Name :__________________ **Quiz: No. 14** **Time: 15 minutes**

Student ID# : _____________ **Course: ME 5160, Fall 2023**

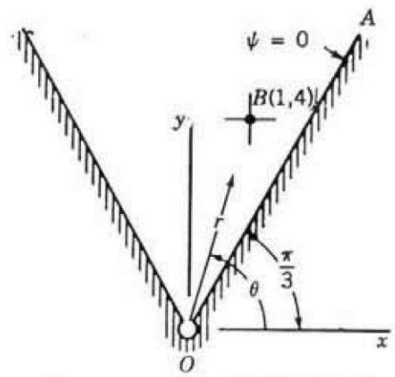
The exam is closed book and closed notes.

An ideal fluid flows between the inclined walls of a two-dimensional channel into a line sink located at the origin, as shown in the Figure below. The velocity potential for this flow field is:

$$\phi = m \ln r$$

where m is a constant. (a) Find the expressions for radial (v_r) and tangential (v_θ) velocity components. (b) Determine the corresponding stream function. Note that the value of the stream function along the wall *OA* (i.e. $\theta = \frac{\pi}{3}$) is zero. (c) Find m if the value of stream function at point *B* located at *x* = 1, *y* = 4 is $\psi_B = -0.71$.

Equations: $v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$; $v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$; $r = \sqrt{x^2 + y^2}$; $\theta = \tan^{-1}\left(\frac{y}{x}\right)$



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Solution:

KNOWN: ϕ, ψ_B

FIND: v_r, v_θ, ψ, m

ASSUMPTIONS: Irrotational flow

ANALYSIS:

(a)

$$v_r = \frac{\partial \phi}{\partial r} = \frac{m}{r} \quad \textbf{(1)}$$

$$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = 0 \quad \textbf{(0.5)}$$

(b)

$$v_r = \frac{m}{r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad \textbf{(1)}$$

$$\frac{\partial \psi}{\partial \theta} = m \quad \textbf{(0.5)}$$

Integrate with respect to θ:

$$\psi = m\theta + f_1(r) \quad \textbf{(1)}$$

On the other hand:

$$v_\theta = 0 = -\frac{\partial \psi}{\partial r} \quad \textbf{(1)}$$

Therefore ψ is not a function of r, so $f_1(r)$ is a constant and the equation for ψ becomes:

$$\psi = m\theta + C_1 \quad \textbf{(1)}$$

Also, $\psi = 0$ for $\theta = \frac{\pi}{3}$:

$$0 = m\frac{\pi}{3} + C_1 \quad \textbf{(1)}$$

$$C_1 = -m\frac{\pi}{3}$$

$$\psi = m\theta - m\frac{\pi}{3} = m\left(\theta - \frac{\pi}{3}\right) \quad \textbf{(0.5)}$$

(c)

At point B:

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 4^2} = 4.12$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(4) = 1.33 \quad \textbf{(1)}$$

The value of stream function at point B:

$$\psi_B = -0.71 = m\left(1.33 - \frac{\pi}{3}\right) \quad \textbf{(1)}$$

Therefore:

$$m \approx -2.51 \quad \textbf{(0.5)}$$