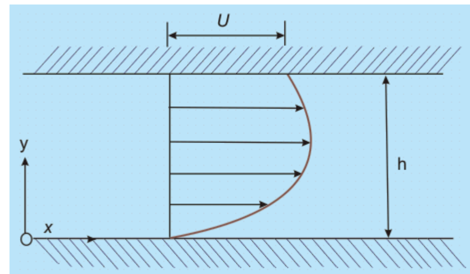
Name :___________________ **Quiz: No. 5** **Time: 15 minutes**

Course: ME 5160, Fall 2024

The exam is closed book and closed notes.

Consider steady, two-dimensional, incompressible viscous flow of a Newtonian fluid between parallel plates a distant h apart, as shown in the Figure below. The upper plate moves at constant velocity U while the lower plate is at rest. The pressure varies linearly in the x direction: $p = p(x) = Cx$. (a) If the plates are very wide and very long and the flow is fully developed, neglect gravity and find the velocity distribution between the plates. (b) Determine the wall shear stress at the top wall.



Incompressible Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Incompressible Navier-Stokes Equations in Cartesian Coordinates:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$\rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$\rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

Name :__________________ **Quiz: No. 5** **Time: 15 minutes**

Course: ME 5160, Fall 2024

Solution:

KNOWN: *p(x)*

FIND: *u(y)*

ASSUMPTIONS: incompressible, steady, Newtonian flow, no gravity effects

ANALYSIS:

The plate is very wide and very long and the flow is fully developed, therefore the flow is essentially axial: $v = w = 0$.

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \textbf{(1)}$$

$$\frac{\partial u}{\partial x} + \underbrace{0}_{v=0} + \underbrace{0}_{w=0} = 0 \qquad \textbf{(1)}$$

$$\Longrightarrow \; u = u(y) \text{ only} \qquad \textbf{(2)}$$

x-momentum:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) \qquad \textbf{(1)}$$

$$\underbrace{0}_{no\ gravity} - \frac{\partial p}{\partial x} + \mu\left(\underbrace{0}_{u=u(y)} + \frac{\partial^2 u}{\partial y^2} + \underbrace{0}_{u=u(y)}\right) = \rho\left(\underbrace{0}_{steady} + \underbrace{0}_{u=u(y)} + \underbrace{0}_{v=0} + \underbrace{0}_{w=0}\right) \qquad \textbf{(1)}$$

$$\frac{\partial p}{\partial x} = C \qquad \textbf{(1)}$$

$$\Longrightarrow \mu\frac{d^2 u}{dy^2} = C$$

Integrate twice:

$$u = \frac{1}{\mu} C \frac{y^2}{2} + C_1 y + C_2 \qquad \textbf{(1)}$$

Name :________________ **Quiz: No. 5** **Time: 15 minutes**

Course: ME 5160, Fall 2024

Boundary conditions:

$$\text{at } y = 0\text{: } u(y) = 0 \quad \rightarrow \quad C_2 = 0 \qquad \textbf{(0.5)}$$

$$\text{at } y = h\text{: } u(y) = U \quad \rightarrow \quad C_1 = \frac{U}{h} - \frac{Ch}{2\mu} \qquad \textbf{(0.5)}$$

Replace and find:

$$u = \frac{1}{\mu} C \frac{y^2}{2} + \left(\frac{U}{h} - \frac{Ch}{2\mu}\right) y \qquad \textbf{(0.5)}$$

Shear stress:

$$\tau = \mu \frac{\partial u}{\partial y}\bigg|_{y=h} = Ch + \mu \left(\frac{U}{h} - \frac{Ch}{2\mu}\right) \qquad \textbf{(0.5)}$$