Name: ------------------ **Exam 1 – Ch3 (Part2)** **Take home assignment**

ME:6260 **Spring 2024**

This exam is open book test only. Class notes or on-line material cannot be used

Following Taylor and Green (1937), consider the two-dimensional vortex flow field with constant density ρ:

$$\mathbf{u} = (u, v) = (Asin(kx)\cos(ky), Bcos(kx)\sin(ky))$$

a) If the flow is steady and inviscid, and A and B are constants, explicitly determine the pressure, $p(x, y)$, in terms of x, y, A, ρ and k from below equations in two dimensions.
b) If the flow field is unsteady and viscous (with viscosity μ), A and B are functions of time t, and $A = A_0$ at $t = 0$, determine $A(t)$, $B(t)$ and $p(x, y, t)$ so that the given $\mathbf{u}$ is an exact solution of below equations in two dimensions.
c) How long does it take for $A(t)$ to fall to $A_0/2$?

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Hint: (a) Use the continuity equation first, and find the relation between A and B.
Use this result along with given velocity profiles in the momentum equations and integrate to find a pressure field. Equations will have f(y) and g(x) that can be eliminated by comparison.

(b) Use the continuity equation first, and find the relation between A and B.
Assume the pressure field from part (a) is still valid but with A(t), i.e., $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$ and $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$; thus, $\frac{\partial u}{\partial t} = +\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ and $\frac{\partial v}{\partial t} = \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$; and use this result to solve for A(t)

(c) set t=1/2 in result for (b)

Extra credit: prove that the 2nd part hint (b) is correct, i.e., solve for the unsteady pressure field.