Transition, Pressure Gradients, and BL Separation

Laminar BL theory altered when transition occurs to turbulent flow or BL separation both of which are affected by p_x, especially BL separation. However, turbulent BL more resistant separation due adverse p_x.

Transition $f(Re)$ complicated process not fully understood or predictable based current understanding physics or models. For $Re > Re_{crit}$ transition occurs depending geometry and many factors: free stream turbulence, roughness, p_x, vibration, etc.

Re_{crit} ↓ when $\langle u^2 \rangle$ free stream or roughness ↑

τ_w turb > τ_w laminar

$\langle u v \rangle_y$ increased diffusion νu_{yy} alone

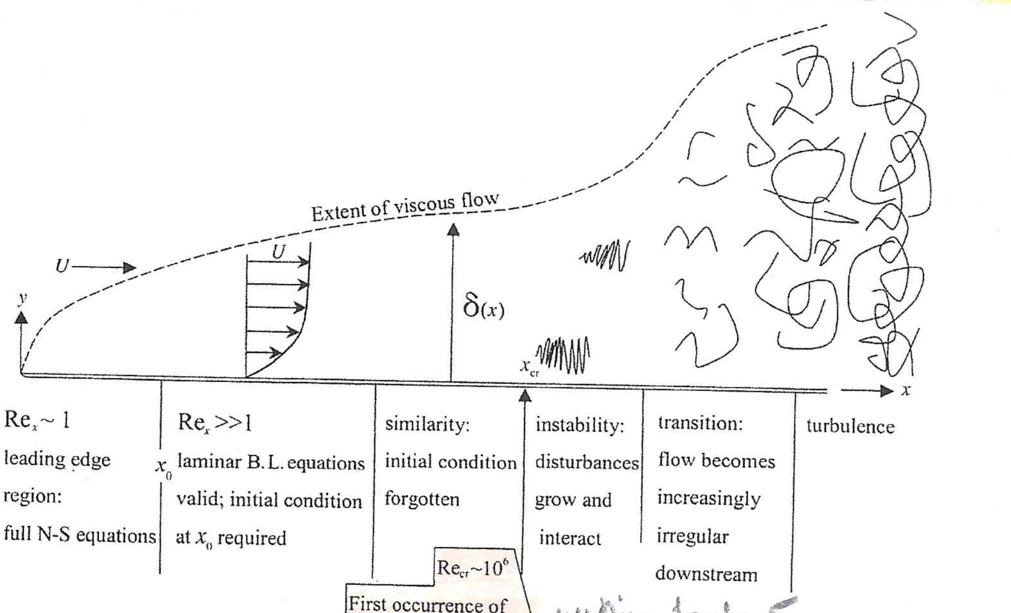


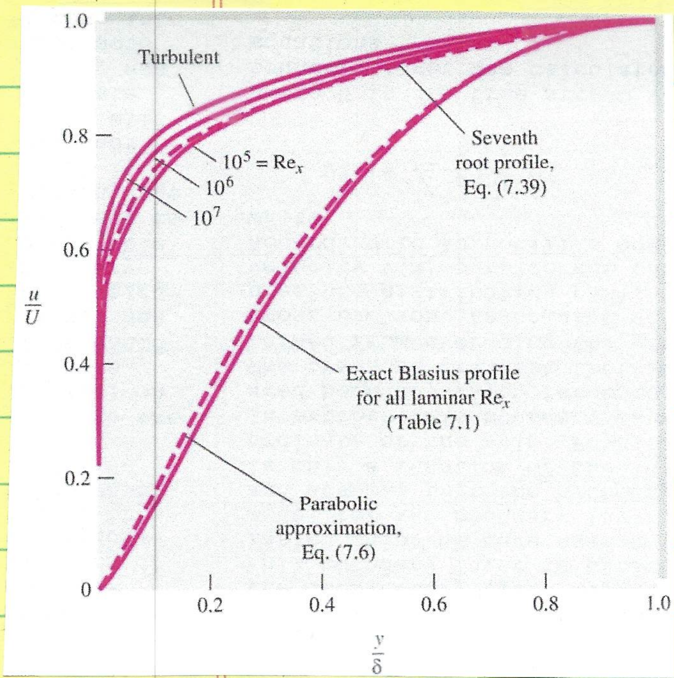
FIGURE 10.11 Schematic depiction of flow over a semi-infinite flat plate. Here, increasing x is synonymous with increasing Reynolds number.

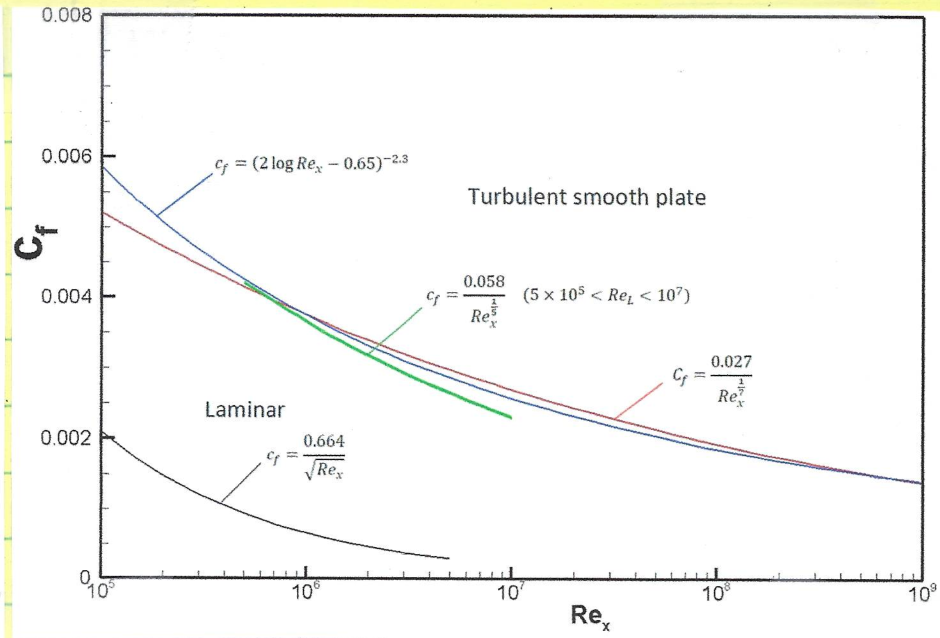
Laminar BL: $\delta \propto x^{1/2}$ $\tau_w \propto x^{-1/2}$ $(U^{3/2})$

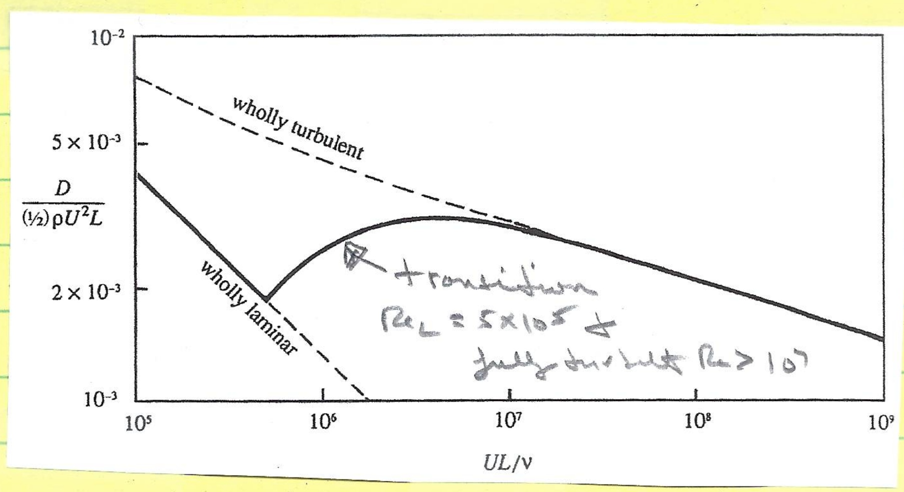
$p_x = 0$

Turbulent BL: $\delta \propto x^{6/7}$ $\tau_w \propto x^{-1/7}$ $(U^{13/7})$

nearly linear — decreases more slowly — nearly quadratic







The role of p_x in inducing separation is revealed via the BL equation at $y=0$

$$\mu u_{yy} = p_x$$

$\boxed{I.}$ $p_x < 0 \Rightarrow u_{yy}\big|_{wall} < 0$ since u_y max at wall and decreases to 0 at δ to merge to

accelerating BL

$\therefore$ $u_{yy} < 0$ across whole δ $\therefore$ no PI

12] $p_x > 0 \Rightarrow u_{yy}\big|_{wall} > 0$ since $u_{yy} < 0$ at δ

decelerating BL

$u(y)$ must have PI within δ, which has important implications stability and transition

note: Blasius $u_{yy} = 0$ at wall.

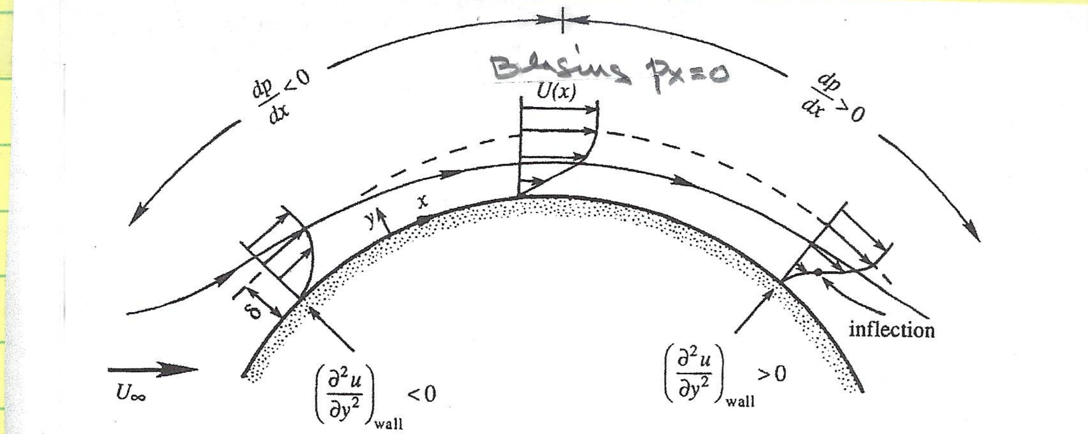


FIGURE 10.13 Velocity profiles across boundary layers with favorable ($dp/dx < 0$) and adverse ($dp/dx > 0$) pressure gradients, as indicated above the flow. The surface shear stress and stream-wise fluid velocity near the surface are highest and lowest in the favorable and adverse pressure gradients, respectively, with the $dp/dx = 0$ case falling between these limits.

Decelerating outer flow $U_x < 0$ and $p_x > 0$ tends to increase δ, as per above and FS soln:

$$\psi(x,y) = [\nu x U_e(x)]^{1/2} f(\eta) \qquad \eta = y/\delta(x) = \frac{y}{x}\sqrt{Re_x} = \sqrt{\frac{a}{\nu}}\, x^{\frac{n-1}{2}}$$

$$f''' + \frac{n+1}{2} f f'' - n f'^2 + n = 0 \quad \text{(as per alternate derivation \#2)}$$

$$U_e(x) = a x^n \qquad -p_x = U_e U_{ex} = n a^2 x^{2n-1}$$

$$\delta(x) = [\nu x / U_e]^{1/2} = [\nu x^{1-n}/a]^{1/2} \qquad \delta \uparrow n<1 \quad \downarrow n>1$$

$n = 1$ n = constant

$$u = U_e f' \qquad v = -\psi_x = -\left[\frac{n+1}{2}(\nu a x^{n-1})^{1/2} f + (\nu a x^{n-1})^{1/2}\frac{n-1}{2}\eta f'\right]$$

$$\frac{v}{a x^n} = \frac{v}{U_e(x)} = \frac{-1}{Re^{1/2}}\left[\frac{n+1}{2} f + \frac{n-1}{2}\eta f'\right]$$

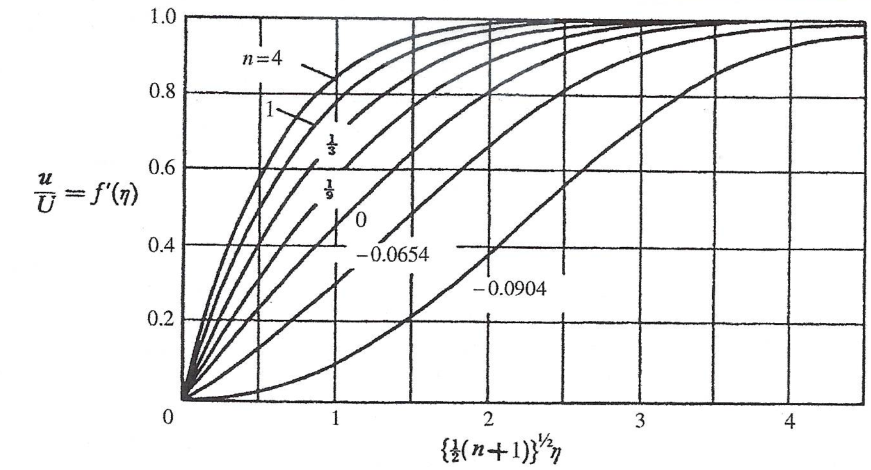


FIGURE 10.8 Falkner-Skan profiles of stream-wise velocity in a laminar boundary layer when the external stream is $U_e = ax^n$. The horizontal axis is the scaled surface-normal coordinate. The various curves are labeled by their associated value of n. When $n > 0$, the free-stream speed increases with increasing x, and $\partial^2 u/\partial y^2$ is negative throughout the boundary layer. When $n = 0$ (the Blasius boundary layer), the free-stream speed is constant, and $\partial^2 u/\partial y^2 = 0$ at the wall and is negative throughout the boundary layer. When $n < 0$, the free-stream speed decreases with increasing x, and $\partial^2 u/\partial y^2$ is positive near the wall but negative higher up in the boundary layer so there is an inflection point in the stream-wise velocity profile at a finite distance from the surface. *Reprinted with the permission of Cambridge University Press, from: G. K. Batchelor,* An Introduction to Fluid Dynamics, *1st ed. (1967).*

$n > 0$ accelerating U_e: $\eta, f, f' > 0$

$\therefore$ $v < 0$ and outer flow drawn towards wall thinning BL and preventing separation

$n > 0$ decelerating U_e: $v > 0$ eg $f < \eta f'$ for $n = 0$ Blasius

thus outer flow displaced from wall, which may lead to separation.

Also: $v(y) = -\int_0^y u_x \, dy$

$p_x = -\rho U U_x$

$U_x < 0 \quad p_x > 0$

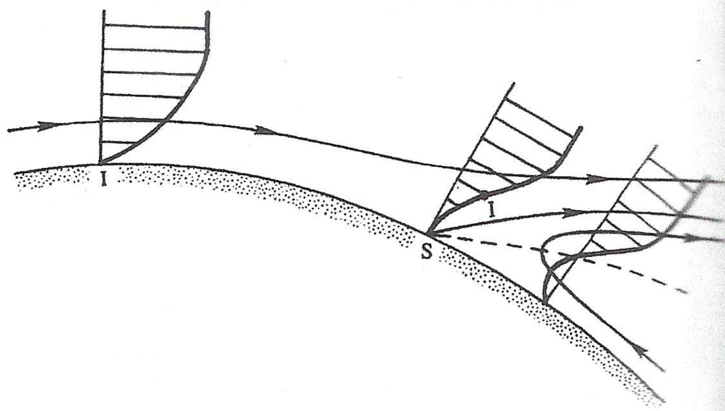
$\Rightarrow -u_x$ in BL

also larger $p_x < 0$

$\therefore$ also larger deceleration $u(y)$ for $y < \delta \Rightarrow v > 0$ larger

$\delta \uparrow$ not only νu_{yy} but also advection away from surface.

FIGURE 10.14 Streamlines and velocity profiles near a separation point S where a streamline emerges from the surface. The usual boundary-layer equations are not valid downstream of S. The inflection point in the stream-wise velocity profile is indicated by I. The dashed line is the locus of $u = 0$.



For $p_x > 0$ BL flow decelerates, $\delta \uparrow$, & PI within BL. When p_x strong enough & acts over sufficient distance, the flow separates & a region of reverse flow develops near the wall. The point S at which forward flow meets reverse flow is a local stagnation point = the separation point. Fluid elements approach S from either side; thus, a separation ψ emerges from the surface at S. Furthermore, τ_w changes sign at S since surface flow changes direction

$\therefore \quad u_y\big|_{wall} = 0 \quad$ at S

Note BL assumptions & equations no longer valid $\therefore$ viscous/inviscid interaction & streamwise diffusion important

EXAMPLE 10.8

Using a third-order two-dimensional power-series expansion near a flat-plate boundary layer's separation point, $x = x_s$ and $y = 0$, determine how the stream function $\psi(x,y)$ depends on $\partial p/\partial x$ and β_s, the angle the separating streamline makes with the horizontal surface as shown in Figure 10.17.

Solution

A third-order power series expansion for $\psi(x,y)$ is:

$$\psi(x,y) = a_0 + a_1x' + a_2y + a_3x'^2 + a_4x'y + Ay^2 + a_5x'^3 + a_6x'^2y + Bx'y^2 + Cy^3.$$

where $x' = x - x_s$, and a_0 through a_6, A, B, and C are undetermined constants. This stream function must satisfy the no-slip boundary condition, $u = v = 0$ on $y = 0$, so $\partial\psi/\partial y = -\partial\psi/\partial x = 0$ on $y = 0$. These two conditions cause a_1 through a_6 to be zero, and if $\psi = 0$ defines the plate surface, then the stream function reduces to $\psi(x,y) = Ay^2 + B(x - x_s)y^2 + Cy^3$. In addition, the surface shear stress, τ_w, is zero at the separation point, so:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0,x=x_s} = \mu\left(\frac{\partial^2\psi}{\partial y^2}\right)_{y=0,x=x_s} = (2A + 2B(x - x_s) + 6Cy)_{y=0,x=x_s} = 2A = 0,$$

and this leaves:

$$\psi(x,y) = B(x - x_s)y^2 + Cy^3.$$

In the vicinity of the separation point, this stream function $\psi(x,y)$ must satisfy two additional conditions. The first comes from the limiting form of (9.1) as $y \to 0$ (see Example 9.1), which for the present coordinate system and stream function is:

$$(\partial p/\partial x)_{y=0} = \mu(\partial^2 u/\partial y^2)_{y=0} = \mu(\partial^3\psi/\partial y^3)_{y=0},$$

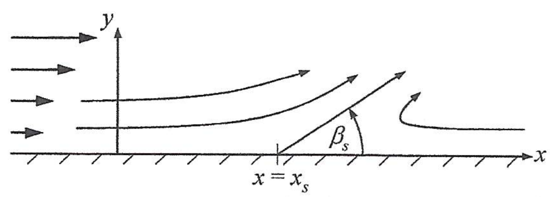


FIGURE 10.17 Streamline pattern near the separation point ($x = x_s$, $y = 0$) on a flat surface.

and this implies $C = (1/6\mu)(\partial p/\partial x)$. The second condition is that the zero-streamline must leave the surface at an angle β_s with respect to the downstream direction. The zero-streamline is given by $\psi(x,y) = 0$, which implies:

$$0 = B(x - x_s)y^2 + \frac{1}{6\mu}\left(\frac{\partial p}{\partial x}\right)y^3, \quad \text{or} \quad -\frac{1}{6\mu}\left(\frac{\partial p}{\partial x}\right)y = B(x - x_s), \quad \text{or} \quad -\frac{1}{6\mu}\left(\frac{\partial p}{\partial x}\right)\frac{dy}{dx} = B.$$

So, with $dy/dx = \tan\beta_s$, the final form for the stream function expansion is:

$$\psi(x,y) = \frac{y^2}{6\mu}\left(\frac{\partial p}{\partial x}\right)(y - (x - x_s)\tan\beta_s).$$

Thus for boundary layer separation from a flat surface, the angle of the separating streamline may be independent of the local pressure gradient. And, when the flow is in the positive x-direction upstream of the separation point (i.e. $\psi > 0$ for $y > 0$), this stream function only makes sense when $\partial p/\partial x$ is locally positive, an adverse pressure gradient.

Separation complex process that is usually 3D and depends Re unsteady.

Bluff body: high p fore body and low p or constant after body called base pressure in separation region at higher Re and size wake depends laminar or turbulent flow.

External flow geometry: bluff, slender, sharp edge vs. smooth surface, form/pressure vs friction C_D.

Internal flow geometry: converget/diverget, bends, and other minor losses.

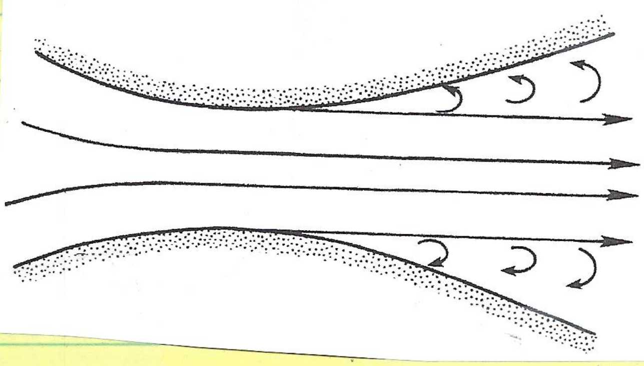


FIGURE 10.16 Separation of flow in a divergent channel. Here, an adverse pressure gradient has lead to boundary-layer separation just downstream of the narrowest part of the channel. Such separated flows are unstable and are exceedingly likely to be unsteady, even if all the boundary conditions are time independent.

Slender body: LE and TE separations

Low Re even bluff body wake can be steady, eg, circular cylinder, $4 < Re < 40$ steady vortex in wake. At higher Re BL on cylinder and smooth surface separation f(Re) eg drag crisis $S = 82^\circ$ vs 125°

S laminar less sensitive Re

turbulent BL more resistant separation due fuller profile higher momentum flow near wall

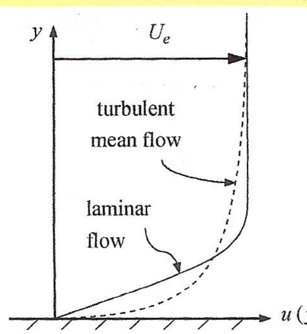


FIGURE 10.15 Nominal comparison of laminar and turbulent-mean-flow stream-wise velocity profiles for boundary layers with nominally equal displacement thickness. Here the primary differences are the presence of higher speed fluid closer to the surface and greater surface shear stress in the turbulent boundary layer.