C5.5 Does an automobile radio antenna vibrate in resonance due to vortex shedding? Consider an antenna of length L and diameter D. According to beam-vibration theory [e.g. Kelly [34], p. 401], the first mode natural frequency of a solid circular cantilever beam is $\omega_n = 3.516[EI/(\rho AL^4)]^{1/2}$, where E is the modulus of elasticity, I is the area moment of inertia, ρ is the beam material density, and A is the beam cross-section area. (a) Show that ω_n is proportional to the antenna radius R. (b) If the antenna is steel, with $L = 60$ cm and $D = 4$ mm, estimate the natural vibration frequency, in Hz. (c) Compare with the shedding frequency if the car moves at 65 mi/h.

Fig 2.13 : $A = \pi R^2$ $I = \pi R^4/4$

(a) $$\omega_n = 3.516\left[\frac{E\pi R^4/4}{\rho\pi R^2 L^4}\right]^{1/2} = 1.758\,[E/\rho]^{1/2}\frac{R}{L^2}$$

(b) $L = 60$ cm $D = 4$ mm $E = 2.1 \times 10^{11}$ Pa $\rho = 7840$ kg/m^3

$$\omega_n = 1.758\left[2.1\times10^{11}/7840\right]^{1/2}\frac{.002}{.6^2}$$

$$= 51 \text{ rad/s} = 8 \text{ Hz}$$

(c) $U = 65$ mph $= 29.1$ m/s $\quad Re_D = \frac{\rho U D}{\mu}$

Fig 5.25 $\quad St = \frac{\omega D}{2\pi U} = .21 \quad = \frac{1.2\,(29.1)\,.004}{.000018}$

$$= \frac{\omega(.004)}{2\pi(29.1)} = .21 \quad = 7800$$

$\omega = 9600$ rad/s

$= 1500$ Hz $>> \omega_n$