$u = f(\mu, \tau_w, \rho, y)$

P6.35 In the overlap layer of Fig. 6.9*a*, turbulent shear is large. If we neglect viscosity, we can replace Eq. (6.24) by the approximate velocity-gradient function

$$\frac{du}{dy} = fcn(\, y, \tau_w, \rho)$$

Show that, by dimensional analysis, this leads to the logarithmic overlap relation (6.28).

$u/u^* = \frac{1}{\kappa} \ln \frac{y u^*}{\nu} + B$

BL approximation

$$\rho \frac{D\bar{u}}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y}$$

$$\tau = \mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'}$$

$$= \tau_{lam} + \tau_{turb}$$

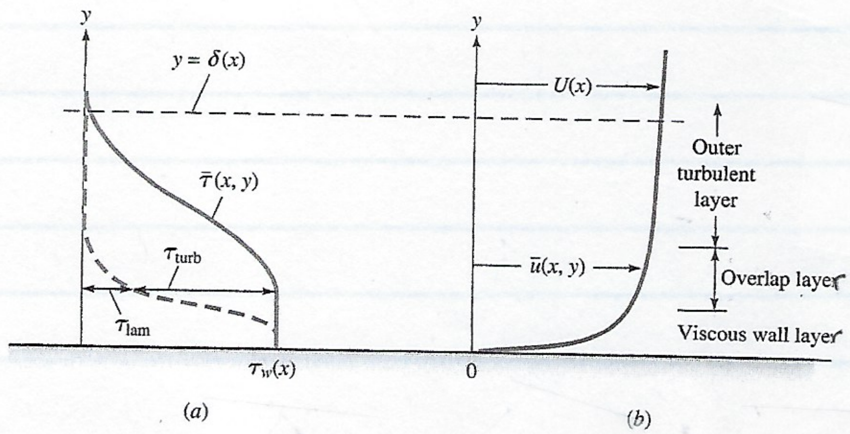

Fig. 6.9 Typical velocity and shear distributions in turbulent flow near a wall: (*a*) shear; (*b*) velocity.

$$\Pi_1 = y^a \tau_w^b \rho^c \frac{du}{dy}$$

$n = 4$
$m = 3$ $\Rightarrow r = 1$

$$= \frac{du}{dy} \left(\frac{y \sqrt{\rho}}{\sqrt{\tau_w}} \right) = \text{constant} = C_1$$

$$\frac{du}{dy} = C_1 \sqrt{\frac{\tau_w}{\rho}} \frac{1}{y} = \frac{C_1 u^*}{y}$$

integrate $\quad u = C_1 u^* \ln y + C_2$

rearrange constants $\quad u/u^* = \frac{1}{\kappa} \ln \frac{u^* y}{\nu} + B$