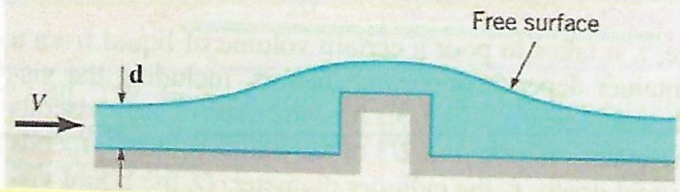
A thin layer of an incompressible fluid flows steadily over a horizontal smooth plate as shown in the Figure. The fluid surface is open to the atmosphere, and an obstruction having a square cross section is placed on the plate as shown. The dimensional variables for this problem are V, g, d, ρ, μ, and surface tension σ. Taking V, d, and ρ as repeating variables, one can find that the dimensionless parameters are Reynolds, Froude, and Weber numbers. (a) Write down the Reynolds and Froude numbers (or find if don't have them memorized) and use Pi theory to find out the equation for Weber number. (b) A model with a length scale of ¼ and a fluid density scale of 2.0 is to be designed to study the mean velocity V at upstream. Find the scale ratios for velocity, viscosity, and surface tension.



$F(\sigma, d, g, V, \rho, \mu) = 0$ $\quad n=6$

$Re = \frac{VL}{\nu}$, $\nu = \mu/\rho$ $\quad m=3$

$Fr = U/\sqrt{gL}$

$\Pi_1 = Re = \rho V d/\mu$ $\quad r = n-m = 3$

$\Pi_2 = Fr = V/\sqrt{gd}$

$\Pi_3 = V^a d^b \rho^c \sigma = M^0 L^0 T^0$

$LT^{-1} \quad L \quad ML^{-3} \quad MT^{-2}$

$a=-2 \quad b=-1 \quad c=-1 \Rightarrow \Pi_3 = \frac{\sigma}{\rho V^2 d}$

or $\Pi_3^{-1} = \rho V^2 d/\sigma = We$

(1) $Fr_m = Fr_p \quad \frac{V_m}{\sqrt{g d_m}} = \frac{V_p}{\sqrt{g d_p}}$ given $\frac{d_m}{d_p} = \frac{1}{4}$

$\therefore \frac{V_m}{V_p} = \sqrt{\frac{d_m}{d_p}} = \frac{1}{2}$

(2) $Re_m = Re_p$ given $\rho_m/\rho_p = 2$

$\Rightarrow \mu_m/\mu_p = 1/4$

$\frac{V_m d_m \rho_m}{\mu_m} = \frac{V_p d_p \rho_p}{\mu_p}$

$\frac{\mu_m}{\mu_p} = \frac{V_m d_m \rho_m}{V_p d_p \rho_p}$

$= \frac{1}{2} \times \frac{1}{4} \times 2$

$= 1/4$

(3) $We_m = We_p \Rightarrow \frac{\sigma_m}{\sigma_p} = \frac{1}{8}$

$\frac{\rho_m V_m^2 d_m}{\sigma_m} = \frac{\rho_p V_p^2 d_p}{\sigma_p}$

$= \frac{\rho_m V_m^2 d_m}{\rho_p V_p^2 d_p}$

$= 2 \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{8}$