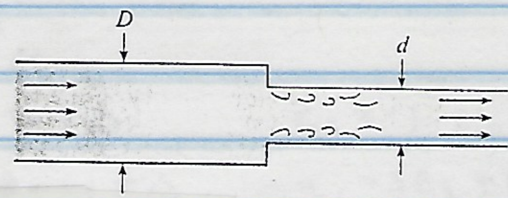
8.19 By dimensional analysis, determine the dimensionless relationship for the change in pressure that occurs when water or oil flows through a horizontal pipe with an abrupt contraction as shown.



$F = ma$

$= MLT^{-2}$

$\Delta p = ML^{-1}T^{-2}$

$\mu = \dfrac{\tau}{du/dy} =$

$\dfrac{ML^{-1}T^{-2}}{T^{-1}}$

$= ML^{-1}T^{-1}$

$F(\Delta p, \underbrace{Q, D}_{\text{repeating variables}}, d, \rho, \mu) = 0$ $\quad n = 6$

$ML^{-1}T^{-2} \quad L^3T^{-1} \quad L \quad L \quad ML^{-3} \quad ML^{-1}T^{-1}$ $\quad m = 3$

$r = n - m = 3$

$Q^{x_1} D^{y_1} \rho^{z_1} \Delta p$

M $\quad z_1 + 1 = 0 \qquad z_1 = -1$

L $\quad 3x_1 + y_1 - 3z_1 - 1 = 0 \qquad y_1 = 3z_1 + 1 - 3x_1 = -3 + 1 + 6 = 4$

T $\quad -x_1 \quad -2 = 0 \qquad x_1 = -2$

$\Pi_1 = Q^{-2} D^4 \rho^{-1} \Delta p = \Delta p D^4 / Q^2 \rho$

discharge pressure coefficient

usual $C_p = \dfrac{\Delta p}{\frac{1}{2}\rho U^2}$

$Q^{x_2} D^{y_2} \rho^{z_2} d$

M $\quad z_2 = 0$

L $\quad 3x_2 + y_2 - 3z_2 + 1 = 0 \Rightarrow y_2 = -1$

T $\quad -x_2 = 0$

$\Pi_2 = D^{-1} d = d/D$ $\qquad$ contraction ratio

$$Q^{x_3} D^{y_3} \rho^{z_3} \mu$$

M	$z_3 + 1 = 0$	$z_3 = -1$
L	$3x_3 + y_3 - 3z_3 - 1 = 0$	$y_3 = 3z_3 + 1 - 3x_3 = -3 + 1 + 3 = 1$
T	$-x_3 - 1 = 0$	$x_3 = -1$

$$\Pi_3 = Q^{-1} D^{1} \rho^{-1} \mu = D\mu / Q\rho$$

$$\Pi_3^{-1} = \frac{Q\rho}{D\mu}$$

discharge Re

usual $Re = \frac{VD\rho}{\mu}$

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

7.19

7.19 One type of viscometer consists of an open reservoir with a small diameter tube at the bottom as illustrated in Fig. P7.19. To measure viscosity the system is filled with the liquid of interest and the time required for the liquid level to fall from level H_i to H_f is determined. Use dimensional analysis to obtain a relationship between the viscosity, μ, and the draining time, τ. Assume that the other variables involved are the initial head, H_i, the final head, H_f, the tube diameter, D, and the specific weight of the liquid, γ.

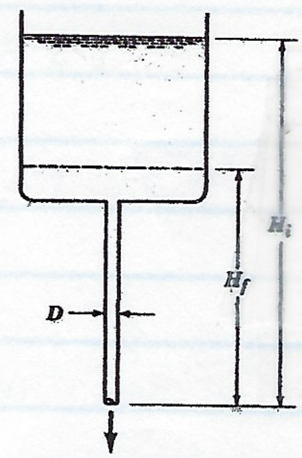


FIGURE P7.19

$$\tau = f(D, H_i, H_f, \mu, \gamma) = \text{draining time}$$

$$F(\tau, D, H_i, H_f, \mu, \gamma) = 0$$

$$T \quad L \quad L \quad L \quad FL^{-2}T \quad FL^{-3}$$

$n = 6$

$m = 3$

$r = n - m = 3$

by inspection $\Pi_1 = \tau\gamma D/\mu = T(FL^{-3})L/FL^{-2}T = F^0L^0T^0$

$$\Pi_2 = H_i/D$$

$$\Pi_3 = H_f/D$$

$$\tau\gamma D/\mu = f(H_i/D, H_f/D)$$

For fixed geometry $\frac{\tau\gamma D}{\mu}$ = constant = K

$$\mu = \tau\gamma D/K$$

$$= K'\gamma\tau \qquad K' = D/K$$

= constant for fixed geometry