Name :___________________ **Quiz: No. 8** **Time: 20 minutes**

Course: ME 5160, Fall 2024

The exam is closed book and closed notes.

The power P generated by a certain windmill design depends upon its diameter D, the air density ρ, the wind velocity V, the rotation rate Ω, and the number of blades n.

(a) Write this relationship in dimensionless form using D, ρ, and V as repeating variables (note that n is already dimensionless and therefore can be used directly as one of the Pi groups).

(b) A model windmill, of diameter 50 cm, develops 2.7 kW at sea level for V = 40 m/s and when rotating at 4800 rev/min. A geometrically and dynamically similar prototype has diameter 5 m and operates in winds of 12 m/s at 2000 m standard altitude. What is the appropriate rotation rate, and the power delivered by the prototype? Use ρ = 1.0067 kg/m^3 at 2000 m altitude and ρ = 1.2255 kg/m^3 at sea level.

		Dimensions	
Quantity	Symbol	$MLT\Theta$	$FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ, τ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω, Ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Thermal expansion coefficient	β	Θ^{-1}	Θ^{-1}

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Solution

KNOWN: dimensional parameters, geometrical and dynamic similarity between model and prototype, $(D, \rho, V, \Omega, P)_m$, $(D, \rho, V)_p$

FIND: Pi groups, $(\Omega, P)_p$

ASSUMPTIONS: the problem is only a function of the above dimensional variables

ANALYSIS:

(a) For the function $P = f(D, \rho, V, \Omega, n)$ the appropriate dimensions are

$\{P\} = \{M\,L^2\,T^{-3}\}$; $\{D\} = \{L\}$; $\{\rho\} = \{M\,L^{-3}\}$; $\{V\} = \{L\,T^{-1}\}$; $\{\Omega\} = \{T^{-1}\}$; $\{n\} = \{1\}$ **(1.5)**

$$n = 6$$
$$j = 3$$

$$k = n - j = 3 \quad \textbf{(1)}$$

Using repeating variables D, ρ, and V find the Pi groups:

$$\Pi_1 = D^a \rho^b V^c P = \{(L)^a (ML^{-3})^b (LT^{-1})^c (ML^2T^{-3})\} = \{M^0 L^0 T^0\} \quad \textbf{(0.5)}$$

$$a = -2;\ \ b = -1;\ \ c = -3$$

$$\Pi_1 = \frac{P}{\rho D^2 V^3} \quad \textbf{(1)}$$

$$\Pi_2 = D^a \rho^b V^c \Omega = \{(L)^a (ML^{-3})^b (LT^{-1})^c (T^{-3})\} = \{M^0 L^0 T^0\} \quad \textbf{(0.5)}$$

$$a = 1;\ \ b = 0;\ \ c = -1$$

$$\Pi_2 = \frac{\Omega D}{V} \quad \textbf{(1)}$$

$$\Pi_3 = D^a \rho^b V^c n = \{(L)^a (ML^{-3})^b (LT^{-1})^c (M^0 L^0 T^0)\} = \{M^0 L^0 T^0\} \quad \textbf{(0.5)}$$

$$a = 0;\ \ b = 0;\ \ c = 0$$

$$\Pi_3 = n \quad \textbf{(1)}$$

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The dimensionless function is

$$\frac{P}{\rho D^2 V^3} = f\left(\frac{\Omega D}{V}, n\right)$$

(b) For the geometric similarity, the number of blades n (Π_3) is the same between model and prototype. For the dynamic similarity, model and prototype have the same advance ratio (Π_2).

$$\left(\frac{\Omega D}{V}\right)_m = \left(\frac{\Omega D}{V}\right)_p \quad \textbf{(1)}$$

$$\frac{(4800\ \text{rev/min})(0.5\ \text{m})}{(40\ \text{m/s})} = \frac{\Omega_p (5\ \text{m})}{(12\ \text{m/s})}$$

$$\Omega_p = 144\ \text{rev/min} \quad \textbf{(0.5)}$$

Model and prototype also have the same non-dimensional power (Π_1).

$$\left(\frac{P}{\rho D^2 V^3}\right)_m = \left(\frac{P}{\rho D^2 V^3}\right)_p \quad \textbf{(1)}$$

$$\frac{(2700\ \text{W})}{(1.2255\ \text{kg/m}^3)(0.5\ \text{m})^2(40\ \text{m/s})^3} = \frac{P_p}{(1.0067\ \text{kg/m}^3)(5\ \text{m})^2(12\ \text{m/s})^3}$$

$$P_p = 5990\ \text{W} \approx 6\ \text{kW} \quad \textbf{(0.5)}$$