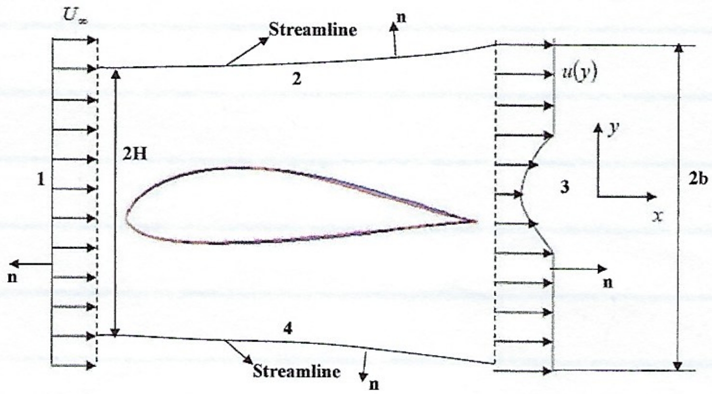
Calculation of drag force on the airfoil using integral analysis

Consider an experiment in which the drag on an airfoil immersed in a steady incompressible flow can be determined from measurement of the velocity distributions far upstream and downstream of the body (figure below).

1. Velocity far upstream is the uniform flow U_∞,
2. Velocity in the wake of the body is measured by Hotwire/Pitot probe to be $u(y)$, which is less than U_∞ due to the drag of the airfoil.
3. Objective: Find the drag force D per unit length of the airfoil.

Method 1: Choose control volume that follows streamline



Solution:

Find relation between H and b using Mass conservation

Since we choose the streamline as the control volume, there is no mass flow across it. n is the unit normal vector.

$$\rho \int_{CS} (V.n)dA = 0 = -2HU_\infty \rho + 2\rho \int_0^b u(y)dy;$$

$$H = \frac{1}{U_\infty}\int_0^b u(y)dy;$$

Momentum balance

The pressure is uniform and so there is no net pressure force. The flow is assumed to be incompressible and steady, so the momentum conservation equation without any unsteady terms applies only across section 1 and 3.

$$\sum F_x = -D = \rho \int_1 u(V.n)dA + \rho \int_3 u(V.n)dA;$$

$$-F_x = 2\rho \int_0^b u^2(y)dy - 2\rho H U_\infty^2; \text{ Substitute H from mass balance equation}$$

$$-F_x = 2\rho \int_0^b u^2(y)dy - 2\rho U_\infty \int_0^b u(y)dy;$$

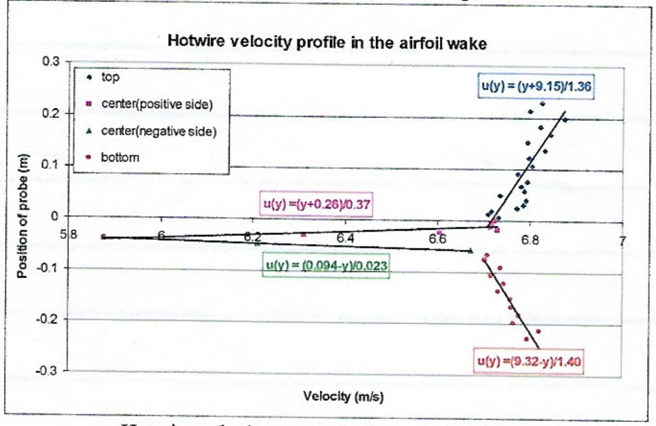
$$F_x = 2\rho \int_0^b u(U_\infty - u)dy$$

$$C_D = \frac{2D}{\rho U_\infty^2 bc} = \frac{4\int_0^b u(U_\infty - u)dy}{U_\infty^2 bc}$$

Where, b is the width of the airfoil span and c is the chord length.

Example

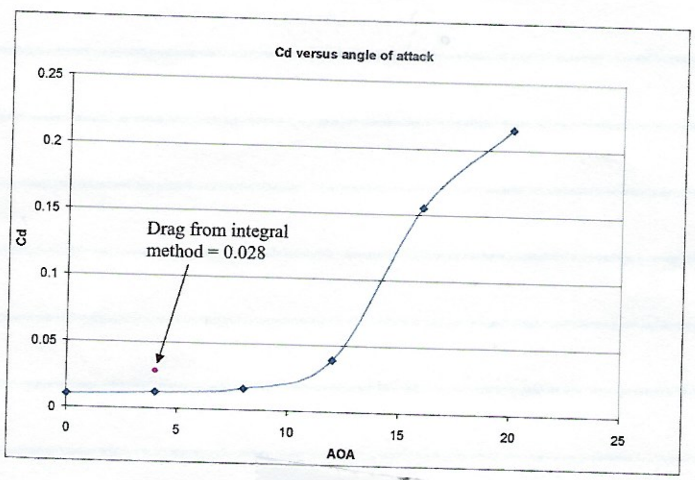
$U_\infty = 7.04\text{m/s}$, $b = 0.762\text{m}$, $c = 0.3048\text{m}$, $\rho = 1.21\ \text{kg/m}^3$



Hotwire velocity profile in the wake for AOA = 4

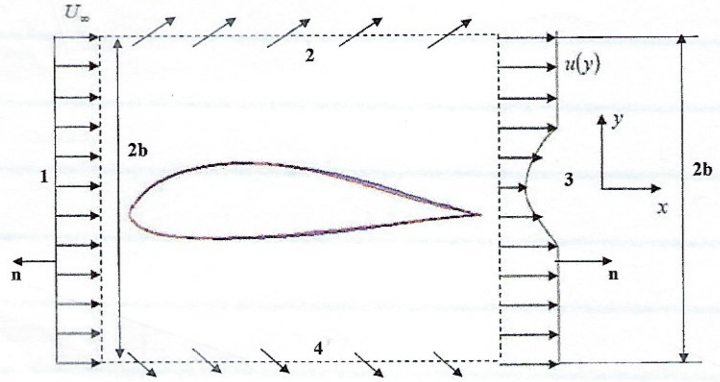
$$C_D = \frac{4\int_0^b u(U_\infty - u)dy}{U_\infty^2 bc} = 2\times 0.174\left[\begin{array}{l}\left(\int_{0.006}^{0.229}(0.735y+6.73)(0.31-0.735y)dy\right)+\\ \left(\int_{-0.039}^{0.0}(2.7y+0.702)(6.34-02.7y)dy\right)+\\ \left(\int_{-0.058}^{-0.039}(4.09y-4.48)(5.52-4.09y)dy\right)+\\ \left(\int_{-0.229}^{-0.067}(6.66y-0.714)(7.75-6.66y)dy\right)\end{array}\right]$$

Note: The velocity profile in the wake is ***not symmetrical*** *due to airfoil shape and angle of attack. Each of the four equations has different* **y limits**.



Comparison of drag data with benchmark

Method 2: Rectangular control volume



Solution:
Use Mass conservation
There is outflow of mass and x-momentum through sections 2 and 4 as well.

$$\rho \int_{CS} (V.n)dA = 0 = -2bU_\infty \rho + 2\rho \int_0^b u(y)dy + \dot{m}_2 + \dot{m}_4;$$

where, $\dot{m}_2$ and $\dot{m}_4$ are the mass fluxes through sections 2 and 4 respectively and $\dot{m}_2 \neq \dot{m}_4$.

Momentum balance
Note: It is assumed that the x-directional velocity at surfaces 2 and 4 are nearly U_∞. This means that the momentum fluxes through sections 2 and 4 in the x-direction are equal to $U_\infty \dot{m}_2$ and $U_\infty \dot{m}_4$ respectively. Multiplying both sides of the mass conservation equation by U_∞ we get; (Mf is the momentum flux)

$$Mf_{x2} + Mf_{x4} = U_\infty(\dot{m}_2 + \dot{m}_4) = 2\rho b U_\infty^2 - 2\rho U_\infty \int_0^b u dy;$$

We already know that;

$$Mf_{x1} = 2\rho b U_\infty^2;$$

$$Mf_{x3} = 2\rho \int_0^b u^2(y)dy;$$

Note: Even if the mass fluxes through sections 2 and 4 are not symmetrical this method is still applicable and gives the same result as the streamline control volume approach.

The momentum equation can be expressed as

$$\sum F_x = -D = -Mf_{x1} + Mf_{x2} + Mf_{x3} + Mf_{x4} = -2\rho \int_0^b u(U_\infty - u)dy;$$

$$D = 2\rho \int_0^b u(U_\infty - u)dy;$$

$$C_D = \frac{2D}{\rho U_\infty^2 bc} = \frac{4\int_0^b u(U_\infty - u)dy}{U_\infty^2 bc}$$