5.80 A prototype ship is 35 m long and designed to cruise at 11 m/s (about 21 kn). Its drag is to be simulated by a 1-m-long model pulled in a tow tank. For Froude scaling find (a) the tow speed, (b) the ratio of prototype to model drag, and (c) the ratio of prototype to model power.

$$Fr_m = \frac{V_m}{\sqrt{gL_m}} = Fr_p = \frac{V_p}{\sqrt{gL_p}}$$

$$\alpha = \frac{L_m}{L_p} = \frac{1}{35}$$

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} = \sqrt{\alpha} \Rightarrow V_m = V_p\sqrt{\alpha} = 1.86\ \frac{m}{s}$$

$$\frac{D_m}{\frac{1}{2}\rho V_m^2 A_m} = \frac{D_p}{\frac{1}{2}\rho V_p^2 A_p} \qquad A = \text{surface area} \propto L^2$$

$$\frac{D_m}{D_p} = \frac{V_m^2}{V_p^2}\frac{A_m}{A_p} = \alpha\frac{L_m^2}{L_p^2} = \alpha^3 = \frac{1}{35^3} = \frac{1}{42{,}900}$$

$$\frac{P_m}{P_p} = \frac{D_m}{D_p} \times \frac{V_m}{V_p} = \alpha^3\sqrt{\alpha} = \alpha^{3.5} = \frac{1}{254{,}000}$$

Solution: Given $\alpha = 1/35$, then Froude scaling determines everything:

$$Fr_m = \frac{V_m^2}{gL_m} = \frac{V_p^2}{gL_p} = Fr_p, \text{ or } \frac{V_m}{V_p} = \left(\frac{L_m}{L_p}\right)^{1/2} = \sqrt{\alpha}$$

$$V_{tow} = V_m = V_p\sqrt{\alpha} = 11/\sqrt{(35)} \approx 1.86 \text{ m/s}$$

$$F_m/F_p = (V_m/V_p)^2(L_m/L_p)^2 = (\sqrt{\alpha})^2(\alpha)^2 = \alpha^3 = (1/35)^3 = \frac{1}{42900} \quad \textit{Ans.}$$

$$P_m/P_p = (F_m/F_p)(V_m/V_p) = \alpha^3(\sqrt{\alpha}) = \alpha^{3.5} = 1/35^{3.5} = \frac{1}{254000}$$

Model Ship Testing:

$C_T(Re, Fr) = C_W(Fr) + C_V(Re)$ Froude Scaling

$$C_{W_p} = C_{W_m} = C_{T_m} - C_{V_m}(1+k)$$

$$C_{T_p} = C_{W_p} + C_{V_p}(1+k)$$

C_{T_m} measured in towing tank & $C_V(Re)$ based on model-ship correlation line, ie, data for flat plate skin friction for $Re_m \le Re \le Re_p$. k = form factor, which accounts for 3D effects on C_V and usually assumed independent Fr and Re. Note C = drag coefficient $= \frac{F}{\frac{1}{2}\rho V^2 S}$

Example: $L_p = 100$ m, $V_p = 10$ m/s, $S_p = 300\ m^2$
$\alpha = 1/25$, $F_{T_m}(V_m) = 60$ N

Find V_m using Fr Scaling:

$$Fr_m = \frac{V_m}{\sqrt{gL_m}} = Fr_p = \frac{V_p}{\sqrt{gL_p}} = \frac{10}{\underbrace{\sqrt{9.81 \times 100}}_{31.32}} = .32$$

$$V_m = V_p\sqrt{\frac{L_m}{L_p}} = V_p\sqrt{\alpha}$$

$$= 10\sqrt{1/25} = 2\ m/s$$

$$Re_m = \frac{U_m L_m}{\nu} = \frac{2 \times 100/25}{10^{-6}} = 8 \times 10^6$$

$\nu = 10^{-6}\ m^2/s$

$\rho = 1000\ kg/m^3$

$$Re_p = \frac{U_p L_p}{\nu} = \frac{10 \times 100}{10^{-6}} = 10^9$$

Use ITTC model-ship correlation or flat plate friction lines, as per text

$C_{V_m} = C_V(Re_m) = .003$ take $k = 0$

$C_{V_p} = C_V(Re_p) = .0015$

$$F_{V_m} = \tfrac{1}{2} \rho U_m^2 S_m C_{V_m} \qquad S_m/S_p = \alpha^2$$

$$= \tfrac{1}{2} \times 1000 \times 2^2 \times (300/25^2) \times .003$$

$$= 2.88\ N$$

$$F_{W_m} = F_{T_m} - F_{V_m} = 60 - 2.88 = 57.12\ N$$

$$F_{W_p} = \tfrac{1}{2} \rho U_p^2 S_p \times \frac{F_{W_m}}{\tfrac{1}{2} \rho U_m^2 S_m} = \alpha^{-3} F_{W_m}$$

$$= 25^3 F_{W_m}$$

$$= 8.92 \times 10^5\ N$$

$$F_{V_p} = \tfrac{1}{2} \rho U_p^2 S_p C_{V_p} = \tfrac{1}{2} \times 1000 \times 10^2 \times 300 \times .0015$$

$$= .225 \times 10^5\ N$$

$$F_{T_p} = 9.14 \times 10^5\ N$$

2.5% over estimate!

If neglect $C_V(Re)$: $F_{T_p} = \tfrac{1}{2} \rho U_p^2 S_p \times \dfrac{F_{T_M}}{\tfrac{1}{2} \rho U_m^2 S_m} = \alpha^{-3} F_{T_M} = 9.37 \times 10^5$

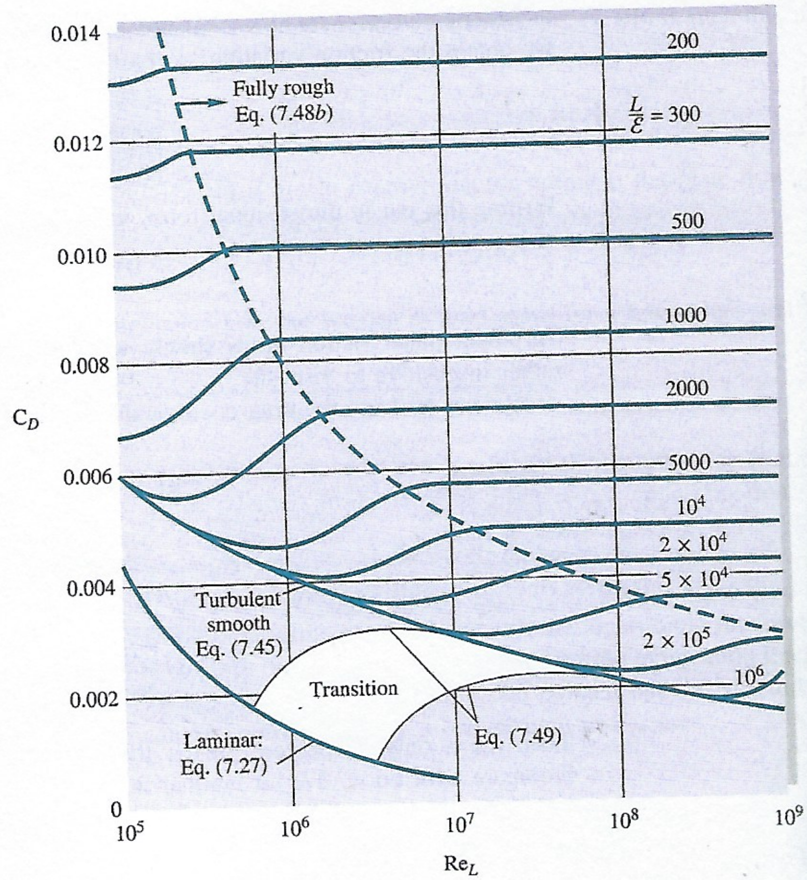

Fig. 7.6 Drag coefficient of laminar and turbulent boundary layers on smooth and rough flat plates. This chart is the flat-plate analog of the Moody diagram of Fig. 6.13.

Equation (7.48*b*) is plotted to the right of the dashed line in Fig. 7.6. The shows the behavior of the drag coefficient in the transition region $5 \times 10^5 <$ 8×10^7, where the laminar drag at the leading edge is an appreciable fra total drag. Schlichting [1] suggests the following curve fits for these tra curves, depending on the Reynolds number Re_{trans} where transition begins:

$$C_D \approx \begin{cases} \dfrac{0.031}{Re_L^{1/7}} - \dfrac{1440}{Re_L} & Re_{trans} = 5 \times 10^5 \\[2ex] \dfrac{0.031}{Re_L^{1/7}} - \dfrac{8700}{Re_L} & Re_{trans} = 3 \times 10^6 \end{cases}$$

EXAMPLE 7.4

A hydrofoil 1.2 ft long and 6 ft wide is placed in a seawater flow of 40 ft/s, slugs/ft³ and $\nu = 0.000011$ ft²/s. (*a*) Estimate the boundary layer thickness the plate. Estimate the friction drag for (*b*) turbulent smooth-wall flow from