces)	Eq. (78)	Eq. (78)	Eq. (78)	$\partial P/\partial\xi_i = 0$	$\partial k/\partial\xi_i = 0$	$\partial\omega/\partial\xi_i = 0$	$\partial\nu_t/\partial\xi_i = 0$
	28	Impermeable slip (no forces)	Eq. (78)	Eq. (78)	Eq. (78)	$\partial P/\partial\xi_i = 0$	$\partial k/\partial\xi_i = 0$	$\partial\omega/\partial\xi_i = 0$	$\partial\nu_t/\partial\xi_i = 0$
	30	Free surface	Eq. (34)	Eq. (34)	Eq. (35)	Eq. (33)	$\partial k/\partial\xi_i = 0$	$\partial\omega/\partial\xi_i = 0$	$\partial\nu_t/\partial\xi_i = 0$

BCs in CFDShip-Iowa

Computational Boundaries	40	Zero gradient	$\partial U/\partial \xi_i = 0$	$\partial V/\partial \xi_i = 0$	$\partial W/\partial \xi_i = 0$	$\partial P/\partial \xi_i = 0$	$\partial k/\partial \xi_i = 0$	$\partial \omega/\partial \xi_i = 0$	$\partial \nu_t/\partial \xi_i = 0$
	41	Translational periodicity, w/ ghost cells	*	*	*	*	*	*	*
	42	Translational periodicity, w/o ghost cells	*	*	*	*	*	*	*
	43	Pole (l-around)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)
	44	Pole (j-around)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)
	45	Pole (k around)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)	Eq. (80)
	50	Cylindrical zero gradient	*	*	*	*	*	*	*
	51	Rotational periodicity, w/ ghost cells	*	*	*	*	*	*	*
	52	Rotational periodicity, w/o ghost cells	*	*	*	*	*	*	*
	60	No-slip/centerplane	*	*	*	*	*	*	*
	61	x-axis symmetry	0	$\partial V/\partial \xi_i = 0$	$\partial W/\partial \xi_i = 0$	$\partial P/\partial \xi_i = 0$	$\partial k/\partial \xi_i = 0$	$\partial \omega/\partial \xi_i = 0$	$\partial \nu_t/\partial \xi_i = 0$
	62	y-axis symmetry	$\partial U/\partial \xi_i = 0$	0	$\partial W/\partial \xi_i = 0$	$\partial P/\partial \xi_i = 0$	$\partial k/\partial \xi_i = 0$	$\partial \omega/\partial \xi_i = 0$	$\partial \nu_t/\partial \xi_i = 0$
	63	z-axis symmetry	$\partial U/\partial \xi_i = 0$	$\partial V/\partial \xi_i = 0$	0	$\partial P/\partial \xi_i = 0$	$\partial k/\partial \xi_i = 0$	$\partial \omega/\partial \xi_i = 0$	$\partial \nu_t/\partial \xi_i = 0$
	91	Multi-block w/ ghost cells	*	*	*	*	*	*	*
	92	Multi-block w/o ghost cells	*	*	*	*	*	*	*
	99	Blanked out points	0	0	0	0	0	0	0

* See text for detailed description

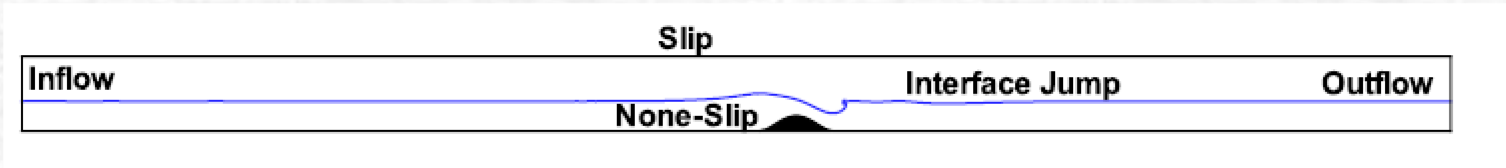
BCs in Ansys Fluent

1 Flow Inlet and Exit Boundary Conditions
2 Using Flow Boundary Conditions
3 Pressure Inlet Boundary Conditions
4 Velocity Inlet Boundary Conditions
5 Mass Flow Inlet Boundary Conditions
6 Inlet Vent Boundary Conditions
7 Intake Fan Boundary Conditions
8 Pressure Outlet Boundary Conditions
9 Pressure Far-Field Boundary Conditions
10 Inputs at Pressure Far-Field Boundaries
11 Outflow Boundary Conditions
12 Outlet Vent Boundary Conditions
13 Exhaust Fan Boundary Conditions
14 Wall Boundary Conditions
15 Symmetry Boundary Conditions
16 Periodic Boundary Conditions
17 Axis Boundary Conditions
18 Fan Boundary Conditions
19 Radiator Boundary Conditions
20 Porous Jump Boundary Conditions

https://www.afs.enea.it/project/neptunius/docs/fluent/html/ug/node236.htm

Simulation Examples using CFDShip-Iowa

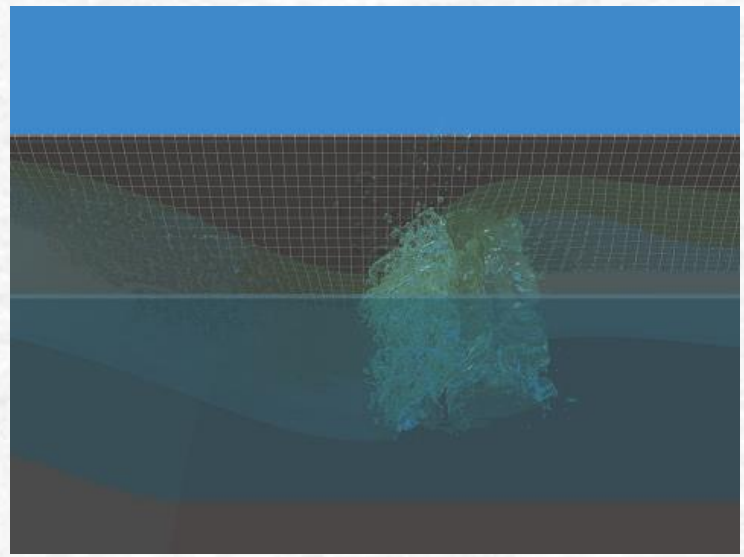
Plunging wave breaking:



- *Inlet:*
  $u = constant, v = w = 0$
- *Exit:*

  $$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = \frac{\partial w}{\partial n} = 0$$

- For pressure, $\frac{\partial p}{\partial n} = 0$ for all the boundaries.
- Mass balance needed at the outlet.



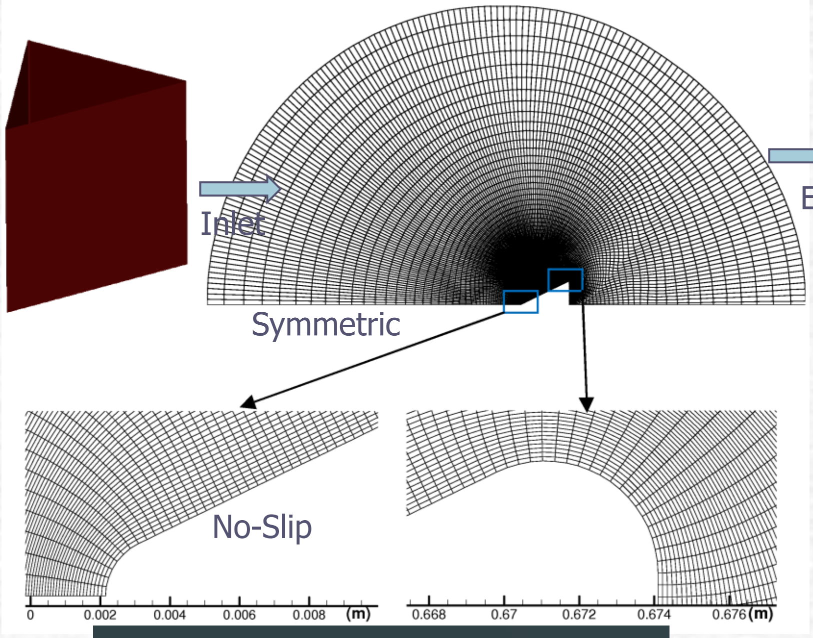
Wave breaking in bump flow simulation: 2.2 billion grid points

Movie

Simulation Examples using CFDShip-Iowa

Wedge flow:

- *The j_{max} boundary is split into two parts: inlet and exit.*
- *Inlet:*
  $u = constant, v = w = 0$
- *Exit:*
  $$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = \frac{\partial w}{\partial n} = 0$$
- Slip BCs at both top and bottom.
- For pressure, $\frac{\partial p}{\partial n} = 0$ for all the boundaries.
- Mass balance needed at the outlet.



Wedge flow simulation, 1 billion grid points

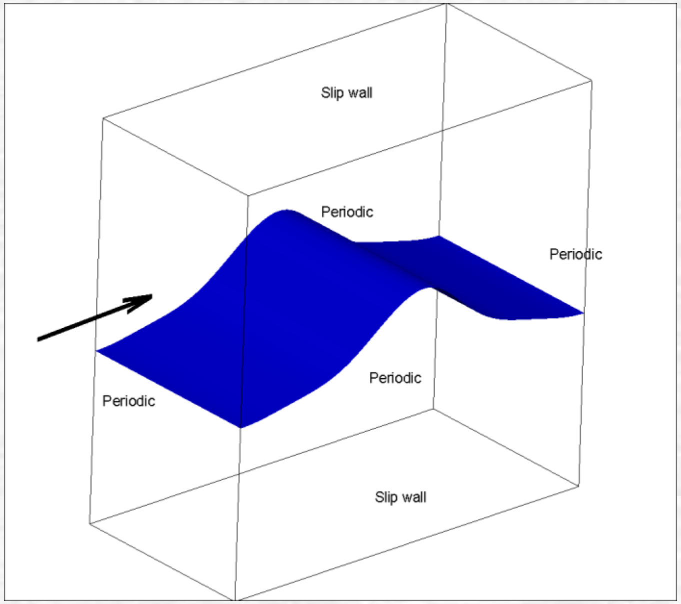
Movie

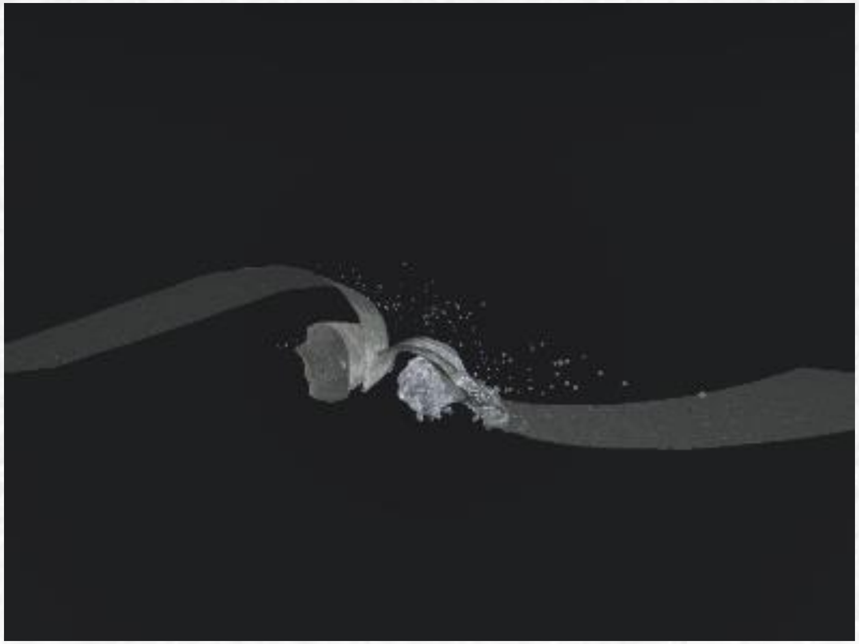
Simulation Examples using CFDShip-Iowa

Stokes wave breaking

Slip wall BCs at top and bottom

Periodic BCs at inlet, exit, and two sides.





Stokes wave breaking: 3.2-12 Billion Grid Points

Movie

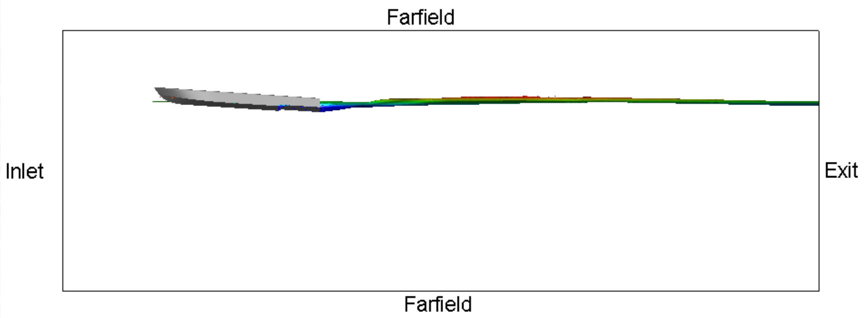
Simulation Examples using CFDShip-Iowa

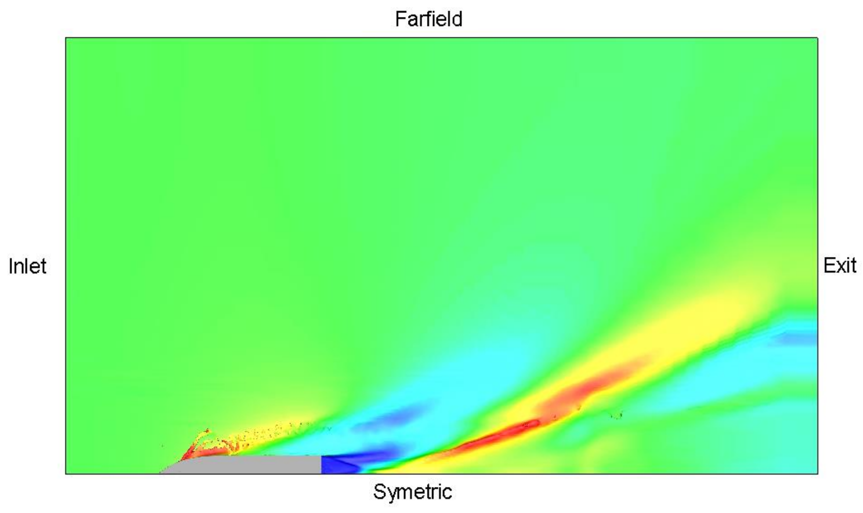
NSWC15E Planing Hull

- Water is moving, ship-fixed system
- *Inlet (10):*

$$u = u_{inflow}, v = w = 0$$

- *Exit(11):* $\frac{\partial^2 U}{\partial n^2} = 0, \frac{\partial p}{\partial n} = 0$





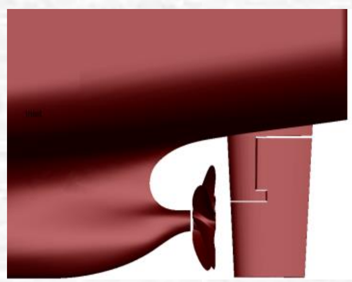
Movies: bottom view   side view

Simulation Examples using CFDShip-Iowa

KCS free running

- Symmetric BC can not be used, use full ship
- Inlet (10): $u = v = w = 0$,

  since the ship is moving, earth fixed system (inertial)
- Exit (11)



Movies: free running