4.75 Given the following steady *axisymmetric* stream function:

$\frac{\partial}{\partial\theta}=0,\ v_\theta=0$

$$\psi = \frac{B}{2}\left(r^2 - \frac{r^4}{2R^2}\right), \quad \text{where } B \text{ and } R \text{ are constants}$$

valid in the region $0 \le r \le R$ and $0 \le z \le L$. (*a*) What are the dimensions of the constant B?

(*b*) Show whether this flow possesses a velocity potential and, if so, find it. (*c*) What might this flow represent? [HINT: Examine the axial velocity v_z.]

(a) $d\psi = dQ \Rightarrow \psi$ same dimensions $Q = \frac{m^3}{s}\ \frac{L^3}{T}$

$\therefore\ B = \frac{L}{T}$

(b) $\underline{V} = \nabla\phi$ exist or not

$v_r = -\frac{1}{r}\frac{\partial\psi}{\partial z} = 0 \qquad v_z = \frac{1}{r}\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{B}{2}\left(2r - \frac{4r^3}{2R^2}\right)$

$\psi \neq f(z)$

$= B\left(1 - \frac{r^2}{R^2}\right) = \frac{B}{R^2}(R^2 - r^2)$

$\nabla \times \underline{V} = \omega_r \hat{e}_r + \omega_\theta \hat{e}_\theta + \omega_z \hat{e}_z$

$\omega_r = \frac{1}{r} v_{z\theta} - v_{\theta z} \qquad \omega_\theta = v_{rz} - v_{zr} \qquad \omega_z = \frac{1}{r}\left[(r v_\theta)_r - v_{r\theta}\right]$

$= 0 \qquad\qquad = -v_{zr} \qquad\qquad = 0$

$= -\left[-\frac{2Br}{R^2}\right]$

$\neq 0$ rotational ϕ does not exist!

(c) $v_z = \frac{B}{R^2}(R^2 - r^2)$

c.f. Poiseuille pipe flow $v_z = \frac{1}{4\mu}\left(-\frac{dp}{dz}\right)(R^2 - r^2)$

$B = \frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right) \qquad \frac{m^2}{\frac{Ns}{m^2}} \times \frac{N}{m^3} = \frac{m}{s}$

$\mu = \frac{\tau}{\frac{\partial u}{\partial y}} = \frac{N/m^2}{\frac{m}{ms}} = \frac{Ns}{m^2} \qquad \frac{dp}{dz} = \frac{N/m^2}{m} = \frac{N}{m^3}$