P4.83 The flow pattern in bearing lubrication can be illustrated by Fig. P4.83, where a viscous oil (ρ, μ) is forced into the gap $h(x)$ between a fixed slipper block and a wall moving at velocity U. If the gap is thin, $h \ll L$, it can be shown that the pressure and velocity distributions are of the form $p = p(x)$, $u = u(y)$, $v = w = 0$. Neglecting gravity, reduce the Navier-Stokes equations (4.38) to a single differential equation for $u(y)$. What are the proper boundary conditions? Integrate and show that

$$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - yh) + U\left(1 - \frac{y}{h}\right)$$

where $h = h(x)$ may be an arbitrary slowly varying gap width. (For further information on lubrication theory, see Ref. 16.)

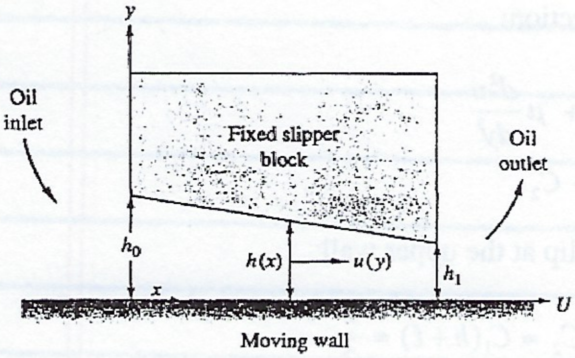

Fig. P4.83

Continuity : $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ $\quad v = w = 0$

$0 = 0 \quad u \neq f(x) \quad u = u(y)$

X - momentum

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$$

$$0 = -\frac{dp}{dx} + \mu\frac{\partial^2 u}{\partial y^2}$$

$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}$ $\qquad$ BC: $u(0) = U$, $u(h) = 0$

$du/dy = \frac{1}{\mu}\frac{dp}{dx}y + c_1$

$u(y) = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + c_1 y + c_2$

$u(0) = U = c_2$

$u(h) = 0 = \frac{1}{2\mu}\frac{dp}{dx}h^2 + c_1 h + U$

$c_1 = -\frac{1}{2\mu}\frac{dp}{dx}h - U/h$

$u(y) = \frac{1}{2\mu}\frac{dp}{dx}y^2 - \left[\frac{1}{2\mu}\frac{dp}{dx}h + U/h\right]y + U$

$= \frac{1}{2\mu}\frac{dp}{dx}(y^2 - yh) + U(1 - y/h)$