P4.7 Consider a sphere of radius R immersed in a uniform stream U_o, as shown in Fig. P4.7. According to the theory of Chap. 8, the fluid velocity along streamline AB is given by

$$\mathbf{V} = u\mathbf{i} = U_o\left(1+\frac{R^3}{x^3}\right)\mathbf{i}$$

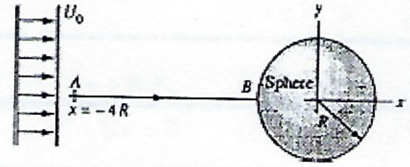

Fig. P4.7

Find (a) the position of maximum fluid acceleration along AB and (b) the time required for a fluid particle to travel from A to B. Note that x is negative along line AB.

(a) $\frac{DV}{Dt} = \frac{\partial V}{\partial t} + V\cdot\nabla V$ $\quad V = u\hat{i} + v\hat{j} + w\hat{k}$

$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$

$a_x = u\frac{\partial u}{\partial x}$

$= U_o(1+R^3/x^3)(-3U_oR^3/x^4)$

$= -3U_o^2R^3(x^{-4}+R^3x^{-7})$ $\quad x \le -R$

max @ $\frac{da_x}{dx} = 0 = -3U_o^2R^3\underbrace{(-4x^{-5}-7R^3x^{-8})}_{=0} \Rightarrow -x = (7R^3/4)^{1/3}$

$x = -1.205R$

(b) $u = \frac{dx}{dt} = U_o(1+R^3/x^3)$

or $\int_{-4R}^{-R} \frac{dx}{1+\frac{R^3}{x^3}} = \int_0^t U_o\,dt$

$U_o t = \left[x - \frac{R}{6}\ln\frac{(x+R)^2}{x^2-Rx+R^2} - \frac{R}{\sqrt{3}}\tan^{-1}\left(\frac{2x-R}{R\sqrt{3}}\right)\right]_{-4R}^{-R} = \infty$

∴ potential flow theory
take ∞ time to reach
stagnation point $u(-R)=0$!