ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2024 – HW11 Solution

P7.6 For the laminar parabolic boundary-layer profile of Eq. (7.6), compute the shape factor "H" and compare with the exact Blasius-theory result, Eq. (7.31).

Solution: Given the profile approximation $u/U \approx 2\eta - \eta^2$, where $\eta = y/\delta$, compute

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy = \delta\int_0^1 (2\eta-\eta^2)(1-2\eta+\eta^2)\, d\eta = \frac{2}{15}\delta$$

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right) dy = \delta\int_0^1 (1-2\eta+\eta^2)\, d\eta = \frac{1}{3}\delta$$

Hence $H = \delta^*/\theta = (\delta/3)/(2\delta/15) \approx$ **2.50** (compared to *2.59* for Blasius solution)

P7.9 Repeat the flat-plate momentum analysis of Sec. 7.2 by replacing Eq. (7.6) with the simple but unrealistic linear velocity profile suggested by Schlichting [1]:

$$\frac{u}{U} \approx \frac{y}{\delta} \qquad \text{for} \quad 0 \le y \le \delta$$

Compute momentum-integral estimates of c_f, θ/x, δ^*/x, and H.

Solution: Carry out the same integrations as Section 7.2. Results are less accurate:

$$\theta = \int_0^\delta \frac{u}{U}(1-\frac{u}{U})\, dy = \int_0^\delta \frac{y}{\delta}(1-\frac{y}{\delta})\, dy = \frac{\delta}{6} \;;\quad \delta^* = \int_0^\delta (1-\frac{u}{U})\, dy = \frac{\delta}{2} \;;\; H = \frac{\delta/2}{\delta/6} = 3.0$$

$$\tau_w = \mu\frac{U}{\delta} = \rho U^2\frac{d\theta}{dx} = \rho U^2\frac{d(\delta/6)}{dx} \;;\; \textit{Integrate}: \frac{\delta}{x} \approx \frac{\sqrt{12}}{\sqrt{\mathrm{Re}_x}} \approx \frac{3.64}{\sqrt{\mathrm{Re}_x}}$$

Substitute these results back for the following inaccurate estimates:

$$c_f = \frac{\theta}{x} = \frac{0.577}{\sqrt{\mathrm{Re}_x}} \;;\qquad \frac{\delta^*}{x} = \frac{1.732}{\sqrt{\mathrm{Re}_x}} \;;\quad H = 3.0 \qquad \textit{Ans.}(a,b,c,d)$$

P7.23 Suppose you buy a 4 × 8-ft sheet of plywood and put it on your roof rack, as in the figure. You drive home at 35 mi/h.
(a) If the board is perfectly aligned with the airflow, how thick is the boundary layer at the end? (b) Estimate the drag if the flow remains laminar. (c) Estimate the drag for (smooth) turbulent flow.

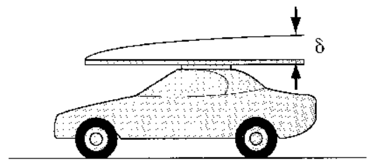

Fig. P7.23

Solution: For air take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E–}5$ kg/m·s. Convert L = 8 ft = 2.44 m and U = 35 mi/h = 15.6 m/s. Evaluate the Reynolds number, is it laminar or turbulent?

$$\mathrm{Re}_L = \frac{\rho UL}{\mu} = \frac{1.2(15.6)(2.44)}{1.8E\text{–}5} = 2.55E6 \quad \textit{probably laminar + turbulent}$$

(a) Evaluate the range of boundary-layer thickness between laminar and turbulent:

$$\textit{Laminar}: \quad \frac{\delta}{L} = \frac{\delta}{2.44\ m} \approx \frac{5.0}{\sqrt{2.55E6}} = 0.00313, \quad or: \quad \delta \approx 0.00765\ m = \mathbf{0.30\ in}$$

$$\textit{Turbulent}: \quad \frac{\delta}{2.44} \approx \frac{0.16}{(2.55E6)^{1/7}} = 0.0195, \quad or: \quad \delta \approx 0.047\ m = \mathbf{1.9\ in} \quad \textit{Ans.}\ \text{(a)}$$

(b, c) Evaluate the range of boundary-layer drag for both laminar and turbulent flow. Note that, for flow over both sides, the appropriate area A = 2bL:

$$F_{lam} = C_D \frac{\rho}{2} U^2 A \approx \left(\frac{1.328}{\sqrt{2.55E6}}\right)\frac{1.2}{2}(15.6)^2(2.44 \times 1.22 \times 2\ sides) = \mathbf{0.73\ N} \quad \textit{Ans.}\ \text{(b)}$$

$$F_{turb} \approx \left(\frac{0.031}{(2.55E6)^{1/7}}\right)\frac{1.2}{2}(15.6)^2(2.44 \times 1.22 \times 2\ sides) = \mathbf{3.3\ N} \quad \textit{Ans.}\ \text{(c)}$$

We see that the turbulent drag is about 4 times larger than laminar drag.

P7.27 Air at 20°C and 1 atm flows at 3 m/s past a sharp flat plate 2 m wide and 1 m long. (*a*) What is the wall shear stress at the end of the plate? (*b*) What is the air velocity at a point 4.5 mm normal to the end of the plate? (*c*) What is the total friction drag on the plate?

Solution: For at 20°C and 1 atm, take ρ = 1.2 kg/m3 and μ = 1.8E-5 kg/m-s. Check the Reynolds number to see if the flow is laminar or turbulent:

$$\mathrm{Re}_L = \frac{\rho UL}{\mu} = \frac{(1.2)(3.0)(1.0)}{1.8\mathrm{E}-5} = 200,000 \qquad \textbf{Laminar}$$

We can proceed with our laminar-flow formulas:

$$c_{f,x=L} = \frac{0.664}{\sqrt{\mathrm{Re}_L}} = \frac{0.664}{\sqrt{200000}} = 0.00148;\ \ \tau_w = c_f\frac{\rho}{2}U^2 = (0.00148)(\frac{1.2}{2})(3)^2 = \mathbf{0.0080}\,Pa\ \ Ans.(a)$$

$$\text{At } y = 4.5\,mm,\ \text{the Blasius}\ \ \eta = y\sqrt{\frac{U}{\nu x}} = (0.0045m)\sqrt{\frac{3.0}{(1.5E-5)(1.0)}} = 2.01$$

$$\text{Table 7.1: at } \eta = 2.0,\ \text{read } \frac{u}{U} \approx 0.63,\quad \text{hence}\ \ u = (0.63)(3.0) \approx \mathbf{1.89}\frac{m}{s}\ \ Ans.(b)$$

Finally, compute the drag for both sides of the plate, $A = 2bL$:

$$C_D = \frac{1.328}{\sqrt{200,000}} = 0.00297,$$

$$\text{or}: \quad F = C_D\frac{\rho}{2}U^2(2bL) = (0.00297)(\frac{1.2}{2})(3.0)^2[2(2.0)(1.0)] = \mathbf{0.064}\ N \quad Ans.(c)$$

NOTE: For part (*b*), we never had to compute the boundary layer thickness, $\delta \approx$ 11.2 mm.

P7.34 Consider turbulent flow past a flat plate of width b and length L. What percentage of the friction drag on the plate is carried by the rear half of the plate?

Solution: The formula for turbulent boundary drag on a plate is Eq. (7.45):

$$C_D = \frac{2D(x)}{\rho U^2 bx} \approx \frac{0.031}{\mathrm{Re}_x^{1/7}} = \frac{0.031\mu^{1/7}}{(\rho U x)^{1/7}}, \quad \text{or:} \quad D(x) = (const)\, x^{6/7}$$

At $x = L$, we obtain a force equal to (const) $L^{6/7}$. At $x = L/2$, we obtain a force equal to (const) $L^{6/7}/2^{6/7} =$ (const)(0.552) $L^{6/7}$, which is 55.2% of the total force. Thus the force on the trailing half of the plate is only (100 – 55.2) = **44.8%** of the total force on the plate. Unlike laminar flow (29.3%), this is nearly half of the total, since turbulent shear drops off much slower with x.