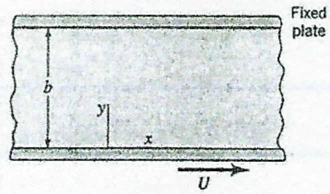
6.96

6.96 The viscous, incompressible flow between the parallel plates shown in Fig. P6.96 is caused by both the motion of the bottom plate and a pressure gradient, $\partial p/\partial x$. As noted in Section 6.9.2, an important dimensionless parameter for this type of problem is $P = -(b^2/2\,\mu U)\,(\partial p/\partial x)$ where μ is the fluid viscosity. Make a plot of the dimensionless velocity distribution (similar to that shown in Fig. 6.32b) for $P = 3$. For this case where does the maximum velocity occur?



■ FIGURE P6.96

$u_x + v_y + w_z = 0$ $\quad v = 0 \quad w = 0 \Rightarrow u_x = 0$

ie 1D fully developed flow $\quad$ ie $u = u(y)$

$$\rho(u_t + u u_x + v u_y + w u_z) = -p_x + \mu(u_{xx} + u_{yy} + u_{zz})$$

$$u_{yy} = p_x/\mu$$

$$u_y = \frac{p_x}{\mu} y + c_1$$

$$u = \frac{p_x}{2\mu} y^2 + c_1 y + c_2$$

$u(0) = U \quad u(b) = 0$

$$u(0) = c_2 = U$$

$$u(b) = \frac{p_x}{2\mu}(b^2) + c_1 b + U = 0$$

$$c_1 = -\frac{p_x}{2\mu} b - \frac{U}{b}$$

$$u(y) = \frac{p_x}{2\mu} y^2 - \left[\frac{p_x}{2\mu} b + \frac{U}{b}\right] y + U$$

$$= \frac{1}{2\mu}(p_x)\left[y^2 - by\right] + U(1 - y/b)$$

make dimensionless: $u/U \quad y/b \quad P = \frac{-b^2}{2\mu U} p_x$

$$u/U = \frac{1}{2\mu U}(p_x)\, b^2 \left[\left(\frac{y}{b}\right)^2 - \frac{y}{b}\right] - \frac{y}{b} + 1$$

$$u/U = -P\,(y/b)(y/b - 1) - y/b + 1$$

max velocity:

$$\frac{d\,u/U}{dy} = 0 = -P\left[\frac{2y}{b^2} - \frac{1}{b}\right] - \frac{1}{b} = 0$$

for $P = 3$: $y/b = 1/3$

u/U	y/b
1	0
1.17	0.1
1.28	0.2
1.33	0.3
1.32	0.4
1.25	0.5
1.12	0.6
0.93	0.7
0.68	0.8
0.37	0.9
0	1

Calculated from Eq. (2) with P =3.